# Market Access, Investment, and Heterogeneous Firms
General Appendix

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## 1 General Framework

The main manuscript presents an equilibrium such that the qualitative selection into foreign investment depends on the relative size of trade costs. The assumptions used were meant to facilitate a model comparable with the new theory on trade and firm heterogeneity. In this appendix, I generalize the domestic and foreign acquisition incentives for arbitrary demand and cost functions, the latter derived from primitive assumptions over production.

To begin, I will derive a number of elasticities related to costs that will be used in the main proofs. All important elasticities will be "boxed". Then, I will set-up a general production framework to derive the effects of productivity on production and demand in each market. Finally, I will derive the optimal level of capital as a function of productivity, discussing how the marginal incentives for domestic and foreign capital change with productivity, and compare to one another. The key results from this appendix will be:

- If there is no foreign production, and assuming variable costs derived from a constant elasticity of substitution production function (over capital and variable factors) the incentives to invest in domestic capital will be increasing in productivity if:

$$\epsilon_{MR_h,q_h} \quad > \quad -\frac{1}{1-\rho}$$

  where $\epsilon_{MR_h,q_h}$ is the elasticity of the marginal revenue curve at home with respect to quantity, and $\frac{1}{1-\rho}$ is the elasticity of substitution between capital and labor in the CES production function.

- If there is foreign production and trade costs are zero, domestic and foreign capital investment are maximized at the same level of productivity.

- If there is foreign production and trade costs are positive, when domestic capital investment is maximized as a function of productivity, foreign capital investment is still increasing in productivity.

Hence, as in the manuscript, domestic incentives for investment are maximized in a middle range of productivity if demand parameters are such that $\epsilon_{MR_h,q_h}$ can be sufficiently low and $\epsilon_{MR_h,q_h}$ is falling in output. Further, foreign investment incentives are skewed to the right of domestic incentives when trade costs are positive, and maximized at the same level of productivity when trade costs are zero.

### 1.1 Production and Costs

Costs in the model are dictated by a production function holding capital factors fixed. Within a general framework including both capital and labor, suppose that production from a given location, $X_j$, is governed by:

$$F(K_j, L_j) = X_j$$

where $L_j$ is effective labor hired by the firm at location $j$ and $K_j$ is capital held by the firm at that location. We assume that the production function has the following properties:

$$\frac{\partial F(K_j, L_j)}{\partial K_j} > 0, \frac{\partial^2 F(K_j, L_j)}{\partial K_j^2} < 0$$
$$\frac{\partial F(K_j, L_j)}{\partial L_j} > 0, \frac{\partial^2 F(K_j, L_j)}{\partial L_j^2} < 0$$
$$\frac{\partial^2 F(K_j, L_j)}{\partial K_j L_j} > 0$$

In the manuscript, firms differ in their variable factor productivity, $\alpha$, and variable costs are simply labor costs (at a numeraire wage). Given this setup, the production function at location $j$ can be augmented to implicitly define the variable cost function in terms of capital, productivity, and output:

$$F(K_j, \alpha \cdot C(X_j, \alpha, K_j)) = X_j \tag{1}$$

Using this implicit definition of costs, we can derive a number of partial elasticities of costs and marginal costs within respect to other variables that will be important for the general relationship between investment incentives and productivity. To begin, note that marginal production costs can be derived as:

$$\frac{\partial C\left(X_j,\alpha,K_j\right)}{\partial X_j}=\frac{1}{\alpha\frac{\partial F\left(K_j,\alpha\cdot C\left(X_j,\alpha,K_j\right)\right)}{\partial L}} \tag{2}$$

