Metzler vs. Melitz: Domestic Characteristics and the Impact of Tariffs

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Abstract

A common prediction within open economy firm heterogeneity models is a Metzler-type paradox in which tariff liberalization decreases competitiveness within the liberalizing import market. This effect is driven by the de-location of firms from the liberalized import market to the foreign market. In this paper, I show that plausible variation in domestic characteristics can reverse this mechanism. Within a two country model with flexible productivity heterogeneity, I show how tariffs, tastes, and levels of intra-national mark-ups (sales taxes, internal distances, or distribution mark-ups) determine the qualitative effects of tariff liberalization. The critical issue is whether the aggregate demand response which occurs subsequent to liberalization affects exporters or domestic firms to a larger degree. Indeed, I show that a sufficient condition to avoid the Metzler-type paradox is that the country with a higher elasticity of entry to shocks (eg. poorly skewed productivity draws) also has the smaller equilibrium demand for each variety. This occurs when the country with less-productive firms also has a lower taste for the differentiated industry, or a larger internal mark-up affecting all varieties.

Key Words: Firm-heterogeneity, Free Entry, Productivity, Tariffs, Domestic Taxes

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1 Introduction

The link between trade liberalization and welfare is arguably the most important question in international trade, and certainly one of the most studied in economics as a whole. Large changes to trade barriers can greatly affect the pattern of trade, the wages and employment patterns of workers, and profits earned by firms. Understandably, it remains an important research question, and is subject to fierce policy debate.

A large portion of the past ten to fifteen years of research on this issue has focused on the role of the firm, expansion to foreign markets, and a link between average productivity and welfare. Specifically, it was at least implied (if not explicitly argued) that selection of more efficient firms would confer additional gains from trade along with the standard gains. Surprisingly, recent work by Arkolakis, Costinot, and Rodriguez-Clare (2011) (ACR) has shown that we actually haven't learned all that much, at least in terms of the long-run, general equilibrium gains from trade within models that employ common assumptions. Precisely, in their work, if utility is CES, trade is balanced, and trade elasticities are constant, then the gains from trade can be calculated using (1) a trade elasticity and (2) a measure of domestically sourced consumption.

However, despite the analytical tractability of modern firm-heterogeneity models and the novel results in ACR, there are two important caveats. First, implicit in the third assumption in ACR - that trade elasticities are constant - and the parametrization of Melitz (2003) in Helpman, Melitz, and Yeaple (2004) and Chaney (2008) is the use of a Pareto distribution to characterize productivity. The critical assumption that facilitates analysis is that the skew of the Pareto distribution from which costs are randomly drawn is identical across countries. While analytically tractable and employed extensively throughout the literature, it begs the question as to how productivity differences across trading partners affect long run equilibria. Indeed, other areas of the literature *require* exogenous differences in productivity distributions and other domestic characteristics that are not elasticity neutral. For example, the entire literature on selection and the extensive margin of trade (as in Helpman, Melitz, and Rubinstein (2008)) relies on heterogeneous bounds of the cost distribution to match the prevalence of zeros in the data. Further, in my earlier paper, Spearot (2013), I show that other differences in the productivity distribution (skew, for example) are required to reconcile the disparate effects of tariffs on imports to the US during the Uruguay Round.

The second caveat relates to the aggregate effects of tariffs in firm-heterogeneity models, where the typical assumptions yield a Metzler-type paradox (Metzler (1949)) when evaluating the effects of unilateral changes to tariffs. Specifically, unilateral liberalization *decreases* competitiveness in the domestic market. The intuition is that lower tariffs reduce protection within that market, and hence, firms exit the liberalized market and enter the foreign market. However, this de-location of firms away from the liberalized market ultimately reduces competition within that market, this yields a lower residual demand curve for each variety, and in some models, lower prices charged by domestic firms. Indeed, similar forces have been shown to have an impact on the incentives for trade agreements in Venables (1985), Bagwell and Staiger (2009), Ossa (2010), and Bagwell and Staiger (2012).

In this paper, I show that the de-location effects that are pervasive across a large collection of models are sensitive to the domestic characteristics of trading partners. In particular, I show that subject to heterogeneity in the productivity distribution that governs entry into each domestic market, the qualitative effects of tariffs are critically linked to the overall nature of trade policy (net tariffs or subsidies), taste heterogeneity across countries, and intra-national mark-ups due to distribution sectors, sales taxes, or internal distance. Under reasonable parameter assumptions, the aforementioned Metzler-type paradox can be overturned. Overall, my contribution is to provide a unified view of how country characteristics and policy parameters dictate the effects of trade shocks in the presence of productivity heterogeneity.

To allow for rich substitution patterns within and across industries, I introduce these issues by employing an extended version of Melitz and Ottaviano (2008). To motivate the distributional assumptions, I first show that by employing the Pareto distribution there is no variation in average firm-size across exporters selling to a common import market without either (1) bilateral distance factors or (2) variation in the Pareto skew parameter. Using Columbian transaction level data for 2003, I show that one can reject the assumption that average firm-size is constant, even when controlling for reported values of transportation costs. Hence, along with the literature summarized above, the data seem to indicate that variation in technology parameters (such as skew) is important in matching the real world characteristics of trade patterns.

Then, using a two-country framework, I address the long run behavior of the model subject to cross-country variation in the parameters of the Pareto distribution. The key issue is that a free entry condition in a given country is linear when transforming demand levels by the skew of the Pareto distribution. If all Pareto parameters are identical across countries, then we have linear system of free-entry conditions in the transformed demand levels, and if non-degenerate, a unique solution. However, if we do not have symmetry in Pareto parameters, the elasticities of entering firms in each country with respect to demand shocks are not equal, and hence, we have a non-linear system free-entry conditions that need not be unique.[1] Crucially, when there are multiple solutions, the aggregate response to tariffs differs qualitatively across these solutions, with one solution yielding a Metzler-type paradox, and another the opposite. However, imposing conditions that the number of entering firms is positive in each country "selects" from the candidate solutions, and the equilibrium is always unique.

The primary contribution of the paper is evaluating how domestic primitives associate with the qualitative direction of tariff liberalization. Similar to Venables (1985), Demidova (2008), Bagwell and Staiger (2009), and Ossa (2010), the entry response to tariffs depends crucially on the relative slope of the expected profit functions in the transformed demand levels. This relative response of profits in each country determines the de-location effects of trade shocks. Within the two country model that I employ, the condition that determines the de-location of firms compares the (geometric) average effect of demand shocks on export profits to the average effect of demand shocks on domestic profits. Intuitively, if the latter is larger than the former, then the reduction in demand following unilateral liberalization will lead to an exit of firms in the liberalizing market and entry into other markets, and ultimately, a Metzler-type paradox. Assuming only productivity heterogeneity across countries, I begin by showing that positive tariffs in each country yield a case in which the former is less than the latter. Indeed, tariffs mitigate the impact of demand shocks on exporters, home firms in the liberalizing market exit, and tariff liberalization yields a Metzler-type paradox. These results are consistent with the Melitz (2003) model with cross-country productivity heterogeneity as in Demidova (2008). However, if countries levy a net subsidy, the former is greater than the latter since subsides amplify the impact of of shocks on exporters, and the only supported equilibrium is one without a Metzler-type paradox.

These results become more complicated when allowing for additional heterogeneity beyond initial trade

[1] In the appendix, I show that with three countries that trade freely, one can find cases in which there are *six* solutions such that free entry is satisfied for all three countries. Generally, the number of solutions to free entry conditions for a given set of parameters is only bounded above by the product of all Pareto parameters scaled by the least-skewed country. This number can be very large.

policies and productivity distributions. Specifically, the effect of demand shocks on export profits can be larger than on domestic profits if the country with the higher elasticity of entry to demand shocks also has a lower demand for varieties domestically. While the former is satisfied by our distributional assumptions, where a distribution skewed toward lower values of productivity yields a higher elasticity of entry to shocks, the latter is not immediate. However, when both are satisfied, a shock in a large export market is amplified for exporters that enter at a more pronounced rate. To make precise the conditions under which this occurs, I evaluate tariff liberalization in a neighborhood free trade, and focus on the role of taste heterogeneity and effective intra-national mark-ups. In terms of the former, I show that the relatively unproductive country can also have a more competitive market when it also has a lower intrinsic taste for the differentiated good.[2] Note again that this normally does not happen in the typical set-up since a less-productive country usually has a less-competitive differentiated sector. However, "competition", as measured by the demand level facing each firm, can be lower by virtue of the average efficiency of firms that compete *and* other domestic characteristics specific to each market. In this case, a sufficiently lower taste for the good reduces demand in the market. And, as described above, these conditions are sufficient to overturn the Metzler-paradox.

In a similar fashion, intra-national mark-ups, which may include domestic sales taxes, distribution costs due to internal distance, or constant distribution mark-ups, can also support an equilibrium in which the less-productive country hosts the more competitive differentiated market.[3] The relationship to the above intuition is that a "tax" on revenues within a differentiated market scales down the effective size of that market, though independently of product origin. Hence, if a less-productive country also has a sufficiently large intra-national mark-up, then the conditions are satisfied such that the Metzler paradox is overturned.

[2] Indeed, this suggests a possible important role for income driven taste heterogeneity, as in Simonovska (2010), in dictating the aggregate impact of tariffs.

[3] These types of mark-ups have found increasing interest in the literature as economists have focused on the role of intra-national distribution and geography as a source of trade frictions (Mayer and Zignago (2011), Atkin and Donaldson (2012) and Cosar and Fajgelbaum (2012)).

2 General Setup and Motivation

To make clear the issues at-hand, I will utilize the framework in Melitz and Ottaviano (2008) as the base model. However, I will adjust this standard firm-heterogeneity model to account for differences in tastes across countries, and for differences in the level of "internal" mark-ups.

Consumers

Consumer preferences in each country are specified according to the following form

$$U_j = x_{0,j} + \theta_j \int_{i\in\Omega} q_{i,j} di - \frac{1}{2}\eta \left(\int_{i\in\Omega} q_{i,j} di\right)^2 - \frac{1}{2}\gamma \int_{i\in\Omega} (q_{i,j})^2 di, \tag{1}$$

where Ω_j represents the measure of varieties available in country j, $q_{i,j}$ is the consumption of variety i in j, and the parameters θ_j (> 0) and η (> 0) determine the substitution pattern between the differentiated industry and the outside good, x_0. Note that I allow θ_j to differ across countries, which implies that countries may differ fundamentally in their valuation for the differentiated good relative to the numeraire. Finally, γ (> 0) represents the degree to which varieties are substitutable. If γ were zero, all firms would price at the same level, since products would be homogeneous in the eyes of the consumer.

The budget constraint faced by consumers in country j is written as:

$$x_{0,j} + \int_{i\in\Omega_j} p^c_{i,j} q_{i,j} di \leq I_j \tag{2}$$

where $p^c_{i,j}$ is the delivered consumer price to j. Note that implicit in this budget constraint is the assumption that the numeraire is freely traded. As in the existing literature, I will assume that parameter values are such that consumers have positive consumption in both the outside good and differentiated industries. Hence, the inverse demand function for a given variety i is derived as:

$$\begin{aligned} p_{i,j} &= \underbrace{\theta - \eta Q_j}_{A_j} - \underbrace{\frac{\gamma}{L_j}}_{b_j} q_{i,j} \\ \Rightarrow \quad p^c_{i,j} &= A_j - b_j q_{i,j} \end{aligned} \tag{3}$$

In (3), Q_j is the total quantity sold by all firms to the representative consumer in j, and A_j contains all aggregate terms within the demand curve for each variety in j. Finally, b_j will measure the slope of the aggregate demand curve for variety i in j, which is γ scaled inversely by the number of consumers in j.

