1. The Dude is an avid bowler. Since he is a master of relaxation, the probability of rolling a strike in any given frame is independent of all the others. Suppose that the probability of rolling a strike is 0.7 for each frame:

   a. A perfect game requires 12 strikes in a row. What is the probability that The Dude rolls 12 consecutive strikes?

   \[ \text{Given that each strike is independent, you simply multiply the Pr(Strike) 12 times. That is Pr(12 strikes)= Pr(Strike)^{12}=0.0138} \]

   b. What is the probability that The Dude fails to roll a strike in at least one of the twelve attempts?

   \[ \text{Failing to get 12 strikes is the complement of getting 12 strikes. So, the probability if not getting a strike in at least one frame is written as:} \]
   \[ \text{Pr(at least one frame without a strike)=1-Pr(12 strikes)=1-0.0138=0.9862} \]

2. Suppose that you join a game in which a coin is flipped three times. The probability of getting heads is 0.6. The random variable X is defined as the number of heads throughout the game.

   a. Please solve for and diagram the probability distribution of X.

   \[ S=\{HHH,HHT,HTH,THH,HTT,THT,TTH,TTT\} \]

   \[ Pr(H=0)=1 \times (0.4 \times 0.4 \times 0.4)=0.064 \]
   \[ Pr(H=1)=3 \times (0.6 \times 0.4 \times 0.4)=0.288 \]
   \[ Pr(H=2)=3 \times (0.6 \times 0.6 \times 0.4)=0.432 \]
   \[ Pr(H=3)=1 \times (0.6 \times 0.6 \times 0.6)=0.216 \]
b. What is the expected value of $X$?

$$E(X) = 0 \times 0.056 + 1 \times 0.288 + 2 \times 0.432 + 3 \times 0.216 = 1.8$$

3. Suppose that there exists a 10-sided die with numbers 1-5 in Red and 6-10 in Blue. Calculate the probability of rolling an odd number given that the number you roll is blue.

$$S = \{1R, 2R, 3R, 4R, 5R, 6B, 7B, 8B, 9B, 10B\}$$

$$Pr(Odd|Blue) = Pr(Odd & Blue) / Pr(Blue) = (2/10) / (5/10) = \frac{2}{5}$$

4. Wave height is distributed normally with mean 5ft and standard deviation 2 ft.

a. What is the probability that a wave is 6ft tall?

Zero. For a continuous random variable, the probability of being any specific value is zero.

b. What is the probability that a wave is between 6 and 7 feet tall?

First, compute $Z$ scores.

$$z(6) = (6 - 5) / 2 = 0.5$$
\[ z(7)=\frac{7-5}{2}=1 \]

\[ Pr(z(6)<Z<z(7)) = Pr(Z<z(7))- Pr(Z<z(6))=0.8413-0.6915=0.1498 \]

c. What is the probability that a wave is between 2 and 7 feet tall?

\[ z(2)=\frac{2-5}{2}=-1.5 \]

\[ Pr(Z<z(2)) = Pr(Z<-1.5)) = Pr(Z>(-1.5))= 1-Pr(Z<1.5) \]

\[ Pr(z(6)<Z<z(7))= Pr(Z<z(7))- (1-Pr(Z<1.5))=0.8413-(1-0.9332)=0.7745 \]

d. Suppose that two waves are coming, and independent from one another. What is the probability that wave one is between 3 and 7 feet tall OR wave two is between 4 and 6 feet tall?

*Call the first wave between 3 and 7 as event A*

*Call the second wave between 4 and 6 as event A*

\[ z(4)=(-0.5) \]

\[ z(6)=(0.5) \]

\[ Pr(A)=Pr(z(4)<Z<z(6)) = Pr(Z<z(6))- Pr(Z<z(4)) = Pr(Z<0.5)- Pr(Z<-0.5) \]

\[ = Pr(Z<0.5))- (1-Pr(Z<0.5)) \]

\[ = 2*Pr(Z<0.5)- 1 \]

\[ = 2*0.6915- 1 \]

\[ = 0.383 \]

\[ z(3)=(-1) \]

\[ z(7)=(1) \]

\[ Pr(B)=Pr(z(3)<Z<z(7)) = Pr(Z<z(7))- Pr(Z<z(3)) = Pr(Z<1)- Pr(Z<-1) \]

\[ = Pr(Z<1))- (1-Pr(Z<1)) \]

\[ = 2*Pr(Z<1)- 1 \]

\[ = 2*0.8413- 1 \]

\[ = 0.6826 \]

\[ Pr(A \text{ OR } B)=Pr(A)+Pr(B)-Pr(A\&B) \]

\[ =Pr(A)+Pr(B)-Pr(A)Pr(B) \]

\[ =0.383+0.6826-0.383*0.6826 \]

\[ =0.8042 \]