By taking logs and fully differentiating, the partial elasticity of marginal costs with respect to productivity can be derived as follows:

$$\frac{\partial\frac{\partial C\left(X_j,\alpha,K_j\right)}{\partial X_j}}{\frac{\partial C\left(X_j,\alpha,K_j\right)}{\partial X_j}}=-\frac{\partial\alpha}{\alpha}-\frac{1}{\frac{\partial F\left(K_j,\alpha\cdot C\left(X_j,\alpha,K_j\right)\right)}{\partial L}}\frac{\partial\frac{\partial F\left(K_j,\alpha\cdot C\left(X_j,\alpha,K_j\right)\right)}{\partial L}}{\partial L}\left(C\left(X_j,\alpha,K_j\right)+\alpha\cdot\frac{\partial C\left(X_j,\alpha,K_j\right)}{\partial\alpha}\right)$$

Note that $\left(C\left(X_j,\alpha,K_j\right)+\alpha\cdot\frac{\partial C\left(X_j,\alpha,K_j\right)}{\partial\alpha}\right)=0$ since differentiating (1) with respect to $\alpha$ yields:

$$\frac{\partial F\left(K_j,\alpha\cdot C\left(X_j,\alpha,K_j\right)\right)}{\partial L}\left(C\left(X_j,\alpha,K_j\right)+\alpha\cdot\frac{\partial C\left(X_j,\alpha,K_j\right)}{\partial\alpha}\right)=0$$

Hence, the elasticity of marginal costs with respect to $\alpha$ can be written as:

$$\boxed{\epsilon_{MC,\alpha}\equiv\frac{\partial\frac{\partial C\left(X_j,\alpha,K_j\right)}{\partial X_j}}{\partial\alpha}\frac{\alpha}{\frac{\partial C\left(X_j,\alpha,K_j\right)}{\partial X_j}}=-1} \tag{3}$$

Next, consider the partial elasticity of marginal costs with respect to output. To derive this, first differentiate (2) with respect to output holding productivity and capital fixed:

$$\frac{\frac{\partial\frac{\partial C\left(X_j,\alpha,K_j\right)}{\partial X_j}}{\partial X_j}}{\frac{\partial C\left(X_j,\alpha,K_j\right)}{\partial X_j}}=-\frac{1}{\frac{\partial F\left(K_j,\alpha\cdot C\left(X_j,\alpha,K_j\right)\right)}{\partial L}}\frac{\partial\frac{\partial F\left(K_j,\alpha\cdot C\left(X_j,\alpha,K_j\right)\right)}{\partial L}}{\partial L}\left(\alpha\cdot\frac{\partial C\left(X_j,\alpha,K_j\right)}{\partial X_j}\right)$$

Substituting $\frac{\partial C\left(X_j,\alpha,K_j\right)}{\partial X_j}$, we get:

$$\frac{\frac{\partial\frac{\partial C\left(X_j,\alpha,K_j\right)}{\partial X_j}}{\partial X_j}}{\frac{\partial C\left(X_j,\alpha,K_j\right)}{\partial X_j}}=-\frac{1}{\frac{\partial F\left(K_j,\alpha\cdot C\left(X_j,\alpha,K_j\right)\right)}{\partial L}}\frac{\partial\frac{\partial F\left(K_j,\alpha\cdot C\left(X_j,\alpha,K_j\right)\right)}{\partial L}}{\partial L}\frac{1}{\frac{\partial F\left(K_j,\alpha\cdot C\left(X_j,\alpha,K_j\right)\right)}{\partial L}}$$

Multiplying both sides by output, and multipying $\frac{L}{L}$ on the RHS, and arranging in terms of elasticities, we get:

$$\frac{\frac{\partial\frac{\partial C\left(X_j,\alpha,K_j\right)}{\partial X_j}}{\partial X_j}X_j}{\frac{\partial C\left(X_j,\alpha,K_j\right)}{\partial X_j}}=-\frac{F}{\frac{\partial F\left(K_j,\alpha\cdot C\left(X_j,\alpha,K_j\right)\right)}{\partial L}L}\frac{\partial\frac{\partial F\left(K_j,\alpha\cdot C\left(X_j,\alpha,K_j\right)\right)}{\partial L}}{\partial L}\frac{L}{\frac{\partial F\left(K_j,\alpha\cdot C\left(X_j,\alpha,K_j\right)\right)}{\partial L}}$$

Hence, the elasticity of marginal costs with respect to output is:

$$\boxed{\epsilon_{MC,X} = -\frac{\epsilon_{FL,L}}{\epsilon_{F,L}} > 0}$$

As in the manuscript, marginal costs are increasing in productivity.