Firms

The characterization of firms in each country is relatively simple. Firms enter under uncertainty, paying a fixed entry cost F, which is constant across countries (this is not necessary, though helpful for exposition). Upon entry, firms from country l draw a marginal cost c from a country-specific Pareto distribution with the following pdf:

$$g(c) = k_l \frac{c^{k_l - 1}}{(c_l^m)^{k_l}} \tag{4}$$

In (4), there are two-layers of heterogeneity across countries. First, slightly more standard in the literature is variation in the upper-bound of the distribution c_l^m, which we henceforth assume to be non-binding for any country selling to any market. The second-layer, which is non-standard (other than a similar type of heterogeneity in Demidova (2008) and Demidova and Rodriguez-Clare (2011)), is variation in the Pareto parameter, k_l. Variation in this parameter across countries will be crucial for the results, and we will discuss the empirical implications of this parameter shortly.

Each firm may sell one variety to each market, potentially paying an ad-valorem tariff τ_j on the value of each unit sold from abroad to j. Note that we allow this tariff to be negative, in that case implying an import subsidy. Further, all firms selling to market j will be subject to an ad-valorem sales tax $\tilde{s}_j$. Though we refer to it as a sales tax for exposition, I will later discuss how other domestic characteristics have a similar effect on demand. Hence, the relationship between the consumer price detailed in (3) and the price that foreign producers receive is $p^c_{l,j} = (1+\tau_l)(1+\tilde{s}_l)p^s_{l,j}$. This yields the following inverse demand function that foreign suppliers use to optimally set production for the import market in j:

$$p_i^s = \frac{1}{t_j s_j}\left(A_j - \frac{\gamma}{L_j} q_{i,j}\right)$$

where, $t_j = (1+\tau_j)$ and $s_j = (1+\tilde{s}_j)$. For the domestic case, firms do not pay the tariff, and hence, $t_j = 1$.

Firms choose quantities to maximize profits, where the maximization problem for firm i exporting to j is written as:

$$\pi_{i,j}\left(c_i\right) = \max_{q_{i,j}} \left\{ \frac{1}{t_j s_j} \left(A_j - \frac{\gamma}{L_j} q_{i,j} \right) \cdot q_{i,j} - c_i q_{i,j} \right\}.$$

Suppressing i's for the remainder of the paper, firms in j selling to their home market do so without paying a tariff, and hence do so profitability if $c < A_j$. Assuming that this is the case, the quantity of domestic sales to j is written as

$$q_{jj}(c) = \frac{A_j - cs_j}{2b_j}.$$

Further, producer revenues are written as

$$v_{jj}(c) = \frac{A_j^2 - (cs_j)^2}{4b_j s_j},$$

and profits are written as

$$\pi_{jj}(c) = \frac{\left(A_j - (cs_j)\right)^2}{4b_j s_j}.$$

Next, consider a country $l \neq j$ that exports to j. As described above, exporting to j requires the payment of an ad-valorem tariff τ_j, where the tariff factor relevant for pricing is $t_j = 1 + \tau_j$. Hence, quantity and profits earned in selling to j are written as

$$q_{lj}(c) = \frac{A_j - ct_j s_j}{2b_j},$$

producer revenues are written as

$$v_{lj}(c) = \frac{A_j^2 - (ct_j s_j)^2}{4b_j t_j s_j},$$

and profits are written as

$$\pi_{lj}(c) = \frac{(A_j - ct_js_j)^2}{4b_jt_js_j}.$$

What does Pareto imply?

To motivate the more nuanced differences in productivity distributions, I now solve for the average firm-level export value from country l to import market j. To solve for this, we integrate the firm-level import values over $[0, \frac{A_j}{t_js_j}]$:

$$\begin{aligned} \overline{v}_{lj} &= \int_0^{\frac{A_j}{t_js_j}} \frac{A_j^2 - (ct_js_j)^2}{4b_jt_js_j} \frac{g_l(c)}{G_l(\frac{A_j}{t_js_j})} \\ &= \frac{A_j^2}{2b_js_jt_j(k_l+2)} \end{aligned} \tag{5}$$

In (5), after imposing the Pareto distribution, we see that the average firm-level export *does not* depend on the upper bound of the Pareto parameter. This is due to average export value being a truncated average (conditional on export status). Thus, when imposing the Pareto distribution, average exporter size within an industry does not vary across exporters unless the Pareto parameter k_l differs across l.

A quick look at Columbian import data from 2003 rejects the hypothesis that within industries, average firm-level import values are the same. Specifically, using the *DIAN* transaction-level import data, I aggregate to exporter-10 digit groups, and run the following regression:

$$\ln(\overline{v}_{l,j,t+1}) = \beta_v \ln(V_{l,j,t}) + \beta_f \ln(f_{l,j,t}) + \alpha_j + \epsilon_{l,j,t+1} \tag{6}$$

Here, the dependent variable is the average reported *firm-level* import value from country l to market j in period $t+1$, $\ln(V_{l,j,t})$ is the log of the total import value from country l in product j in year t, and $\ln(f_{l,j,t})$ is the log of average reported trade costs from exporter l in industry j in year t. According to the strict predictions of Pareto outlined above, the coefficient on β_v should not be significantly different from zero. The alternative hypothesis is that it is non-zero, and in particular, positive, where countries more engaged with Columbia in industry j should have, on average, larger firms in serving j. In terms

Table 1: Average Firm-level Import Values

	$\ln(\overline{v}_{lj,t+1})$	$\ln(\overline{v}_{lj,t+1})$	$\ln(\overline{v}_{lj,t+1})$
$\ln(V_{lj,t})$	0.581***	0.567***	0.602***
	(0.003)	(0.003)	(0.003)
$\ln(f_{l,j,t})$		-0.411***	-0.396***
		(0.026)	(0.028)
Observations	41,263	41,263	41,244
R^2	0.692	0.699	0.743
Exporter Dummies	No	No	Yes

Notes: HS10 product fixed effects. t is the first six months of 2003, $t+1$ is the second six months of 2003. *** p<0.01, ** p<0.05, * p<0.1

of $\ln(f_{l,j,t})$, there is no effect of trade costs in the model, though adding a per-unit transport cost would scale down average firm-size. On the other hand, adding a fixed cost would scale up average firm-size in exporting to Columbia. Hence, the prediction for β_f is ambiguous.

Using a within-product estimator, Table 1 presents the estimates of (6) aggregating imports to Colombia in 2003 to two periods - t for the first six months, and $t+1$ for the second six months. In the first column, we see a positive and significant association between exporting country size in product j to Colombia and average firm-size in exporting to Columbia. Adding transport costs in the second column, we find a negative effect on average firm size, which suggests that bilateral per-unit transportation costs are relevant for trade to Columbia. In the third column, we add exporter dummies to capture broad differences in firm-size across exporters. Here, we find that the previously outlined relationships are still valid

Hence, the basic country heterogeneity which is reflected in Pareto differences related to the upper-bound on the cost distribution cannot capture these features of the data. While there are no doubt other issues at play here, one consideration which has received little attention is variation in the Pareto skew parameter, k_j. It is not implausible to view cost draws in the US to not only have a more favorable support compared with a developing country, but they may also have a distribution which is skewed toward higher productivity firms (lower k_j). What are the equilibrium implications of allowing for differences in the Pareto parameter and the upper bound on costs? How do these productivity factors interact with other domestic policies and characteristics? To examine these issues, I now detail the long-run equilibrium of the model.

Long Free Entry

With the basics of the model and the motivation for varying Pareto parameters in-hand, I now turn to the aggregate equilibrium conditions. The first involves long run free-entry, in which firms enter until the expected profits from entry are equal to a fixed cost of entry. By imposing the Pareto assumption in (4), the expected profits of selling from j to j are written as:

$$\overline{\pi}_{jj} = \frac{A_j^{k_j+2}}{b_j(k_j+2)(k_j+1)(c_j^m)^{k_j} s_j^{k_j+1}} = \frac{A_j^{k_j+2}}{\delta_j s_j^{k_j+1}}$$

where $\delta_j = b_j(k_j+2)(k_j+1)(c_j^m)^{k_j}$. Selling from j to another market l, expected profits are written as:

$$\overline{\pi}_{jl} = \frac{b_j A_l^{k_j+2}}{b_l t_l^{k_j+1} s_l^{k_j+1} \delta_j}$$

Aggregating over all available markets, the free entry condition for country j is written as:

$$\frac{A_j^{k_j+2}}{s_j^{k_j+1}} + \sum_{l \neq j} \frac{b_j A_l^{k_j+2}}{b_l t_l^{k_j+1} s_l^{k_j+1}} = F\delta_j$$

Defining $\rho_{jl} = \frac{b_l}{b_j} t_l^{k_j+1}$, a more "compact" form of the free entry condition for country j is written as:

$$\frac{A_j^{k_j+2}}{s_j^{k_j+1}} + \sum_{l \neq j} \frac{A_l^{k_j+2}}{s_l^{k_j+1}} \rho_{jl} = F_j \delta_j \tag{7}$$

In (7), the key issue to note is that if k_j's are all identical across countries and equal to k, than the long-run equilibrium consists of a system of equations which are linear in A_z^{k+2} for all z. However, if the k_j's are different, then we have a system of free entry conditions which exhibit a different degree of non-linearity for each country.

The final component of the equilibrium is the number of firms that enter each country, which given the solution(s) to the system of free entry conditions, are pinned down using the definition of A_j. Precisely, each A_j is defined by the following:

$$A_j = \theta - \eta \left(\sum_l N_l \overline{q}_{l,j} \right) \tag{8}$$

where N_l is the number of firms that have entered l and $\overline{q}_{l,j}$ is the expected quantity sold to j by a given entrant in l. For selling from j to j, expected quantity conditional on entry is written as:

$$\overline{q}_{jj} = \frac{A_j^{k_j+1}}{\gamma(k_j+1)(c_j^m)^{k_j} s_j^{k_j}} = \frac{A_j^{k_j+1}\,(k_j+2)\,b_j}{\gamma \delta_j s_j^{k_j}}$$

For selling from l to j, expected quantity is written as:

$$\overline{q}_{l,j} = \frac{A_j^{k_l+1}\,(k_l+2)\,b_l}{\gamma t_j^{k_l} s_j^{k_l} \delta_l}$$

Hence, A_j will be implicitly defined by the following:

$$A_j = \theta - \eta \left(N_j \frac{A_j^{k_j+1}\,(k_j+2)\,b_j}{\gamma \delta_j s_j^{k_j}} + \sum_{l \neq j} N_l \frac{A_j^{k_l+1}\,(k_l+2)\,b_l}{\gamma t_j^{k_l} s_j^{k_l} \delta_l} \right) \tag{9}$$

All other countries l have a similar condition. As I will show below, this condition will help select from multiple candidate long-run solutions that arise from the free entry profit conditions outlined in (7).[4]

3 Two-country Model

To make the issues at hand clear, I now restrict the model to trade between two countries, labeled 1 and 2. The order of analysis will be the following. First, I will evaluate candidate long run solutions using (7).

[4] While I use Melitz and Ottaviano (2008) to outline the issues of the model, note that using Melitz (2003) leads to the same qualitative form of the free-entry condition. Precisely, in the appendix, I show that the free entry condition is

$$B_j^{\frac{k_j}{\sigma-1}} + \sum_{l \neq j} B_l^{\frac{k_j}{\sigma-1}} \psi_{jl} = H_j$$

where B_j is an equilibrium measure of market potential for each variety in market j, k_j the Pareto parameter, σ the elasticity of substitution within the differentiated industry, ψ_{jl} a measure of fixed overhead costs, fixed export costs, and tariffs for market j selling to l, and H_j an adjusted fixed cost of entry. Hence, the examples that I discuss below related to free-entry conditions will also be relevant for the Melitz model.

Then, I will describe how the implied numbers of entering firms from (9) select a unique equilibrium when multiple candidates are present.

