Homework 2 – Answer Key

1a.
Pr(S_n) = .8

Given that each roll is independent, we can multiply the probability of getting a strike by itself three times.

Pr(S_1 \cap S_2 \cap S_3) = .8 \cdot .8 \cdot .8 = .512

1b.

Not getting the Turkey is the complement of getting the Turkey.

In math terms we can say, Pr(Turkey) + Pr(No Turkey) = 1.

Pr(No Turkey) = 1 - .512 = .488

2a.

S = \{HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTTH, TTHH, HTHT, THTH, THHT, HTTT, THTT, TTHT, TTTH, TTTT\}

Pr(X=4) = .3 \cdot .3 \cdot .3 \cdot .3 = .0081

Pr(X=3) = (4)(.3 \cdot .3 \cdot .3 \cdot .7) = .0756

Pr(X=2) = (6)(.3 \cdot .3 \cdot .7 \cdot .7) = .2646

Pr(X=1) = (4)(.3 \cdot .7 \cdot .7 \cdot .7) = .4116

Pr(X=0) = (.7 \cdot .7 \cdot .7 \cdot .7) = .2401
2b.

\[ E[X] = \sum X_i \cdot Pr(X_i) = 4 \cdot 0.0081 + 3 \cdot 0.0756 + 2 \cdot 0.2646 + 1 \cdot 0.4116 + 0 \cdot 0.2401 = 1.2 \]

3.

\[ \Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)} \]

Define event A to be drawing from the all-red-card bowl.

Define Event B to be drawing a red card.

\[ \Pr(A \cap B) = \frac{4}{8} \] because there are 4 instances of a red card coinciding with the all-red-card bowl (The starred entries in the diagram below).

<table>
<thead>
<tr>
<th>All-Red-Card Bowl</th>
<th>Other Bowl</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red*</td>
<td>Red</td>
</tr>
<tr>
<td>Red*</td>
<td>Black</td>
</tr>
<tr>
<td>Red*</td>
<td>Black</td>
</tr>
<tr>
<td>Red*</td>
<td>Black</td>
</tr>
</tbody>
</table>

\[ \Pr(B) = \frac{5}{8} \] because there are 5 red cards out of 8 total.

\[ \Rightarrow \Pr(A|B) = \frac{(4/8)}{(5/8)} = 4/5 = 0.8 \]
4a.

The probability that a wave is 6 ft tall is equal to 0. With all continuous variables, the probability of getting any single value is 0.

4b.

First we calculate the z-scores.

\[ z(6) = \frac{(6-6)}{2} = 0 \]

\[ z(9) = \frac{(9-6)}{2} = 1.5 \]

\[ \Pr[0 < Z < 1.5] = \Pr(Z < 1.5) - \Pr(Z < 0) \]

Then we look up the values for \( \Pr(Z < 1.5) \) and \( \Pr(Z < 0) \) on our tables and plug them in.

\[ \Pr(0 < Z < 1.5) = .9332 - .5 = .4332 \]

4c.

Calculate z-scores:

\[ z(2) = \frac{(2-6)}{2} = -2 \]

\[ z(7) = \frac{(7-6)}{2} = .5 \]

\[ \Pr[-2 < Z < .5] = \Pr(Z < .5) - [1 - \Pr(Z < -2)] \]

Look up values and plug-in:

\[ .6915 - .0227 = .6688 \]

4d.

Define the first wave being between 4ft and 7ft as Event A.

Define the second wave being between 4ft and 9ft as Event B.

Find \( \Pr(A) \):

\[ z(4) = \frac{(4-6)}{2} = -1 \]

\[ z(7) = \frac{(7-6)}{2} = .5 \]

\[ \Pr[-1 < Z < .5] = \Pr(Z < .5) - [1 - \Pr(Z < -1)] \]

\[ = .6915 - .1586 = .5329 = \Pr(A) \]
Find $\Pr(B)$:

$z(4) = (4-6)/2 = -1$

$z(9) = (9-6)/2 = 1.5$

$\Pr[z(4)<Z<z(9)] = \Pr(-1<Z<1.5) = \Pr(Z<1.5) - [1 - \Pr(Z<1)]$

$= .9332 - .1587 = .7745$

Use formula for $\Pr(A \text{ or } B)$:

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

Because the events are independent we can say:

$\Pr(A \cap B) = \Pr(A) \cdot \Pr(B)$

Plugging values into $\Pr(A \text{ or } B)$:

$\Pr(A \cup B) = .5329 + .7745 - (.5329 \cdot .7745) = .8947$