Next, we evaluate the marginal effect of capital through the cost function at location $j$. Differentiating (1) with respect to $K_j$, the marginal effect of capital on costs, $\frac{\partial C(X_j,\alpha,K_j)}{\partial K_j}$, is defined via:

$$-\frac{\partial C\left(X_j,\alpha,K_j\right)}{\partial K_j} = \frac{\frac{\partial F(K_j,\alpha\cdot C(X_j,\alpha,K_j))}{\partial K}}{\alpha\frac{\partial F(K_j,\alpha\cdot C(X_j,\alpha,K_j))}{\partial L}} = \frac{\partial F\left(K_j,\alpha\cdot C\left(X_j,\alpha,K_j\right)\right)}{\partial K}\frac{\partial C\left(X_j,\alpha,K_j\right)}{\partial X_j} > 0 \tag{4}$$

Taking logs, and differentiating with respect to $X$, we get:

$$\frac{1}{\frac{\partial C(X_j,\alpha,K_j)}{\partial K_j}}\frac{\partial\frac{\partial C(X_j,\alpha,K_j)}{\partial K_j}}{\partial X_j} = \frac{1}{\frac{\partial F(K_j,\alpha\cdot C(X_j,\alpha,K_j))}{\partial K}}\frac{\partial\frac{\partial F(K_j,\alpha\cdot C(X_j,\alpha,K_j))}{\partial K}}{\partial L}\alpha\frac{C\left(X_j,\alpha,K_j\right)}{\partial X_j}$$
$$+\frac{1}{\frac{\partial C(X_j,\alpha,K_j)}{\partial X_j}}\frac{\partial\frac{\partial C(X_j,\alpha,K_j)}{\partial X_j}}{\partial X_j}$$

Substituting $\alpha\frac{\partial C(X_j,\alpha,K_j)}{\partial X_j} = \frac{1}{\frac{\partial F\left(K_j,\alpha\cdot C\left(X_j,\alpha,K_j\right)\right)}{\partial L}}$ from the definition of marginal costs, we get:

$$\frac{1}{\frac{\partial C(X_j,\alpha,K_j)}{\partial K_j}}\frac{\partial\frac{\partial C(X_j,\alpha,K_j)}{\partial K_j}}{\partial X_j} = \frac{1}{\frac{\partial F(K_j,\alpha\cdot C(X_j,\alpha,K_j))}{\partial K}}\frac{\partial\frac{\partial F(K_j,\alpha\cdot C(X_j,\alpha,K_j))}{\partial K}}{\partial L}\frac{1}{\frac{\partial F(K_j,\alpha\cdot C(X_j,\alpha,K_j))}{\partial L}}$$
$$+\frac{1}{\frac{\partial C(X_j,\alpha,K_j)}{\partial X_j}}\frac{\partial\frac{\partial C(X_j,\alpha,K_j)}{\partial X_j}}{\partial X_j}$$