The free entry conditions for countries 1 and 2 are written as:

$$\begin{aligned}
\frac{A_1^{k_1+2}}{s_1^{k_1+1}} + \frac{A_2^{k_1+2}}{s_2^{k_1+1}}\rho_{12} &= F\delta_1 \\
\frac{A_1^{k_2+2}}{s_1^{k_2+1}}\rho_{21} + \frac{A_2^{k_2+2}}{s_2^{k_2+1}} &= F\delta_2
\end{aligned}$$

To simplify this system of equations, define $\lambda = \frac{k_2+2}{k_1+2}$, which is precisely the relative skew of the profit distribution in country two relative to country one. Without loss of generality, I assume that $\lambda > 1$ for the remainder of this section, and hence, the country 2 cost distribution is skewed toward higher cost firms. Furthermore, defining $X_1 = A_1^{k_1+2}$, and $X_2 = A_2^{k_1+2}$, we can write the system of free entry conditions as:

$$\frac{X_1}{s_1^{k_1+1}} + \frac{X_2}{s_2^{k_1+1}} \cdot \rho_{12} = F\delta_1 \tag{10}$$

$$\frac{X_1^{\lambda}}{s_1^{k_2+1}} \cdot \rho_{21} + \frac{X_2^{\lambda}}{s_2^{k_2+1}} = F\delta_2 \tag{11}$$

In (10), the relationship between X_1 and X_2 is linear, and summarized by:

$$\begin{aligned}
\left(\frac{\partial X_2}{\partial X_1}\right)_1 &= -\frac{1}{\rho_{12}}\left(\frac{s_2}{s_1}\right)^{k_1+1} < 0 \\
\left(\frac{\partial^2 X_2}{\partial X_1^2}\right)_1 &= 0
\end{aligned}$$

In contrast, the locus of values that satisfy (11) is non-linear, and is dependent on λ:

$$\begin{aligned}
\left(\frac{\partial X_2}{\partial X_1}\right)_2 &= -\rho_{21}\frac{X_1^{\lambda-1}}{X_2^{\lambda-1}}\left(\frac{s_1}{s_2}\right)^{k_2+1} < 0 \\
\left(\frac{\partial^2 X_2}{\partial X_1^2}\right)_2 &= -\rho_{21}\left(\frac{s_2}{s_1}\right)^{k_2+1}(\lambda-1)\,\frac{\left(X_1^{\lambda-2}X_2^{\lambda-1} + X_2^{\lambda-2}X_1^{\lambda-1}\rho_{2,1}\left(\frac{s_2}{s_1}\right)^{k_2+1}\frac{X_1^{\lambda-1}}{X_2^{\lambda-1}}\right)}{\left(X_2^{\lambda-1}\right)^2}
\end{aligned}$$

Clearly, the locus of values that satisfies (11) is downward sloping, and given the assumption that $\lambda > 1$,

$\left(\frac{\partial^2 X_2}{\partial X_1^2}\right)_2 < 0$. Hence, the free entry locus for country 2 is concave from the origin. Intuitively, given the relatively poor skew of country 2's cost distribution, the elasticity of entry in response to demand shocks is higher. Hence, when measured relative to country 1, relative changes to demand have a more pronounced effect on the free entry condition for country 2.

Further, note the limits of the slope of the locus in (11) at the intercepts:

$$\begin{aligned}
\lim_{X_1 \to 0} \left(\frac{\partial X_2}{\partial X_1}\right)_2 &= 0 \\
\lim_{X_2 \to 0} \left(\frac{\partial X_2}{\partial X_1}\right)_2 &= -\infty
\end{aligned}$$

Given the shape of each free-entry locus, the possible equilibrium cases are determined by the relative ranking of X_1 and X_2 intercepts. By evaluating the intercepts of the loci, we have the first result of the paper.

Lemma 1 *(Free Entry Uniqueness)*
If $\lambda > 1$, then there exists a range of $F \in [\underline{F}, \overline{F}]$ such that a solution to (10) and (11) exists in which $X_1 > 0$ and $X_2 > 0$. Further there exists a threshold value $\widehat{F}$ such that for $F \leq \widehat{F}$, there is one solution to (10) and (11), and for $F > \widehat{F}$ there are two solutions to (10) and (11).

Proof. See Appendix ■

Lemma 1 states that if fixed costs aren't too low or high, then there exist values of X_1 and X_2 such that (10) and (11) are satisfied. This highlights one of the first novel features of the varying-Pareto version of the standard firm-heterogeneity model, which is that even when fixed costs are common across countries, fixed costs that are too low or two high yield a long-run equilibrium in which only one free-entry condition can be satisfied. An additional feature of Lemma 1 is that there may be multiple solutions to (10) and (11) such that $X_1 > 0$ and $X_2 > 0$. Specifically, within the region such that any solution exists, when fixed costs are relatively high, this yields two candidate solutions such that free entry conditions are satisfied in each country.

The possible solutions outlined in Lemma 1 are presented in Figure 1. First, consider the two figures on the left-hand side of Figure 1. For the solutions to (10) and (11), the ranking of the horizontal and vertical intercepts switch between countries 1 and 2. In "type A" equilibria, the country 2 iso-profit locus is steeper than the iso-profit locus for country 1. Though I will derive this rigorously in a moment as it relates to the degree and type of import protection, the ranking of slopes governs the response by firms to demand shocks. When this particular slope ranking persists, a shock in country 2 (country 1) has a larger effect on country 2 (country 1) firms. In type B equilibria, this response flips, where a shock in country 2 (country 1) has a larger effect on country 1 (country 2) firms. The precise definitions of type A and type B equilibria are summarized as follows:

Definition 1 *(Equilibrium Type)*

Type A equilibria are defined as any solution to (10) and (11) such that

$$1 > \rho_{12}\rho_{21}\left(\frac{s_2}{s_1}\right)^{k_2-k_1}\left(\frac{X_1}{X_2}\right)^{\lambda-1}$$

Type B equilibria are defined as any solution to (10) and (11) such that

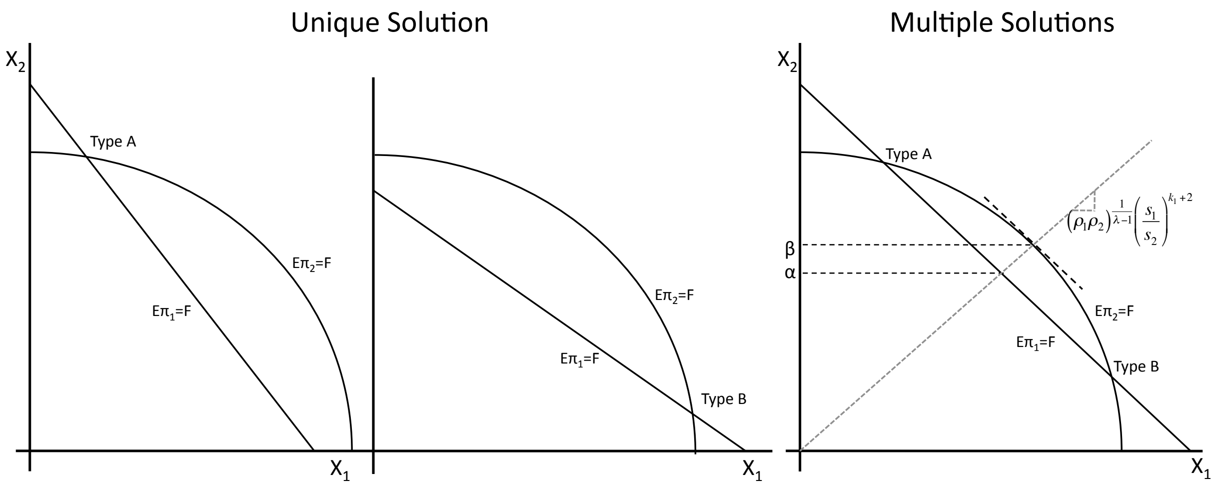
$$1 < \rho_{12}\rho_{21}\left(\frac{s_2}{s_1}\right)^{k_2-k_1}\left(\frac{X_1}{X_2}\right)^{\lambda-1}$$

Finally, consider the right-hand plot in Figure 1, which illustrates the case in which there are two solutions to (10) and (11). In this case, both the vertical and horizontal intercepts for country 2 are less than country 1, but the country 2 free entry condition is bowed out to such an extent that (11) intersects (10) at two points, one of type A and another of type B. This is guaranteed when β, the value of X_2 on (11) at the point of tangency between (11) and (10), is greater than α, the value of X_2 on (10) at the point of tangency between (11) and (10). A result of the intermediate value theorem, when there are two solutions, each equilibrium must lie in different regions as sectioned by the ray from the origin that identifies tangency of the two free entry loci. This bisected space identifies Type A (upper-left) and Type B (lower-right) equilibria.

As a final point in this subsection, consider the following corollary when there is free trade and all taxes are set to zero.

Corollary 2 *(Free Entry Uniqueness under no tariffs and sales taxes)*

Figure 1: Possible Equilibria in the Two-country Long-run Equilibrium



If $\lambda > 1$ and $\rho_{12} = \rho_{21} = s_1 = s_2 = 1$, then within $F \in [\underline{F}, \overline{F}]$ there are always two solutions to (10) and (11) such that $X_1 > 0$ and $X_2 > 0$.

Proof. See Appendix ■

Corollary 2 makes very clear the differences between the current setup and the cannonical setup with common Pareto parameters. Under the typical setup with common Pareto parameters under free trade, there is a differentiated industry in both countries only if they are identical, and there are an infinite number of solutions to the demand levels via the free entry conditions. In the varying Pareto model, when a solution exists under free trade, the free entry conditions yield exactly two solutions (which we will select from in the next section), and the countries need not be identical to yield these solutions.

3.1 Tariff Liberalization

I will now evaluate the effects of tariffs on the candidate equilibria as dictated by the free entry conditions in (10) and (11). The analysis of tariffs is relatively straightforward given the two country setup. For example, consider what happens when tariffs in country 1 are liberalized. In the model, liberalization by country 1 is reflected in an increase in ρ_{21}. Differentiating (10) and (11) with respect to ρ_{21} yields the following:

$$\frac{\partial X_1}{\partial \rho_{21}} = \frac{\rho_{12} X_1^{\lambda}}{\lambda X_2^{\lambda-1}\left(1-\rho_{12}\rho_{21}\left(\frac{s_2}{s_1}\right)^{k_2-k_1}\left(\frac{X_1}{X_2}\right)^{\lambda-1}\right)} \tag{12}$$

$$\frac{\partial X_2}{\partial \rho_{21}} = -\frac{X_1^{\lambda}}{\lambda X_2^{\lambda-1}\left(1-\rho_{12}\rho_{21}\left(\frac{s_2}{s_1}\right)^{k_2-k_1}\left(\frac{X_1}{X_2}\right)^{\lambda-1}\right)} \tag{13}$$

In (12) and (13), we see that the sign of each derivative depends on condition that determines equilibrium type, which again, is determined by the relative slope of the expected profit functions in each demand level. The following Lemma summarizes the effects of tariffs subject to this condition:

Lemma 2 *For equilibria of type A, import liberalization/subsidization in country 1 (higher ρ_{21}) increases competitiveness in 2 and reduces competitiveness in 1. Precisely,*

$$\frac{\partial X_2}{\partial \rho_{21}} < 0\ ,\ \frac{\partial X_1}{\partial \rho_{21}} > 0$$

For equilibria of type B, import liberalization/subsidization in country 2 (higher ρ_{21}) increases competitiveness in 1 and reduces competitiveness in 2. Precisely,

$$\frac{\partial X_2}{\partial \rho_{21}} > 0\ ,\ \frac{\partial X_1}{\partial \rho_{21}} < 0$$

Proof. See Appendix ■

The intuition for this Lemma is that the relative slope of the profit functions determines the strength of the de-location effects that result from liberalization. When country 1 liberalizes, the initial aggregate effect is to make the market more competitive resulting in a reduction to X_1. The critical question for entry is who this affects to a larger degree. To answer this question, the term $1-\rho_{12}\rho_{21}\left(\frac{s_2}{s_1}\right)^{k_2-k_1}\left(\frac{X_1}{X_2}\right)^{\lambda-1}$ can be generally written as:

$$1-\rho_{12}\rho_{21}\left(\frac{s_2}{s_1}\right)^{k_2-k_1}\left(\frac{X_1}{X_2}\right)^{\lambda-1}=1-\frac{\frac{\partial E\Pi_{12}}{\partial X_2}\frac{\partial E\Pi_{21}}{\partial X_1}}{\frac{\partial E\Pi_{11}}{\partial X_1}\frac{\partial E\Pi_{22}}{\partial X_2}}. \tag{14}$$

The term in (14) evaluates the geometric "average" of demand shocks on exporters, $\frac{\partial E\Pi_{12}}{\partial X_2}\cdot\frac{\partial E\Pi_{21}}{\partial X_1}$, against the geometric average of demand shocks on home firms $\frac{\partial E\Pi_{11}}{\partial X_1}\cdot\frac{\partial E\Pi_{22}}{\partial X_2}$. Within the parameterized model, this comparison will be a function of the effective size of the shock through relative demand, and the elasticity of entering firms through the Pareto skew parameter. If skew parameters are the same across countries, then the demand shock in country 1 is effectively the same for country 1 and 2, and the term in (14) is determined by whether or not tariffs mute the effect of the demand shock on exporters. However, if country 2 has an inferior skew of the productivity distribution, the effective demand shock will not be the same due to the elasticity of entry being larger in country 2. A similar issue is also present for any subsequent shocks in country 2, and hence, the question is how the relative shock in each market interacts with the elasticity of entry for exporters and domestic firms.

Continuing, suppose for example we are in equilibrium type A, where $1>\rho_{12}\rho_{21}\left(\frac{s_2}{s_1}\right)^{k_2-k_1}\left(\frac{X_1}{X_2}\right)^{\lambda-1}$. This condition is satisfied when tariffs are relatively high (ρ's low), internal taxes are relatively low in country 2, and the market is relatively competitive in country 1 (X_1 is small compared to X_2). We discuss each in-turn after the main propositions detailed in the next section. But, supposing that this is satisfied, the increase in competitiveness subsequent to liberalization has a larger effect on profits in country 1, thereby reducing the number of entrants into country 1. The opposite happens in country 2, where additional firms enter by virtue of higher export profits and lower competition from country 1 firms. This cycle continues until a new equilibrium is reached, resulting in a higher X_1 and lower X_2. However, when the economy is in equilibrium B, the initial increase in competitiveness in 1 has a larger effect on firms in country 2, and the cycle is reversed, resulting in a lower X_1 and higher X_2.

Given the contrasting results in Lemma 2, this begs the question as to the conditions under which Type A and Type B equilibria can arise. I now derive the precise conditions such that we are either in an equilibrium of type A or B as a function of when there is a positive number of entrants in each country.

3.2 Equilibrium "Selection"

In Lemma 2, it was proven that the effects of liberalization/subsidization depend on whether the equilibrium is of type A or B. In analyzing the equilibrium effects of tariffs, I assumed that there were positive numbers of entrants in both countries such that the free entry conditions in equations (10) and (11) were relevant. Now precisely evaluating when there are a positive number of entering firms, I begin by presenting the problem generally, and then evaluate polar cases as a function of tariffs, country specific taste differences, and finally, country-specific sales taxes.

To derive the conditions on positive entry in each country, we first rearrange (9) for country 1 as follows:

$$\frac{\theta_1 - A_1}{\eta} = N_1 \frac{A_1^{k_1+1}\left(k_1+2\right)b_1}{\gamma\delta_1 s_1^{k_1}} + N_2 \frac{A_1^{k_2+1}\left(k_2+2\right)b_2}{\gamma t_{21}^{k_2} s_1^{k_2}\delta_2}$$

Multiplying both sides by A_1 and $s_1^{k_1}$ imposes our change of variables to get X_1 on the right-hand side.

$$\frac{\theta_1 - A_1}{\eta} A_1 s_1^{k_1} = N_1 \frac{X_1\left(k_1+2\right)b_1}{\gamma\delta_1} + N_2 \frac{X_1^{\lambda}\left(k_2+2\right)b_2}{\gamma t_{21}^{k_2} s_1^{k_2-k_1}\delta_2} \tag{15}$$

A similar equation can be written for country 2:

$$\frac{\theta_2 - A_2}{\eta} A_2 s_2^{k_2} = N_1 \frac{X_2\left(k_1+2\right)b_1}{\gamma t_{12}^{k_1} s_2^{k_1-k_2}\delta_1} + N_2 \frac{X_2^{\lambda}\left(k_2+2\right)b_2}{\gamma t_{21}^{k_2}\delta_2} \tag{16}$$

Defining $Y_j = \frac{\theta_j - A_j}{\eta} A_j s_j^{k_j}$, we can write (15) and (16) in matrix form:

$$\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = \begin{pmatrix} X_1 & \frac{X_1^{\lambda}}{t_{21}^{k_2} s_1^{k_2-k_1}} \\ \frac{X_2}{t_{12}^{k_1} s_2^{k_1-k_2}} & X_2^{\lambda} \end{pmatrix} \cdot \begin{pmatrix} N_1 \frac{(k_1+2)b_1}{\gamma\delta_1} \\ N_2 \frac{(k_2+2)b_2}{\gamma\delta_2} \end{pmatrix} \tag{17}$$

Inverting the square matrix on the right-hand side of (17) yields the following solutions for N_1 and N_2.

$$N_1 = \frac{\gamma\delta_1}{(k_1+2)\, b_1 X_1 X_2^{\lambda}\left(1 - \frac{1}{t_{12}^{k_1}}\frac{1}{t_{21}^{k_2}}\left(\frac{s_2}{s_1}\right)^{k_2-k_1}\left(\frac{X_1}{X_2}\right)^{\lambda-1}\right)}\left(Y_1 X_2^{\lambda} - Y_2 \frac{X_1^{\lambda}}{t_{21}^{k_2} s_1^{k_2-k_1}}\right) \tag{18}$$

$$N_2 = \frac{\gamma\delta_2}{(k_2+2)\, b_2 X_1 X_2^{\lambda}\left(1 - \frac{1}{t_{12}^{k_1}}\frac{1}{t_{21}^{k_2}}\left(\frac{s_2}{s_1}\right)^{k_2-k_1}\left(\frac{X_1}{X_2}\right)^{\lambda-1}\right)}\left(-Y_1 \frac{X_2}{t_{12}^{k_1} s_2^{k_1-k_2}} + Y_2 X_1\right) \tag{19}$$

In the denominator of both N_1 and N_2 we have the term $\left(1 - \frac{1}{t_{12}^{k_1}}\frac{1}{t_{21}^{k_2}}\left(\frac{s_2}{s_1}\right)^{k_2-k_1}\left(\frac{X_1}{X_2}\right)^{\lambda-1}\right)$, which is the determinant of the Jacobian in (17). This is a similar condition, though not identical to the condition that selects between equilibria A and B (they would be identical if tariffs were instead iceberg costs). By evaluating the solutions to N_1 and N_2, we have the following necessary and sufficient conditions for a positive number of entrants in each country.

Lemma 3 $N_1 > 0$ *and* $N_2 > 0$ *if and only if either of the following conditions hold:*

$$\left(\frac{X_1}{X_2}\right)^{\lambda}\frac{1}{t_{21}^{k_2} s_1^{k_2-k_1}} < \frac{Y_1}{Y_2} < t_{12}^{k_1} s_2^{k_1-k_2}\frac{X_1}{X_2} \tag{20}$$

$$t_{12}^{k_1} s_2^{k_1-k_2}\frac{X_1}{X_2} < \frac{Y_1}{Y_2} < \left(\frac{X_1}{X_2}\right)^{\lambda}\frac{1}{t_{21}^{k_2} s_1^{k_2-k_1}} \tag{21}$$

Proof. See Appendix ■

Lemma 3 essentially traces out a "cone" of diversification in the differentiated industry which depends on policy parameters and demand levels. Note that parameters such as the upper bound on the productivity distribution, again, do not affect the number of firms (outside of determining the demand level from the free entry loci). To build intuition about what these conditions imply, consider the case in which $\lambda = 1$, and hence, the skew of productivity in each country are identical. In this case, the conditions in Lemma 3 become:

$$\frac{1}{t_{21}^{k_2}} < \frac{(\theta_1 - A_1)\, A_2^k s_1^k}{(\theta_2 - A_2)\, A_1^k s_2^k} < t_{12}^{k_1} \tag{22}$$

$$t_{12}^{k_1} < \frac{(\theta_1 - A_1)\, A_2^k s_1^k}{(\theta_2 - A_2)\, A_1^k s_2^k} < \frac{1}{t_{21}^{k_2}} \tag{23}$$

Obviously, equation (22) is only relevant when $\frac{1}{t_{21}^{k_2}} < t_{12}^{k_1}$ (net tariffs in serving both countries), and (23) when the exact opposite holds (net subsidies in serving both countries). Equally important within the inequality is the fraction $\frac{(\theta_1 - A_1)A_2^k s_1^k}{(\theta_2 - A_2)A_1^k s_2^k}$, which gives us the parameters and endogenous variables that cannot be too different. Specifically, holding θ's fixed, if demand levels or sales taxes are too different, one of the countries will have an equilibrium negative number of entering firms, which implies that this country will be specialized in the numeraire good. Or put differently, when holding A's and sales taxes fixed, select values of θ's in each country can yield a diversified equilibrium.

When allowing for variation in the skew parameters of productivity across countries, the answer becomes decidedly more complex since the upper and lower bounds on the "cone" also depend on the demand levels and levels of domestic taxation in each country. To build intuition over a variety of the cases allowing for this heterogeneity across countries, I now evaluate the conditions under which Lemma 3 leads to a unique equilibrium as a function of tariffs, tastes, and sales taxes, and the response to tariffs within each case.

4 Tariff liberalization and Domestic Characteristics

4.1 Tariffs and Subsidies

I am now in a position to prove the three main results in the paper. The first is related to tariffs, and is summarized in the following Proposition.

Proposition 1 *Suppose that $\theta_l = \theta$ and $s_l = 1$ for all l. If $t_1 > 1$ and $t_2 > 1$, then any trading equilibrium must be of type A, and unilateral tariff liberalization by either country yields a Metzler-type paradox. If in contrast $t_1 < 1$ and $t_2 < 1$, then any trading equilibrium must be of type B, and a Metzler-type paradox does not occur.*

Proof. See Appendix ■

The proof for Proposition 1 follows via contradiction, where I show that there exists no parameter values such that (20) or (21) is satisfied under a type B equilibrium after assuming that $t_1 > 1$ and $t_2 > 1$. However, the intuition for this result follows a notion of internal consistency between the extent of tariff protection and the equilibrium response of entry to demand shocks. As discussed above, the distinguishing characteristics between type A and type B equilibria can be explained by the response of entry to demand shocks. If the demand response in country j affects country j firms more than country l firms, then a type A equilibrium exists. Crucially, this equilibrium response is internally consistent when countries protect their home firms from foreign export competition, where any demand shock at home is absorbed fully by home firms and partially by foreign firms (at a percentage of the tariff). The Melitz-type Metzler paradox arises in this case, where additional protection provides a positive demand shock for home firms, thereby increasing entry at home and reducing entry abroad. However, this logic only follows when the demand shock resulting from the increased tariff protection is effectively larger for home firms. When there is net subsidization of some type (for example, $t_1 < 1$ and $t_2 < 1$), this logic flips with foreign firms benefiting to a larger degree from the secondary demand shock resulting from increased protection. Hence, for this case, the Melitz-type Metzler paradox does not arise.