Multiplying both sides by output, and multiplying the RHS by $\frac{L}{L}$, we get:

$$\frac{X_j}{\frac{\partial C(X_j,\alpha,K_j)}{\partial K_j}}\frac{\partial\frac{\partial C(X_j,\alpha,K_j)}{\partial K_j}}{\partial X_j} = \frac{L}{\frac{\partial F(K_j,\alpha\cdot C(X_j,\alpha,K_j))}{\partial K}}\frac{\partial\frac{\partial F(K_j,\alpha\cdot C(X_j,\alpha,K_j))}{\partial K}}{\partial L}\frac{1}{\frac{L}{F(K_j,\alpha\cdot C(X_j,\alpha,K_j))}\frac{\partial F(K_j,\alpha\cdot C(X_j,\alpha,K_j))}{\partial L}}$$
$$+\frac{X_j}{\frac{\partial C(X_j,\alpha,K_j)}{\partial X_j}}\frac{\partial\frac{\partial C(X_j,\alpha,K_j)}{\partial X_j}}{\partial X_j}$$

Defining in terms of elasticities, and using the result in (4), we get:

$$\boxed{\epsilon_{MK,X} = \frac{\epsilon_{FK,L}}{\epsilon_{F,L}} - \frac{\epsilon_{FL,L}}{\epsilon_{F,L}} > 0} \tag{5}$$

Next, taking logs of (4), and differentiating with respect to $\alpha$, we get:

$$\begin{aligned}
\frac{1}{\frac{\partial C(X_j,\alpha,K_j)}{\partial K_j}} \frac{\partial \frac{\partial C(X_j,\alpha,K_j)}{\partial K_j}}{\partial \alpha} &= \frac{1}{\frac{\partial F(K_j,\alpha \cdot C(X_j,\alpha,K_j))}{\partial K}} \frac{\partial \frac{\partial F(K_j,\alpha \cdot C(X_j,\alpha,K_j))}{\partial K}}{\partial L} \left( C\left(X_j,\alpha,K_j\right) + \alpha \frac{C\left(X_j,\alpha,K_j\right)}{\partial X_j} \right) \\
&+ \frac{1}{\frac{\partial C(X_j,\alpha,K_j)}{\partial X_j}} \frac{\partial \frac{\partial C(X_j,\alpha,K_j)}{\partial X_j}}{\partial \alpha}
\end{aligned}$$

Since $\left( C\left(X_j,\alpha,K_j\right) + \alpha \frac{C(X_j,\alpha,K_j)}{\partial X_j} \right) = 0$ from above, we have:

$$\boxed{\epsilon_{MK,\alpha} = -1} \tag{6}$$

Finally, we derive $\epsilon_{MK,K} = \frac{K_j}{\frac{\partial C\left(X_j,\alpha,K_j\right)}{\partial K_j}} \frac{\partial \frac{\partial C\left(X_j,\alpha,K_j\right)}{\partial K_j}}{\partial K_j}$, which is the elasticity of the marginal effect of capital on costs with respect to capital. Taking logs and differentiating (4) with respect to $K_j$, we get:

$$\begin{aligned}
\frac{K_j}{\frac{\partial C(X_j,\alpha,K_j)}{\partial K_j}} \frac{\partial \frac{\partial C(X_j,\alpha,K_j)}{\partial K_j}}{\partial K_j} &= \frac{K_j}{\frac{\partial F(K_j,\alpha \cdot C(X_j,\alpha,K_j))}{\partial K_j}} \frac{\partial \frac{\partial F(K_j,\alpha \cdot C(X_j,\alpha,K_j))}{\partial K}}{\partial K_j} + \frac{K_j}{\frac{\partial C(X_j,\alpha,K_j)}{\partial X_j}} \frac{\partial \frac{\partial C(X_j,\alpha,K_j)}{\partial X_j}}{\partial K_j} \\
\epsilon_{MK,K} &= \epsilon_{FK,K} + \epsilon_{MC,K}
\end{aligned}$$