In term of the existing literature, Proposition 1 is similar to Venables (1985), Bagwell and Staiger (2009), Ossa (2010), and Bagwell and Staiger (2012). However, in the presence of a net-subsidy, the effective cost of serving each domestic market is less for exporters than domestic firms, and the result flips.

4.2 Taste Differences

The second result abstracts from the size tariffs, but allowing for taste heterogeneity across countries. In the model, I allow for exogenous taste differences in terms of the level of substitution between the numeraire and the differentiated industry.[5] Subject to these exogenous differences in tastes, the equilibrium of the model is characterized in the following proposition.

Proposition 2 *Suppose that $t_l = 1$ and $s_l = 1$ for all l. There exists a range of $\theta_2 > \theta_1$ such that any trading equilibrium must be of type A, and unilateral tariff liberalization by either country (from free trade) yields a Metzler-type paradox. If contrast, there exists a range of $\theta_1 > \theta_2$ such that any trading equilibrium must be of type B, and a Metzler-type paradox does not occur.*

Proof. See Appendix ■

[5] Of course, these taste differences might naturally relate to income differences (as in Simonovska (2010)), though this is left for a topic of future research.

The implications of this Proposition are fairly provocative. The difference between type A and type B equilibria essentially boils down to the relative competitiveness of country 1 to country 2 in the differentiated industry. That is, when all taxes and tariffs are zero, the condition for Type B equilibria is $X_1 > X_2$. In other words, the more productive country has the *less* competitive differentiated industry. Though this may seem paradoxical, as detailed in Proposition 2, a less productive country can have a more competitive differentiated sector given that competitiveness as reflected in the demand level is in part a function of the intrinsic taste for the differentiated industry. In terms of Lemma 2, the Metzler paradox is eliminated when the geometric average shock to export market profits is larger than the average shock to domestic markets profits. The shock to export markets relative to home markets for country 2 firms is $\frac{dX_1}{dX_2}\left(\frac{X_1}{X_2}\right)^{\lambda-1}$, and for country 1, $\frac{dX_1}{dX_2}$. Since $X_1 > X_2$ and $\lambda > 1$, then $\left(\frac{X_1}{X_2}\right)^{\lambda-1} > 1$, and the Metzler-type paradox is overturned.

4.3 Sales Tax Differences and Intra-national Mark-ups

The third result relates to the level of sales taxes in each country. At first, the role of sales taxes might seem innocuous since they affect all sales in a given country equally. However, the effect of sales taxes on the extensive margin differs according to the shape of the Pareto distribution in each country. Alternatively, even if countries do not levy explicit sales taxes, the modeling setup can be viewed as one with a constant mark-up via the distribution sector. This mark-up could be due to costs of delivery due to internal geography, or imperfect competition through a concentrated intermediary sector. The point is that there is a cost to take *all* products to the final consumer that drives a wedge between supplied prices to the market and supplied prices to the consumer. The role of these internal taxes is summarized in the following Proposition.

Proposition 3 *Suppose that $t_l = 1$ and $\theta_l = \theta$ for all l. If $s_1 > s_2$, then there exists feasible values of the θ such that any trading equilibrium must be of type A with $X_1 > X_2$, and unilateral tariff liberalization by either country yields a Metzler-type paradox. In contrast, if $s_2 > s_1$, then there exists feasible values of θ such that any trading equilibrium must be of type B with $X_2 > X_1$, and a Metzler-type paradox does not occur.*

Proof. See Appendix ■

An natural and intuitive result in Proposition 3 is that in any diversified equilibrium, the less competitive differentiated market (higher X) is the market with the higher internal tax. However, depending on which country has the higher internal tax, there may or may not be a Metzler-type paradox. The subtle intuition behind this result is similar to Proposition 2, though one must adjust for the fact that the demand shock in each country is mitigated for all firms by the internal tax. In this case, the effective market in country 1 is larger than country 2 when accounting for the internal tax, and the relative export to domestic shock for country 2 firms is larger than for country 1. Hence, the Metzler-paradox is overturned.

5 Conclusion

This paper has evaluated the effect of tariffs when allowing for variation in a broad set of domestic characteristics, especially the skew of the productivity distribution. The analysis shows that the characterization of long-run free entry becomes complicated with productivity heterogeneity, even with relatively small amounts of heterogeneity in the skew parameter. The effects of tariffs on competitiveness differ across these solutions.

In the two country model, a Metzler-type paradox arises when evaluating tariffs in the neighborhood of free trade if the country with the better skew of firms (toward lower costs) has a lower taste for the differentiated industry. In contrast, when the the country with better firms also has a higher ex-ante valuation of the differentiated industry, then the Metzler-paradox vanishes. Finally, I show that the level of a intra-national mark-up which may result from taxes or distribution sector heterogeneity can also yield similar effects. Overall, the results detail how heterogeneity in domestic characteristics can dictate the qualitative impact of tariff liberalization.

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A Proofs

A.1 Lemma 1

Note that intercepts for the FE condition for country 1 are written as,

$$\begin{aligned} X_1^1 &= s_1^{k_1+1} F\delta_1 \\ X_2^1 &= s_2^{k_1+1} \frac{F\delta_1}{\rho_{12}}, \end{aligned}$$

and for country 2

$$\begin{aligned} X_1^2 &= \left(s_1^{k_2+1} \frac{F\delta_2}{\rho_{21}}\right)^{1/\lambda} \\ X_2^2 &= \left(s_2^{k_2+1} F\delta_2\right)^{1/\lambda}. \end{aligned}$$

For a unique equilibrium, since both loci are downward sloping we need the ranking of the intercepts to switch from axis to axis. For example,

$$s_1^{k_1+1} F\delta_1 < \left(s_1^{k_2+1} \frac{F\delta_2}{\rho_{21}}\right)^{1/\lambda}$$
$$\left(s_2^{k_2+1} F\delta_2\right)^{1/\lambda} < s_2^{k_1+1} \frac{F\delta_1}{\rho_{12}}$$

would yield a case when the X_1 intercept for country 1 is smaller than for country 2, but this ranking switch on the X_2 axis. Putting these two conditions together, we have:

$$s_2^{\frac{\lambda-1}{\lambda}} \rho_{12} < F^{\frac{\lambda-1}{\lambda}} \frac{\delta_1}{\delta_2^{\frac{1}{\lambda}}} < s_1^{\frac{\lambda-1}{\lambda}} \left(\frac{1}{\rho_{21}}\right)^{\frac{1}{\lambda}}$$

The opposite case occurs if:

$$s_1^{\frac{\lambda-1}{\lambda}} \left(\frac{1}{\rho_{21}}\right)^{\frac{1}{\lambda}} < F^{\frac{\lambda-1}{\lambda}} \frac{\delta_1}{\delta_2^{\frac{1}{\lambda}}} < s_2^{\frac{\lambda-1}{\lambda}} \rho_{12}$$

Hence, the condition which is feasible depends on the whether ρ_{21} and ρ_{12} are less than or greater than one. Hence, the composite condition is:

$$\min\left\{s_2^{\frac{\lambda-1}{\lambda}}\rho_{12}, s_1^{\frac{\lambda-1}{\lambda}}\left(\frac{1}{\rho_{21}}\right)^{\frac{1}{\lambda}}\right\} < F^{\frac{\lambda-1}{\lambda}}\frac{\delta_1}{\delta_2^{\frac{1}{\lambda}}} < \max\left\{s_2^{\frac{\lambda-1}{\lambda}}\rho_{12}, s_1^{\frac{\lambda-1}{\lambda}}\left(\frac{1}{\rho_{21}}\right)^{\frac{1}{\lambda}}\right\}$$

For the case with multiple solutions with free entry conditions, since the free entry condition for country 2 is bowed out ($\lambda > 1$), two conditions need to be satisfied. First, the X_1 and X_2 intercepts for the country 2 free entry condition must be below that of country 1. This occurs if:

$$\begin{aligned}
\left(s_1^{k_2+1}\frac{F\delta_2}{\rho_{21}}\right)^{1/\lambda} &< s_1^{k_1+1}F\delta_1 \\
\left(s_2^{k_2+1}F\delta_2\right)^{1/\lambda} &< s_2^{k_1+1}\frac{F\delta_1}{\rho_{12}} \\
\Rightarrow \quad \max\left\{s_2^{\frac{\lambda-1}{\lambda}}\rho_{12}, s_1^{\frac{\lambda-1}{\lambda}}\left(\frac{1}{\rho_{21}}\right)^{\frac{1}{\lambda}}\right\} &< F^{\frac{\lambda-1}{\lambda}}\frac{\delta_1}{\delta_2^{\frac{1}{\lambda}}}
\end{aligned}$$

The second condition is that at some values of X_1 and X_2, the country 2 locus is placed toward higher values of X_1 and X_2 relative to the country 1 locus. To evaluate when this happens, we calculate the values of each free entry curve at the point of tangency between the free entry loci. Precisely, for country 1 free entry, the value of X_2 such that there is tangency between the two free entry loci is:

$$X_2^{T1} = \frac{s_2^{k_1+2}F\delta_1}{s_2\rho_{12} + s_1\left(\frac{1}{\rho_{12}\rho_{21}}\right)^{\frac{1}{\lambda-1}}}$$

Similarly for country 2 free entry, the value of X_2 such that there is tangency between the two free entry loci is:

$$X_2^{T2} = \frac{s_2^{k_1+2}\left(F\delta_2\right)^{\frac{1}{\lambda}}}{\left(s_2 + s_1\rho_{21}\left(\frac{1}{\rho_{12}\rho_{21}}\right)^{\frac{1\lambda}{\lambda-1}}\right)^{\frac{1}{\lambda}}}$$

If $X_2^{T2} > X_2^{T1}$, then it must be the case that at the tangency point between country 1 and 2 free entry loci, the country 2 locus is placed toward higher values of X_1 and X_2.

$$F^{\frac{\lambda-1}{\lambda}}\frac{\delta_1}{\delta_2^{\frac{1}{\lambda}}} < \frac{s_2\rho_{12} + s_1\left(\frac{1}{\rho_{12}\rho_{21}}\right)^{\frac{1}{\lambda-1}}}{\left(s_2 + s_1\rho_{21}\left(\frac{1}{\rho_{12}\rho_{21}}\right)^{\frac{\lambda}{\lambda-1}}\right)^{\frac{1}{\lambda}}}$$

Putting both conditions together, there are multiple solutions to the free-entry equilibrium if:

$$\max\left\{s_2^{\frac{\lambda-1}{\lambda}}\rho_{12}, s_1^{\frac{\lambda-1}{\lambda}}\left(\frac{1}{\rho_{21}}\right)^{\frac{1}{\lambda}}\right\} < F^{\frac{\lambda-1}{\lambda}}\frac{\delta_1}{\delta_2^{\frac{1}{\lambda}}} < \frac{s_2\rho_{12} + s_1\left(\frac{1}{\rho_{12}\rho_{21}}\right)^{\frac{1}{\lambda-1}}}{\left(s_2 + s_1\rho_{21}\left(\frac{1}{\rho_{12}\rho_{21}}\right)^{\frac{\lambda}{\lambda-1}}\right)^{\frac{1}{\lambda}}}$$

A.2 Lemma 2

Recall that the Free entry conditions are written as:

$$\begin{aligned} X_1 + X_2 \cdot \rho_{12}\frac{s_1^{k_1+1}}{s_2^{k_1+1}} &= F\delta_1 s_1^{k_1+1} \\ X_1^{\lambda} \cdot \rho_{21}\frac{s_2^{k_2+1}}{s_1^{k_2+1}} + X_2^{\lambda} &= F\delta_2 s_2^{k_2+1} \end{aligned}$$

Differentiating with respect to ρ_{21}

$$\begin{aligned} \frac{\partial X_1}{\partial \rho_{21}} + \frac{\partial X_2}{\partial \rho_{21}} \cdot \rho_{12}\frac{s_1^{k_1+1}}{s_2^{k_1+1}} &= 0 \\ \frac{\partial X_1}{\partial \rho_{21}} X_1^{\lambda-1} \cdot \rho_{21}\frac{s_2^{k_2+1}}{s_1^{k_2+1}} + \frac{\partial X_2}{\partial \rho_{21}} X_2^{\lambda-1} &= -\frac{1}{\lambda} X_1^{\lambda}\frac{s_2^{k_2+1}}{s_1^{k_2+1}} \end{aligned}$$

Putting in matrix form:

$$\begin{pmatrix} 1 & \rho_{12}\frac{s_1^{k_1+1}}{s_2^{k_1+1}} \\ X_1^{\lambda-1} \cdot \rho_{21}\frac{s_2^{k_2+1}}{s_1^{k_2+1}} & X_2^{\lambda-1} \end{pmatrix} \cdot \begin{pmatrix} \frac{\partial X_1}{\partial \rho_{21}} \\ \frac{\partial X_2}{\partial \rho_{21}} \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{1}{\lambda} X_1^{\lambda}\frac{s_2^{k_2+1}}{s_1^{k_2+1}} \end{pmatrix}$$

inverting

$$\begin{pmatrix} \frac{\partial X_1}{\partial \rho_{21}} \\ \frac{\partial X_2}{\partial \rho_{21}} \end{pmatrix} = \frac{1}{X_2^{\lambda-1} - \rho_{12}\rho_{21} \frac{s_2^{k_2+1}}{s_1^{k_2+1}} \frac{s_1^{k_1+1}}{s_2^{k_1+1}} X_1^{\lambda-1}} \begin{pmatrix} X_2^{\lambda-1} & -\rho_{12} \frac{s_1^{k_1+1}}{s_2^{k_1+1}} \\ -X_1^{\lambda-1} \cdot \rho_{21} \frac{s_2^{k_2+1}}{s_1^{k_2+1}} & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -\frac{1}{\lambda} X_1^{\lambda} \frac{s_2^{k_2+1}}{s_1^{k_2+1}} \end{pmatrix}$$

$$= \frac{X_2^{1-\lambda}}{1 - \rho_{12}\rho_{21} \left(\frac{s_2}{s_1}\right)^{k_2-k_1} \left(\frac{X_1}{X_2}\right)^{\lambda-1}} \begin{pmatrix} \frac{\rho_{12}}{\lambda} X_1^{\lambda} \left(\frac{s_2}{s_1}\right)^{k_2-k_1} \\ -\frac{1}{\lambda} X_1^{\lambda} \left(\frac{s_2}{s_1}\right)^{k_2+1} \end{pmatrix}$$

As detailed in Definition 1, if $1 - \rho_{12}\rho_{21} \left(\frac{s_2}{s_1}\right)^{k_2-k_1} \left(\frac{X_1}{X_2}\right)^{\lambda-1} > 0$, then we have a type A equilibrium. In this case, liberalization by country 1 (higher ρ_{21}) decreases competitiveness is 1, and increasing competitiveness in 2 ($\frac{\partial X_1}{\partial \rho_{21}} > 0$, $\frac{\partial X_2}{\partial \rho_{21}} < 0$). On the other hand, if we have a type B equilibrium, $1 - \rho_{12}\rho_{21} \left(\frac{s_2}{s_1}\right)^{k_2-k_1} \left(\frac{X_1}{X_2}\right)^{\lambda-1} < 0$ and the results are switched.

A.3 Lemma 3

Recall the equations for the number of entering firms.

$$N_1 = \frac{\gamma\delta_1}{(k_1+2)\, b_1 X_1 X_2^{\lambda} \left(1 - \frac{1}{t_{12}^{k_1}} \frac{1}{t_{21}^{k_2}} \left(\frac{X_1}{X_2}\right)^{\lambda-1}\right)} \left(Y_1 X_2^{\lambda} - Y_2 \frac{X_1^{\lambda}}{t_{21}^{k_2}}\right)$$

$$N_2 = \frac{\gamma\delta_2}{(k_2+2)\, b_2 X_1 X_2^{\lambda} \left(1 - \frac{1}{t_{12}^{k_1}} \frac{1}{t_{21}^{k_2}} \left(\frac{X_1}{X_2}\right)^{\lambda-1}\right)} \left(-Y_1 \frac{X_2}{t_{12}^{k_1}} + Y_2 X_1\right)$$

To prove sufficiency, first consider (20):

$$\left(\frac{X_1}{X_2}\right)^{\lambda} \frac{1}{t_{21}^{k_2}} < \frac{Y_1}{Y_2} < t_{12}^{k_1} \frac{X_1}{X_2},$$

This condition is derived from $Y_1 X_2^{\lambda} > Y_2 \frac{X_1^{\lambda}}{t_{21}^{k_2}}$ and $Y_2 X_1 > Y_1 \frac{X_2}{t_{12}^{k_1}}$. Further, this condition requires that $\left(\frac{X_1}{X_2}\right)^{\lambda} \frac{1}{t_{21}^{k_2}} < t_{12}^{k_1} \frac{X_1}{X_2}$, which is rearranged as $1 > \frac{1}{t_{12}^{k_1}} \frac{1}{t_{21}^{k_2}} \left(\frac{X_1}{X_2}\right)^{\lambda-1}$. Together this implies that $N_1 > 0$ and $N_2 > 0$.

To prove sufficiency via (20),

$$t_{12}^{k_1}\frac{X_1}{X_2} < \frac{Y_1}{Y_2} < \left(\frac{X_1}{X_2}\right)^{\lambda}\frac{1}{t_{21}^{k_2}},$$

this condition is derived from $Y_1X_2^{\lambda} < Y_2\frac{X_1^{\lambda}}{t_{21}^{k_2}}$ and $Y_2X_1 < Y_1\frac{X_2}{t_{12}^{k_1}}$. Further, this condition requires that $\left(\frac{X_1}{X_2}\right)^{\lambda}\frac{1}{t_{21}^{k_2}} > t_{12}^{k_1}\frac{X_1}{X_2}$, which is rearranged as $1 < \frac{1}{t_{12}^{k_1}}\frac{1}{t_{21}^{k_2}}\left(\frac{X_1}{X_2}\right)^{\lambda-1}$. Together this implies that $N_1 > 0$ and $N_2 > 0$.

To show that these are also necessary conditions, suppose that $N_1 > 0$ and $N_2 > 0$. This requires that if $1 > \frac{1}{t_{12}^{k_1}}\frac{1}{t_{21}^{k_2}}\left(\frac{X_1}{X_2}\right)^{\lambda-1}$, then $Y_1X_2^{\lambda} > Y_2\frac{X_1^{\lambda}}{t_{21}^{k_2}}$ and $Y_2X_1 > Y_1\frac{X_2}{t_{12}^{k_1}}$. As above, these two inequalities yield (20). On the other hand, if $1 < \frac{1}{t_{12}^{k_1}}\frac{1}{t_{21}^{k_2}}\left(\frac{X_1}{X_2}\right)^{\lambda-1}$, then $Y_1X_2^{\lambda} < Y_2\frac{X_1^{\lambda}}{t_{21}^{k_2}}$ and $Y_2X_1 < Y_1\frac{X_2}{t_{12}^{k_1}}$, which yield (21). This completes the proof.

A.4 Proposition 1

We complete the proof by contradiction. First, suppose that $t_{1,2} > 1$ and $t_{2,1} > 1$. The Proposition indicates that any equilibria subject to these parameter restrictions must be of type A. Suppose not, where we have an equilibrium of type B subject to these parameter restrictions. By the definition of equilibrium of type B, $t_{1,2} > 1$ and $t_{2,1} > 1$ implies that $1 < \frac{A_1}{A_2} < \frac{X_1}{X_2} < \left(\frac{X_1}{X_2}\right)^{\lambda}$.

Next, recall the two conditions that (individually) yield a positive number of entering firms in each country

$$\begin{aligned}\left(\frac{X_1}{X_2}\right)^{\lambda}\frac{1}{t_{2,1}^{k_2}} &< \frac{Y_1}{Y_2} < t_{1,2}^{k_1}\frac{X_1}{X_2}\\ t_{1,2}^{k_1}\frac{X_1}{X_2} &< \frac{Y_1}{Y_2} < \left(\frac{X_1}{X_2}\right)^{\lambda}\frac{1}{t_{2,1}^{k_2}}\end{aligned}$$

Taking the first condition, multiplying the left and right inequalities yields together yields $\left(\frac{X_1}{X_2}\right)^{\lambda-1}\frac{1}{t_{1,2}^{k_1}t_{2,1}^{k_2}} < 1$, which violates the condition for being in a type B equilibrium. Doing the same for the second condition yields the opposite inequality, which includes equilibrium values defined by the space of type B equilibria.

Imposing the values of Y_1 and Y_2, we get:

$$t_{1,2}^{k_1}\frac{X_1}{X_2} \quad < \quad \frac{(\theta - A_1)\,A_1}{(\theta - A_2)\,A_2} < \left(\frac{X_1}{X_2}\right)^{\lambda}\frac{1}{t_{2,1}^{k_2}}$$

The question now becomes is there value of $\theta > \max\{A_1, A_2\}$ such that this equality can be satisfied? To see whether this is the case, differentiate $\frac{(\theta-A_1)A_1}{(\theta-A_2)A_2}$ with respect to θ to get:

$$\frac{\partial \frac{Y_1}{Y_2}}{\partial \theta} = \frac{(A_1 - A_2)\,A_1}{(\theta - A_2)^2\,A_2}$$

Given the parameter restrictions yield $A_1 > A_2$, this derivative is positive. Further, the limit of $\lim_{\theta\to\infty}\frac{Y_1}{Y_2} = \frac{A_1}{A_2}$. So, if the limit of $\frac{Y_1}{Y_2}$ is above the lower bound of the inquality $t_{1,2}^{k_1}\frac{X_1}{X_2}$, then there exists a range of θ such that $t_{1,2} > 1$ and $t_{2,1} > 1$ supports a type B equilibrium.

$$t_{1,2}^{k_1}\frac{X_1}{X_2} < \frac{A_1}{A_2}$$

But, this condition cannot hold since $t_{1,2}^{k_1} > 1$ and $\frac{X_1}{X_2} = \left(\frac{A_1}{A_2}\right)^{k_1+2} > \frac{A_1}{A_2}$. Hence, we have a contradiction, and $t_{1,2} > 1$ and $t_{2,1} > 1$ can only support type A equilibria.

The proof for $t_{1,2} < 1$ and $t_{2,1} < 1$ follows the same logic, when noting that all type A equilibria require that $1 > \frac{A_1}{A_2} > \frac{X_1}{X_2} > \left(\frac{X_1}{X_2}\right)^{\lambda}$.

A.5 Proposition 2

First, restricting the model to free trade, $t_{1,2} = 1$ and $t_{2,1} = 1$, we find that type A equilibria occur when $1 > \left(\frac{X_1}{X_2}\right)^{\lambda-1}$, which implies that $\left(\frac{X_1}{X_2}\right)^{\lambda-1} < \frac{X_1}{X_2} < \frac{A_1}{A_2} < 1$. Again recalling the two conditions that (individually) yield a positive number of entering firms in each country

$$\left(\frac{X_1}{X_2}\right)^{\lambda} \quad < \quad \frac{Y_1}{Y_2} < \frac{X_1}{X_2}$$
$$\frac{X_1}{X_2} \quad < \quad \frac{Y_1}{Y_2} < \left(\frac{X_1}{X_2}\right)^{\lambda}$$

Since $\left(\frac{X_1}{X_2}\right)^{\lambda-1} < \frac{X_1}{X_2}$, only the first condition can be satisfied for type A equilibria. The question then becomes, under what conditions is $\frac{Y_1}{Y_2}$ within this range. Precisely,

$$\left(\frac{X_1}{X_2}\right)^{\lambda} \quad < \quad \frac{(\theta_1 - A_1)A_1}{(\theta_2 - A_2)A_2} < \frac{X_1}{X_2}$$

Dividing both bounds by A_1/A_2, we get:

$$\frac{\left(\frac{X_1}{X_2}\right)^{\lambda}}{\frac{A_1}{A_2}} \quad < \quad \frac{\theta_1 - A_1}{\theta_2 - A_2} < \underbrace{\frac{\frac{X_1}{X_2}}{\frac{A_1}{A_2}}}_{<1}$$

Since the right-hand bound is now below 1, $\frac{\theta_1 - A_1}{\theta_2 - A_2} < 1$ for there to be a positive number of firms entering in each country in equilibrium. Hence, since $\frac{A_1}{A_2} < 1.$, the only values of θ_1 and θ_2 that satisfy $\frac{\theta_1 - A_1}{\theta_2 - A_2} < 1$ are those such that $\theta_1 < \theta_2$.