To solve for $\epsilon_{MC,K}$, take logs and differentiate (2) with respect to $K$:

$$\begin{aligned}
\frac{1}{\frac{\partial C(X_j,\alpha,K_j)}{\partial X_j}} \frac{\partial \frac{\partial C(X_j,\alpha,K_j)}{\partial X_j}}{\partial K_j} &= -\frac{1}{\frac{\partial F(K_j,\alpha \cdot C(X_j,\alpha,K_j))}{\partial L}} \frac{\partial \frac{\partial F(K_j,\alpha \cdot C(X_j,\alpha,K_j))}{\partial L}}{\partial K_j} \\
&- \frac{1}{\frac{\partial F(K_j,\alpha \cdot C(X_j,\alpha,K_j))}{\partial L}} \frac{\partial \frac{\partial F(K_j,\alpha \cdot C(X_j,\alpha,K_j))}{\partial L}}{\partial L} \alpha \frac{\partial C\left(X_j,\alpha,K_j\right)}{\partial K_j}
\end{aligned}$$

Writing in terms of elasticities:

$$\epsilon_{MC,K} = -\epsilon_{FL,K} - \epsilon_{FL,L}\epsilon_{C,K}$$

where, $\epsilon_{C,K}$ is the elasticity of costs with respect to capital. Using (1), $\epsilon_{C,K}$ can be derived as:

$$\epsilon_{C,K} = -\frac{\epsilon_{F,K}}{\epsilon_{F,L}}$$

Putting it all together, $\epsilon_{MK,K}$ is written as:

$$\boxed{\epsilon_{MK,K} = \epsilon_{FK,K} - \epsilon_{FL,K} + \frac{\epsilon_{FL,L}}{\epsilon_{F,L}}\epsilon_{F,K} < 0}$$

## 1.2 Optimal Production

In the staging of the model, investment decisions are made prior to production decisions. Hence, I will first characterize the general form of the problem in the final stage during which firms optimally supply to home, and if profitable, foreign markets. Precisely, profits are written as:

$$\Pi(q_h, q_x, q_f, \alpha, K_h, K_f) = P_h(q_h) \cdot q_h + P_f(q_f + q_x) \cdot (q_f + q_x) \tag{7}$$

$$-t \cdot q_x - C_h\left(\underbrace{q_h + q_x}_{X_h}, \alpha, K_h\right) - C_f\left(\underbrace{q_f}_{X_f}, \alpha, K_f\right) \tag{8}$$

Here, $P_h()$ and $P_f()$ are home and foreign inverse demand functions, and $C_h$ and $C_f$ are home and foreign cost functions, and $t$ is a per-unit trade barrier.

In the manuscript, I detailed how foreign investment can increase efficiency not only by reducing transport costs but by also reducing export production that pushes up marginal costs at home. Therein lies one of the novel aspects of the paper that distinguishes the paper from all other foreign investment models, though also one of the challenging aspects. Indeed, production and investment in either country will affect the other location if there are linkages through exports. In (8), I assume that this linkage is through exports from home to foreign. Hence, investment and production in foreign will affect the marginal production costs at home if there are exports from home to foreign (and costs have the properties derived above).

To begin the analysis of these linkages, I derive a general system of first order conditions such that there are active margins of exporting and direct foreign production. Precisely, this system of first order conditions can be written as,

$$\begin{aligned} MR_h(q_h) &= MC_h(X_h, \alpha, K_h) \\ MR_f(Q_f) &= MC_h(X_h, \alpha, K_h) + t \\ MR_f(Q_f) &= MC_f(q_f, \alpha, K_f) \end{aligned} \tag{9}$$

$st:$

$$\begin{aligned} X_h &= q_h + q_x \\ Q_f &= q_f + q_x \end{aligned}$$

where $MR_j$ is the marginal revenue function for market $j$, and $MC_j$ is the marginal cost of producing from capital located in market $j$. In a moment, I will differentiate these conditions to evaluate the effects of productivity on investment incentives, domestic and foreign.

## 1.3 Investment

### 1.3.1 Domestic

The question for this appendix is under what conditions investment incentives are increasing in productivity, and how trade barriers affect this relationship. To begin, we will focus on optimal home capital holdings, which subject to a capital cost $r$, are derived from:

$$\frac{\partial \Pi(q_h, q_x, q_f, \alpha, K_h, K_f)}{\partial K_h} = r$$

Since production takes place conditional on capital holdings, the envelope theorem yields a convenient simplification of this condition:

$$-\frac{\partial C_h(X_h, \alpha, K_h)}{\partial K_h} = r$$

To derive the relationship between optimal $K_h$ and $\alpha$, differentiate with respect to $\alpha$ and arrange in terms of elasticities,

$$-\frac{\partial \frac{\partial C_h(X_h,\alpha,K_h)}{\partial K_h}}{\partial X_h}\frac{\partial X_h}{\partial \alpha} - \frac{\partial \frac{\partial C_h(X_h,\alpha,K_h)}{\partial K_h}}{\partial \alpha} - \frac{\partial \frac{\partial C_h(X_h,\alpha,K_h)}{\partial K_h}}{\partial K_h}\frac{\partial K_h}{\partial \alpha} = 0$$

$$\Rightarrow \qquad \epsilon_{MK_h,K_h}\epsilon_{K_h,\alpha} = -\epsilon_{MK_h,\alpha} - \epsilon_{MK_h,X_h}\epsilon_{X_h,\alpha} \tag{10}$$

where $\epsilon_{X_h,\alpha}$ is the elasticity of production at home with respect to a change in productivity. Since as derived above, $\epsilon_{MK_h,K_h} < 0$, we have the following relationship between investment as a function of productivity and cost/output elasticities:

$$If \quad -\epsilon_{MK_h,\alpha} < \epsilon_{MK_h,X_h}\epsilon_{X_h,\alpha} \;\; then \;\; \epsilon_{K_h,\alpha} > 0$$

As we have derived $\epsilon_{MK,\alpha} = -1$ and $\epsilon_{MK,X} = \frac{\epsilon_{FK,L}}{\epsilon_{F,L}} - \frac{\epsilon_{FL,L}}{\epsilon_{F,L}} > 0$, we now must derive $\epsilon_{X_h,\alpha}$ by differentiating the conditions in (10). Doing so, writing in terms of elasticities, and placing in matrix form, we have:

$$\begin{pmatrix} \epsilon_{MR_h,q_h} & 0 & 0 & -\epsilon_{MC_h,X_h} & 0 \\ 0 & 0 & 0 & -\epsilon_{MC_h,X_h} & \epsilon_{MR_f,Q_f}\beta \\ 0 & 0 & -\epsilon_{MC_f,q_f} & 0 & \epsilon_{MR_f,Q_f} \\ 0 & -(1-\theta) & -\theta & 0 & 1 \\ -\lambda & -(1-\lambda) & 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} \epsilon_{q_h,\alpha} \\ \epsilon_{q_x,\alpha} \\ \epsilon_{q_f,\alpha} \\ \epsilon_{Q_f,\alpha} \\ \epsilon_{X_h,\alpha} \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ -1 \\ 0 \\ 0 \end{pmatrix} \tag{11}$$

where $\beta = \frac{MR_f}{MR_f - t}$, $\lambda = \frac{q_h}{X_h}$, and $\theta = \frac{q_f}{Q_f}$. These latter parameters will allow a parsimonious shift between the closed and open economy, and within the open economy, free trade to costly trade.

Solving for the quantity elasticities within (11) is obviously pretty difficult given the size of the matrix. However, we will evaluate the incentives to invest at home at two polar cases: prohibitive tariffs and free trade. Under the first, $\lambda = 1$, where 100% of production from home is sold at home. Under this assumption, we have:

$$\epsilon_{X_h,\alpha} = -\frac{1}{\epsilon_{MR_h,q_h} - \epsilon_{MC_h,X_h}} \tag{12}$$

Next, consider the case in which $\lambda = 1/2$, $\beta = 1$ and $\theta = 0$, which is the case of free trade with identical demand curves in home and foreign and no foreign production. Under this case, we also get:

$$\epsilon_{X_h,\alpha} = -\frac{1}{\epsilon_{MR_h,q_h} - \epsilon_{MC_h,X_h}} \tag{13}$$