Moving to type B equilibria, the conditions require that $1 < \left(\frac{X_1}{X_2}\right)^{\lambda-1}$, which implies $1 < \frac{A_1}{A_2} < \frac{X_1}{X_2}\left(\frac{X_1}{X_2}\right)^{\lambda-1}$. The only condition for a positive number of entering firms that is relevant is:

$$\frac{X_1}{X_2} \quad < \quad \frac{Y_1}{Y_2} < \left(\frac{X_1}{X_2}\right)^{\lambda}$$

Imposing the definition of $\frac{Y_1}{Y_2}$, and dividing both bounds by A_1/A_2, we get:

$$\underbrace{\frac{\frac{X_1}{X_2}}{\frac{A_1}{A_2}}}_{>1} \quad < \quad \frac{\theta_1 - A_1}{\theta_2 - A_2} < \frac{\left(\frac{X_1}{X_2}\right)^{\lambda}}{\frac{A_1}{A_2}}$$

Since $A_1 > A_2$, the only values of θ_1 and θ_2 that satisfy $\frac{\theta_1 - A_1}{\theta_2 - A_2} > 1$ are those such that $\theta_1 > \theta_2$

A.6 Proposition 3

First, recall the conditions for type A and type B equilibrium, respectively:

$$\left(\frac{s_2}{s_1}\right)^{k_2}\left(\frac{X_1}{X_2}\right)^{\lambda} < \frac{(\theta - A_1)\,A_1}{(\theta - A_2)\,A_2} < \left(\frac{s_2}{s_1}\right)^{k_1}\frac{X_1}{X_2}$$
$$\left(\frac{s_2}{s_1}\right)^{k_1}\frac{X_1}{X_2} < \frac{(\theta - A_1)\,A_1}{(\theta - A_2)\,A_2} < \left(\frac{s_2}{s_1}\right)^{k_2}\left(\frac{X_1}{X_2}\right)^{\lambda}$$

To prove proposition three, we first group the parameter and equilibrium space into four cases according to the relative values on X_1, X_2, s_2, and s_1. To begin, we consider in case in which $X_1 > X_2$ and $s_2 > s_1$. Clearly, type A equilibrium cannot occur under these circumstances since $\left(\frac{s_2}{s_1}\right)^{k_2}\left(\frac{X_1}{X_2}\right)^{\lambda} > \left(\frac{s_2}{s_1}\right)^{k_1}\frac{X_1}{X_2}$. Consider next a type B equilibrium, where clearly the bounds on the type B conditions, $\left(\frac{s_2}{s_1}\right)^{k_2}\left(\frac{X_1}{X_2}\right)^{\lambda} > \left(\frac{s_2}{s_1}\right)^{k_1}\frac{X_1}{X_2}$, are feasible. We now must evaluate whether $\frac{(\theta-A_1)A_1}{(\theta-A_2)A_2}$ can fall within this range. Since $X_1 > X_2$, this implies that $A_1 > A_2$. From the proof of Proposition 1 $A_1 > A_2$ implies that $\frac{(\theta-A_1)A_1}{(\theta-A_2)A_2} \in [0, \frac{A_1}{A_2}]$. However, since $\frac{X_1}{X_2} > \frac{A_1}{A_2}$, this implies that the lower bound of type B equilibria cannot be satisfied. Hence, $X_1 > X_2$ and $s_2 > s_1$ cannot be an equilibrium.

Next, consider the case in which $X_2 > X_1$ and $s_1 > s_2$, which follows a similar logic to above. Since $\left(\frac{s_2}{s_1}\right)^{k_2}\left(\frac{X_1}{X_2}\right)^{\lambda} < \left(\frac{s_2}{s_1}\right)^{k_1}\frac{X_1}{X_2}$, the only feasible equilibrium type is A. However, since $X_2 > X_1$, this implies that $A_2 > A_1$, and hence from Proposition 1 that $\frac{(\theta-A_1)A_1}{(\theta-A_2)A_2} \in [\frac{A_1}{A_2}, \infty]$. However, since $\frac{A_1}{A_2} > \left(\frac{s_2}{s_1}\right)^{k_1}\frac{X_1}{X_2}$, the upper-bound on the condition for type-A equilibria cannot be satisfied. Hence, $X_2 > X_1$ and $s_1 > s_2$ cannot be an equilibrium.

Next, consider the case in which $X_2 > X_1$ and $s_2 > s_1$. Suppose that a type A equilibrium can be supported by these values. If so, it must be the case that the upper-bound is greater than the lower-bound on the type-A condition, which requires that $\left(\frac{s_2}{s_1}\right)^{k_1}\frac{X_1}{X_2} > \left(\frac{s_2}{s_1}\right)^{k_2}\left(\frac{X_1}{X_2}\right)^{\lambda}$. Rewriting, we have $\frac{s_1}{s_2} > \frac{A_1}{A_2}$. Next, since $A_2 > A_1$, it must also be the case that $\frac{(\theta-A_1)A_1}{(\theta-A_2)A_2} \in [\frac{A_1}{A_2}, \infty]$. Hence, the upper bound on the type A condition must be above $\frac{A_1}{A_2}$; $\frac{X_1}{X_2}\left(\frac{s_2}{s_1}\right)^{k}_{1} > \frac{A_1}{A_2}$. This is simplified as $\left(\frac{A_1}{A_2}\right)^{k_1+1} > \left(\frac{s_1}{s_2}\right)^{k_1}$ Putting together the two conditions, we get:

$$\left(\frac{A_1}{A_2}\right)^{\frac{k_1+1}{k_1}} > \frac{s_1}{s_2} > \frac{A_1}{A_2}$$

But, since $\frac{k_1+1}{k_1} > 1$ and $\frac{A_1}{A_2} < 1$, this is not possible. Hence, type A equilibrium cannot arise when $X_2 > X_1$ and $s_2 > s_1$.

Now considering a type B equilibrium when $X_2 > X_1$ and $s_2 > s_1$, the conditions on the equilibrium type imply that $\frac{A_1}{A_2} > \frac{s_1}{s_2}$. Further, since $\frac{(\theta - A_1)A_1}{(\theta - A_2)A_2} \in [\frac{A_1}{A_2}, \infty]$, the upper bound on the type B condition must be above $\frac{A_1}{A_2}$. This requires that $\left(\frac{A_1}{A_2}\right)^{k_2+2}\left(\frac{s_2}{s_1}\right)^{k_2} > \frac{A_1}{A_2}$, or simplified, $\left(\frac{A_1}{A_2}\right)^{\frac{k_2+1}{k_2}} > \frac{s_1}{s_2} > \frac{A_1}{A_2}$. Hence, for a type B equlibrium, it must be the case that

$$\frac{s_1}{s_2} < \left(\frac{A_1}{A_2}\right)^{\frac{k_1+1}{k_1}} < \frac{A_1}{A_2}$$

which is feasible for some region of θ. Hence, $\mathbf{X_2 > X_1}$ **and** $\mathbf{s_2 > s_1}$ **is a feasible outcome of equilibrium type B.**

Finally, consider the case in which $X_1 > X_2$ and $s_1 > s_2$. As above, a type B equilibrium requires that $\frac{A_1}{A_2} > \frac{s_1}{s_2}$. Further, since $A_1 > A_2$, $\frac{(\theta - A_1)A_1}{(\theta - A_2)A_2} \in [0, \frac{A_1}{A_2}]$. Hence the lower bound of type B equilibrium must be below $\frac{A_1}{A_2}$, or more specifically, $\left(\frac{A_1}{A_2}\right)^{\frac{k_1+1}{k_1}} > \frac{s_1}{s_2}$

$$\left(\frac{A_1}{A_2}\right)^{\frac{k_1+1}{k_1}} < \frac{s_1}{s_2} < \frac{A_1}{A_2}$$

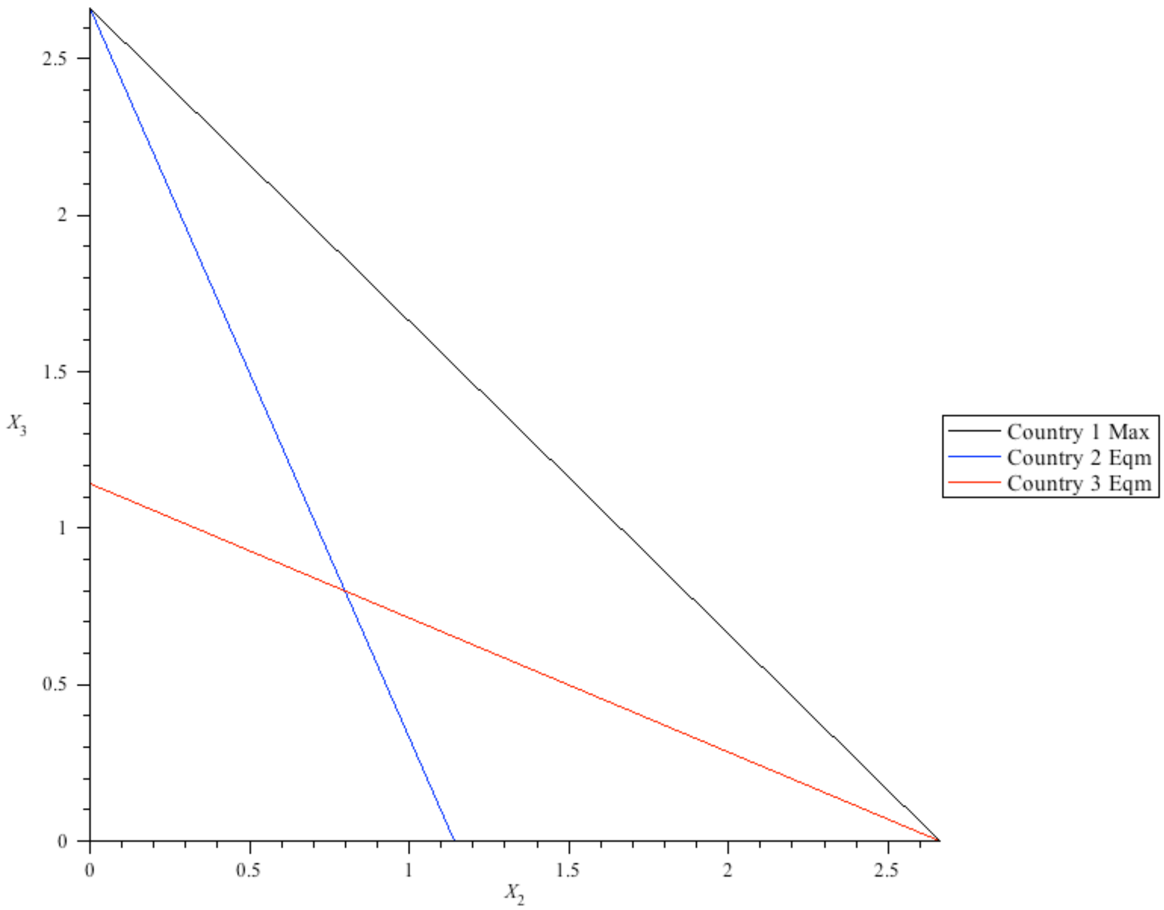
However, since $A_1 > A_2$, this cannot be an equilibrium.