In both cases, $\epsilon_{X_h,\alpha}$ has the same relationship with the elasticity of home marginal revenue curve and home marginal costs. Substituting $\epsilon_{X_h,\alpha}$ in the equation in (10), $\epsilon_{K_h,\alpha} > 0$ if:

$$\epsilon_{MK_h,\alpha} > \epsilon_{MK_h,X_h}\frac{1}{\epsilon_{MR_h,q_h} - \epsilon_{MC_h,X_h}} \tag{14}$$

Substituting the equations for cost elasticities, we have that $\epsilon_{K_h,\alpha} > 0$ if:

$$\epsilon_{MR_h,q_h} \quad > \quad -\frac{\epsilon_{FK_h,L_h}}{\epsilon_{F_h,L_h}} \tag{15}$$

Hence, the incentives to add capital are increasing in productivity if the erosion of marginal revenue is not too large relative to the improvement in marginal cost. The balance of these conditions will be determined by the primitives of costs and demand. As examples, first consider a generalized Cobb-Douglas production function, where $\epsilon_{FK_h,L_h} = \epsilon_{F_h,L_h} = 1 - \eta$, with $\eta$ being the weight on capital in the production function. For this case, $\epsilon_{K_h,\alpha} > 0$ if:

$$\epsilon_{MR_h,q_h} \quad > \quad -1 \qquad\qquad (Cobb - Douglas\ Production\ Function)$$

Thus, independent of the convexity of the production cost function implied by the Cobb-Douglas parameter on variable factors, the incentives to invest are the same. As a separate example, consider a CES production function of the form $Y^\rho = K^\rho + L^\rho$, where $\rho \in (0, 1)$ to guarantee that labor increases marginal productivity of capital (and vice versa). In this case, $\epsilon_{F_h,L_h} = \frac{L^\rho}{Y^\rho}$, $\epsilon_{FK_h,L_h} = (1 - \rho)\frac{L^\rho}{Y^\rho}$, and thus $\epsilon_{K_h,\alpha} > 0$ if:

$$\epsilon_{MR_h,q_h} \quad > \quad -\frac{1}{1-\rho} \qquad\qquad (CES\ Production\ Function)$$

Hence, as $\rho$ approaches zero and CES converges to Cobb-Douglass in the limit, the incentives collapse to the same under Cobb-Douglas. However, as $\rho$ approaches one, it is always the case that $\epsilon_{MR_h,q_h} > -\frac{1}{1-\rho}$ and hence $\epsilon_{K_h,\alpha} > 0$ for all demand systems. Of course, this corresponds to when capital does not affect labor productivity, and hence, the model is uninteresting in that case.

**Comparison of Autarky and Free Trade**

As summarized above, the condition that governs domestic investment incentives is the exact same when moving between autarky and free trade. Does this imply that incentives to invest domestically are exactly the same under free trade as they are under prohibitive trade? No. The intuition is subtle, but important. For free trade, the marginal revenue curve at home must exhibit certain properties for optimal capital to be increasing in productivity. This condition is more likely to be satisfied when a firm, previously in autarky, adds another identical market. Specifically, additional production will push up the marginal cost of serving both markets, and hence, less will be produced solely for the home market. If the elasticity of the marginal revenue curve falls with quantity (for the linear case it falls in quantity from 0 to $-\infty$), then incentives will differ. Hence, for $\epsilon_{MR_h,q_h} > -\frac{1}{1-\rho}$ to be violated under free trade, a firm must have a higher intrinsic productivity compared to the level of productivity at which the same condition is violated under autarky.

### 1.3.2 Foreign

Next, we examine the case of foreign investment, and the relative effect of trade costs on investment incentives as a function of productivity. To abstract from issues of cost-elasticity differences across locations, we will henceforth assume that all cost elasticities are identical across countries. This would be satisfied for CES or Cobb-Douglas (as described above) production functions used in both locations. Finally, as in the paper, I assume that the cost of capital in foreign is the same as the cost of capital in home ($r$).

Similar to domestic investment, the optimal foreign allocation is determined by:

$$-\frac{\partial C_f\left(q_f, \alpha, K_f\right)}{\partial K_f} = r$$

where $\epsilon_{K_f,\alpha}$ is pinned down as follows:

$$\epsilon_{MK_f,K_f}\epsilon_{K_f,\alpha} = -\epsilon_{MK_f,\alpha} - \epsilon_{MK_f,q_f}\epsilon_{q_f,\alpha} \tag{16}$$

As with domestic, a crucial aspect of (16) is $\epsilon_{q_f,\alpha}$, which is derived from the system of first order conditions in (11).

In Proposition 3 in the paper, I argue that the incentives for foreign acquisitions relative to domestic are increasing in productivity when trade costs are positive, and that this governs the choice between the two options. Further, I show that the incentives to acquire abroad collapse to domestic incentives when trade costs are zero. Assuming that cost functions are common across borders and derived from the same constant-elasticity production functions, we can write the incentive for foreign investment relative to domestic as:

$$\left(\epsilon_{K_f,\alpha} - \epsilon_{K_h,\alpha}\right) = -\frac{\overset{+}{\epsilon_{MK,X}}}{\underset{-}{\epsilon_{MK,K}}}\left(\epsilon_{q_f,\alpha} - \epsilon_{X_h,\alpha}\right)$$

Solving for the difference in quantity elasticities using (11):

$$\epsilon_{q_f,\alpha} - \epsilon_{q_h,\alpha} \quad = \quad \frac{1}{D}\left(\frac{\epsilon_{MR_f,Q_f}\cdot\epsilon_{MR_h,q_h}}{\epsilon_{MC,X}}\cdot(1-\lambda\theta)\cdot(\beta-1)\right) \geq 0$$

where

$$\begin{aligned} D &= -\epsilon_{MR_h,q_h}\cdot\epsilon_{MC,X}(1-\lambda) + \epsilon_{MR_h,q_h}\cdot\epsilon_{MR_f,Q_f}\cdot(\theta(1-\lambda)+\beta(1-\theta)) - \epsilon_{MR_f,Q_f}\cdot\epsilon_{MC,X}\beta\cdot\lambda\cdot(1-\theta) \\ &> 0 \end{aligned}$$

Hence:

$$\left(\epsilon_{K_f,\alpha} - \epsilon_{K_h,\alpha}\right) = -\frac{\overset{+}{\epsilon_{MK,X}}}{\underset{-}{\epsilon_{MK,K}}}\frac{1}{\underset{+}{D}}\left(\frac{\epsilon_{MR_f,Q_f}\cdot\epsilon_{MR_h,q_h}}{\epsilon_{MC,X}}\cdot(1-\lambda\theta)\cdot(\beta-1)\right) \geq 0 \tag{17}$$

To examine equation (17) first consider the polar case of free trade. In this case, $\beta = 1$ and $\epsilon_{K_f,\alpha} = \epsilon_{K_h,\alpha}$. This condition implies that whenever trade costs are zero and $\frac{\partial K_h}{\partial \alpha} = 0$ it must be the case that $\frac{\partial K_f}{\partial \alpha} = 0$ and hence additional investment is not optimal in either location.

In contrast, when trade costs are positive and $\beta > 1$, when $\frac{\partial K_h}{\partial \alpha} = 0$ it must be the case that $\frac{\partial K_f}{\partial \alpha} > 0$ and hence additional investment in foreign is optimal when additional investment at home is not. This is due to the market access incentive for foreign investment, which provides an additional incentive beyond that of cost-reduction for relatively high productivity firms.