Considering type A for $X_1 > X_2$ and $s_1 > s_2$, we must have from the definition of type A equilibrium that $\frac{s_1}{s_2} > \frac{A_1}{A_2}$. Further, since $A_1 > A_2$, $\frac{(\theta - A_1)A_1}{(\theta - A_2)A_2} \in [0, \frac{A_1}{A_2}]$. Hence, we must also have that the lower-bound on the condition for a type A equilibrium is below $\frac{A_1}{A_2}$; $\frac{s_1}{s_2} > \left(\frac{A_1}{A_2}\right)^{\frac{k_2+1}{k_2}}$ Putting the conditions together, we have

$$\frac{A_1}{A_2} < \left(\frac{A_1}{A_2}\right)^{\frac{k_2+1}{k_2}} < \frac{s_1}{s_2}$$

which is feasible for some region of θ. Hence, $\mathbf{X_1 > X_2}$ **and** $\mathbf{s_1 > s_2}$ **is a feasible outcome of equilibrium type A.**

Figure 2: Three-country Long-run Equilibrium - MO Model



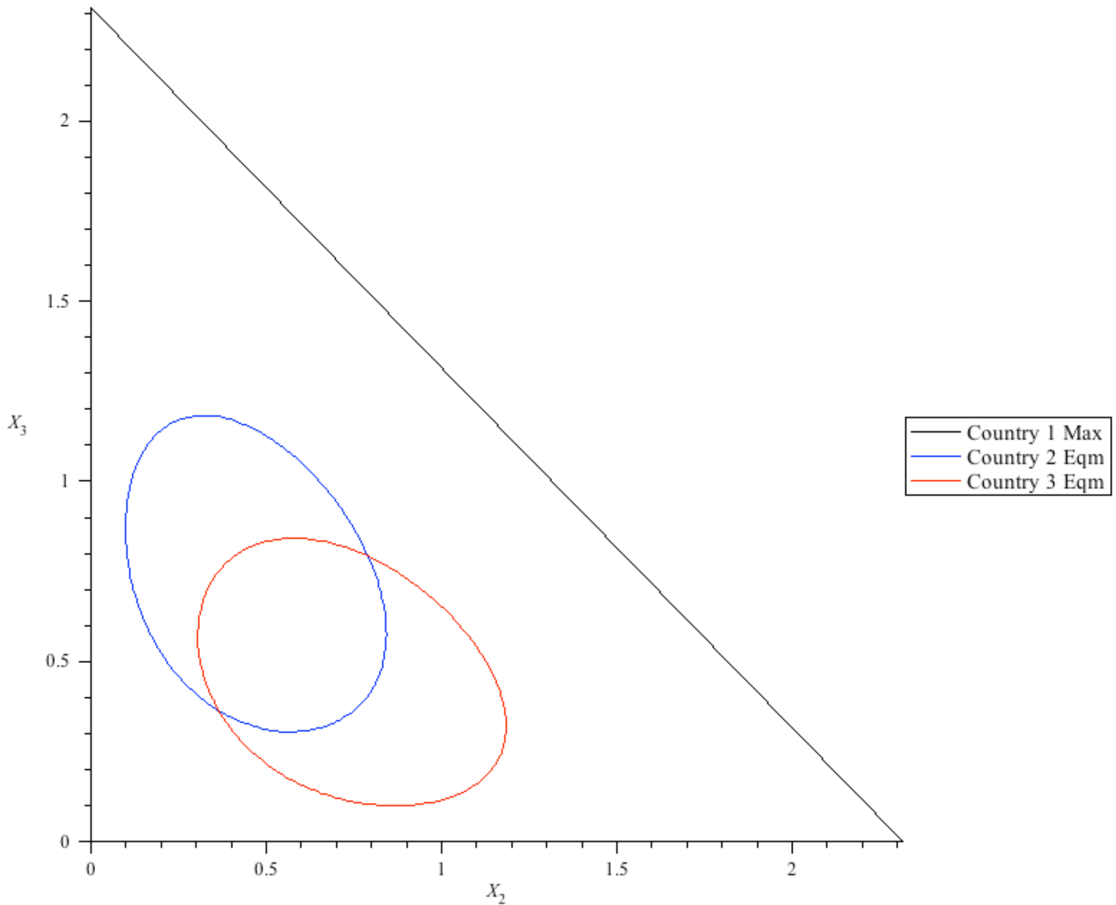
Notes: $\lambda_2 = 1.5, \lambda_3 = 1.5, t_1 = 1.05, t_2 = 1.05, t_3 = 1.05$

B Three-country model

In the two country model, we explored two equilibrium types and the parameter values which support them. In this section, we build on the two country model to evaluate a three country model, and in particular, whether the basic intuition regarding tastes can extend to a multi-country model with arbitrary productivity heterogeneity.

Within the three country setup, we again measure the skew of each country's productivity draws relative to country 1. Again, we define $\lambda_l = \frac{k_l+2}{k_1+2}$ for country l. Further, we allow for the fixed costs of entry to

Figure 3: Three-country Long-run Equilibrium - Similar to Two-country



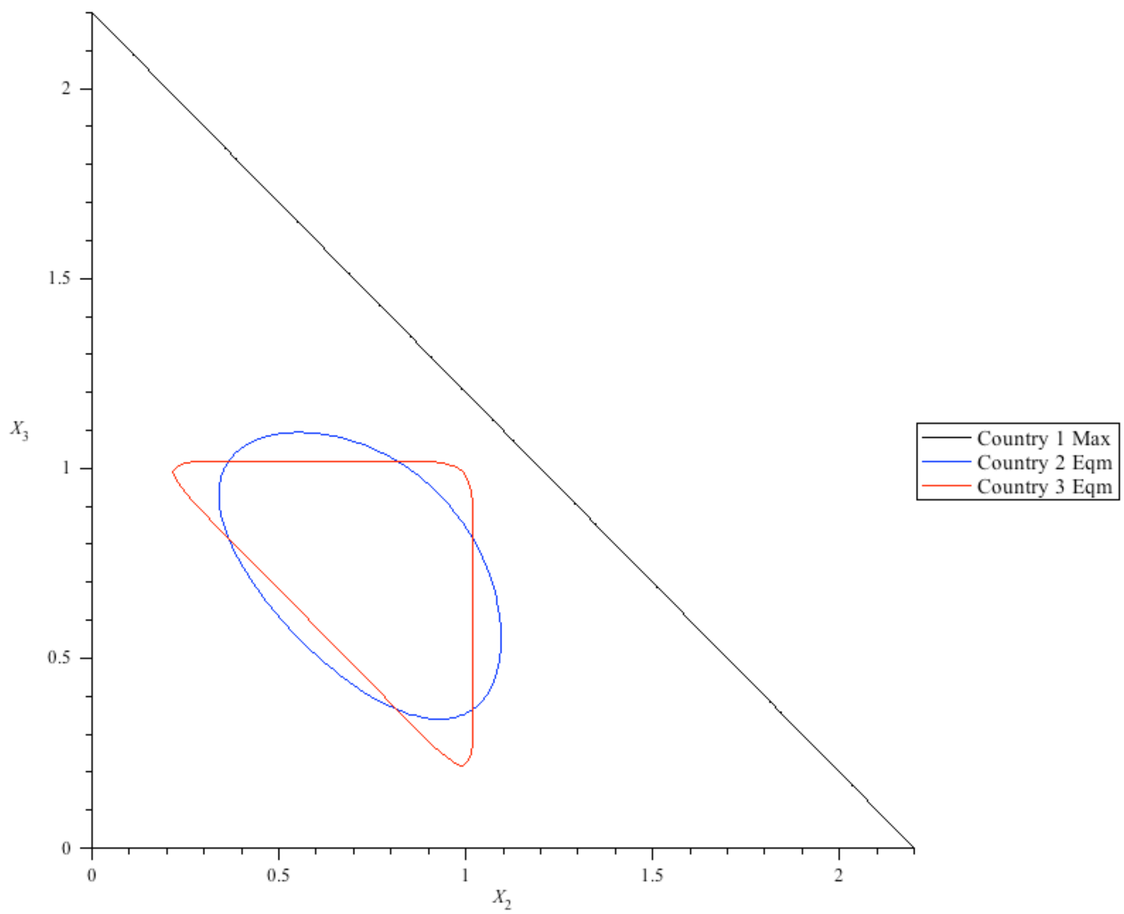
Notes: $\lambda_2 = 1.5, \lambda_3 = 1.5, t_1 = 1.05, t_2 = 1.05, t_3 = 1.05$

differ by country (which is helpful for the analysis). With these assumptions, the free entry conditions of countries one, two and three can be written as:

$$\begin{aligned} X_1 + X_2 \cdot \rho_{12} + X_3 \cdot \rho_{13} &= F_1\delta_1 \\ X_1^{\lambda_2} \cdot \rho_{21} + X_2^{\lambda_2} + X_3^{\lambda_2} \cdot \rho_{23} &= F_2\delta_2 \\ X_1^{\lambda_3} \cdot \rho_{31} + X_2^{\lambda_3} \cdot \rho_{32} + X_3^{\lambda_3} &= F_3\delta_3. \end{aligned}$$

To aid in a graphical analysis of equilibrium outcomes, we can rearranging the problem by solving for X_1

Figure 4: Three-country Long-run Equilibrium - Severe differences is skew



Notes: $\lambda_2 = 3, \lambda_3 = 25, t_1 = 1, t_2 = 1, t_3 = 1$

from the free entry condition for country one and substitute into the free-entry conditions for countries two and three. Doing so yields:

$$X_2^{\lambda_2} + X_3^{\lambda_2} \cdot \rho_{31} + (F_1\delta_1 - X_2 \cdot \rho_{12} + X_3 \cdot \rho_{13})^{\lambda_2} \cdot \rho_{21} = F_2\delta_2 \tag{24}$$

$$X_2^{\lambda_3} \cdot \rho_{32} + X_3^{\lambda_3} + (F_1\delta_1 - X_2 \cdot \rho_{12} + X_3 \cdot \rho_{13})^{\lambda_3} \cdot \rho_{31} = F_3\delta_3 \tag{25}$$

$$s.t. \qquad X_2 \cdot \rho_{12} + X_3 \cdot \rho_{13} \leq F_1\delta_1$$

Here, (26) is the condition such that X_1 is positive via the free-entry condition for country one. Subject to this space, which restricts the values of X_2 and X_3, (24) and (25) identify the values of X_2 and X_3 such that free entry conditions yield a positive value of X_1, X_2 and X_3.

Figures 2, 3, and 4 display three different outcomes for a variety of parameter values. As a benchmark, Figures 2 displays the free entry conditions for Melitz and Ottaviano (2008) for 3 countries, with tariffs of 50% levied by all countries. Clearly, there is a unique equilibrium outcome, and the free entry conditions are linearized after accounting for the Pareto parameter.

Next, Figure 3 displays an equilibrium that is similar to that discussed in the previous section. Specifically, I assume that countries two and three are identical, but with a productivity distribution skewed toward less productive firms. In this case, we find that there are two equilibria (under the assumed parameter values), similar to section three.

Finally, in Figure 4, I assume severe differences in productivity skew, such that $\lambda_2 = 3$ and $\lambda_3 = 25$. Further, we assume that trade is free, which centers the free entry manifolds for countries two and three within X_2 and X_3 space. For country 3, we also assume relatively low values of F_3, since otherwise the firms would not be productive enough to earn positive profits in any market. Hence, country 3 could be interpreted as a particularly backward country in terms of productivity skew, but with entry costs which are subsidized. The provocative result from this example is that there are *six* values of X_1, X_2, and X_3, such that free entry conditions are satisfied. Hence, current work in this area is focusing on using the relative ranking of tastes across countries to select from a potentially large number of equilibria.