Economics 217 **Name:**_____
Exam #3

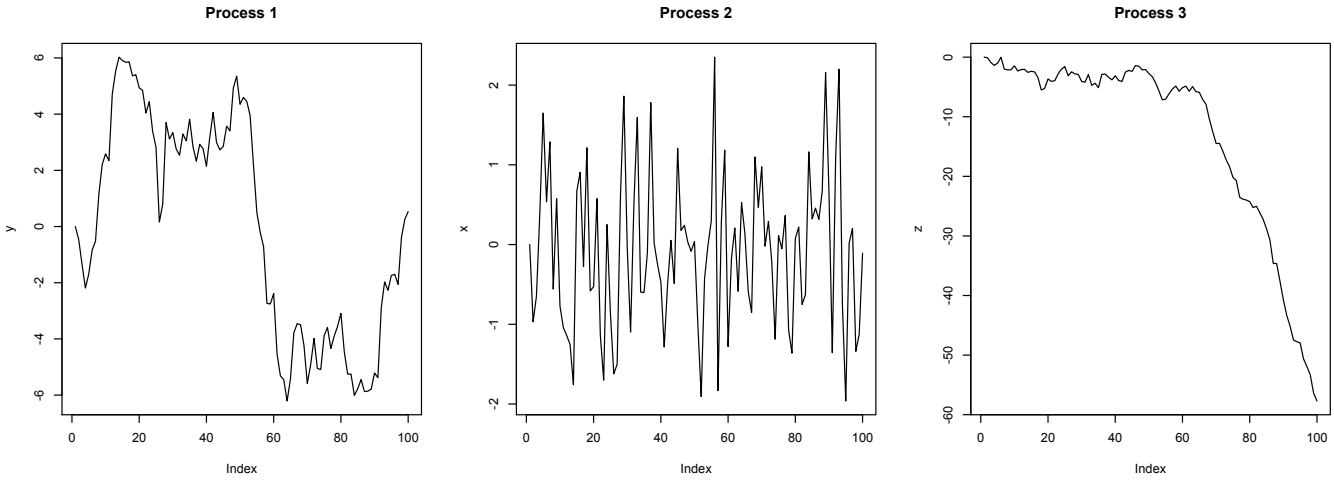
Instructions. Closed book and notes, 180 minutes. Please directly answer on the exam paper. Partial credit will be granted for brief, relevant remarks and for partial results, but not unrelated equations and text from memory. There are 8 questions, 110 points in total.

Problem 1 - AR processes

Please write down the equation for the AR(1) process. In terms of variance, please derive the key necessary condition for stationarity, specifying precisely what happens when this condition is not met. (10 points)

Problem 2 - AR processes

Below, I've illustrated three AR1 processes. Please briefly discuss the properties of each and what this means for the likely value of ϕ (compared with $\phi = 1$). (10 points)



Problem 3 - MA processes

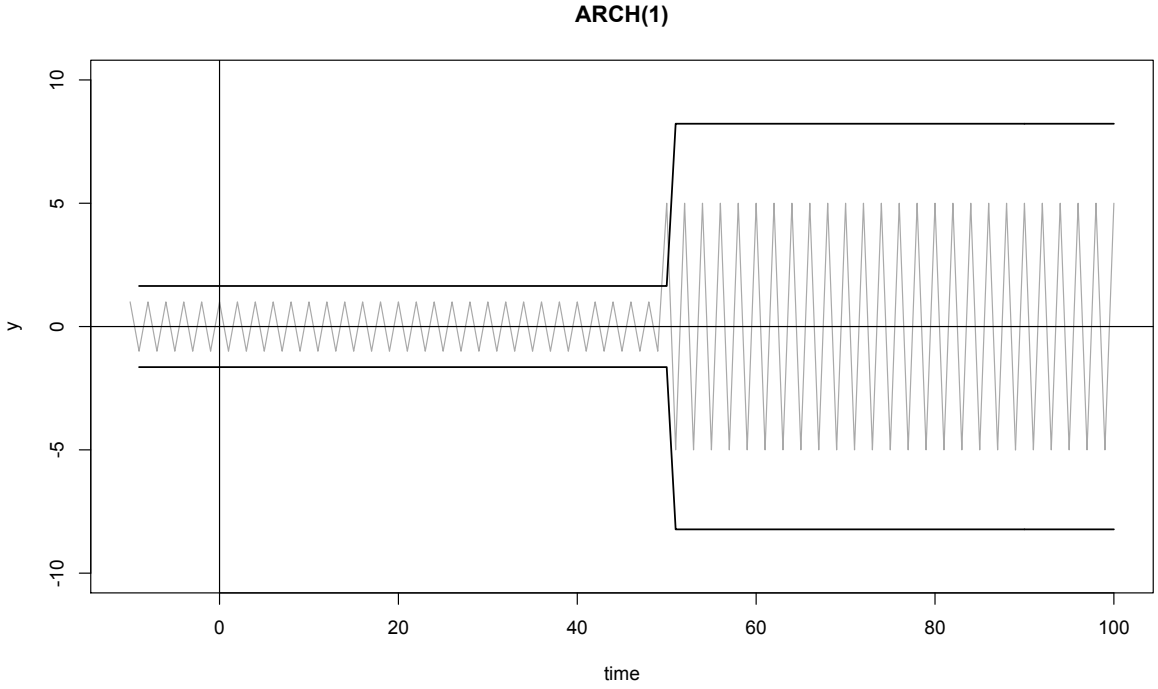
Suppose that we have the following (restricted) MA(2) model:

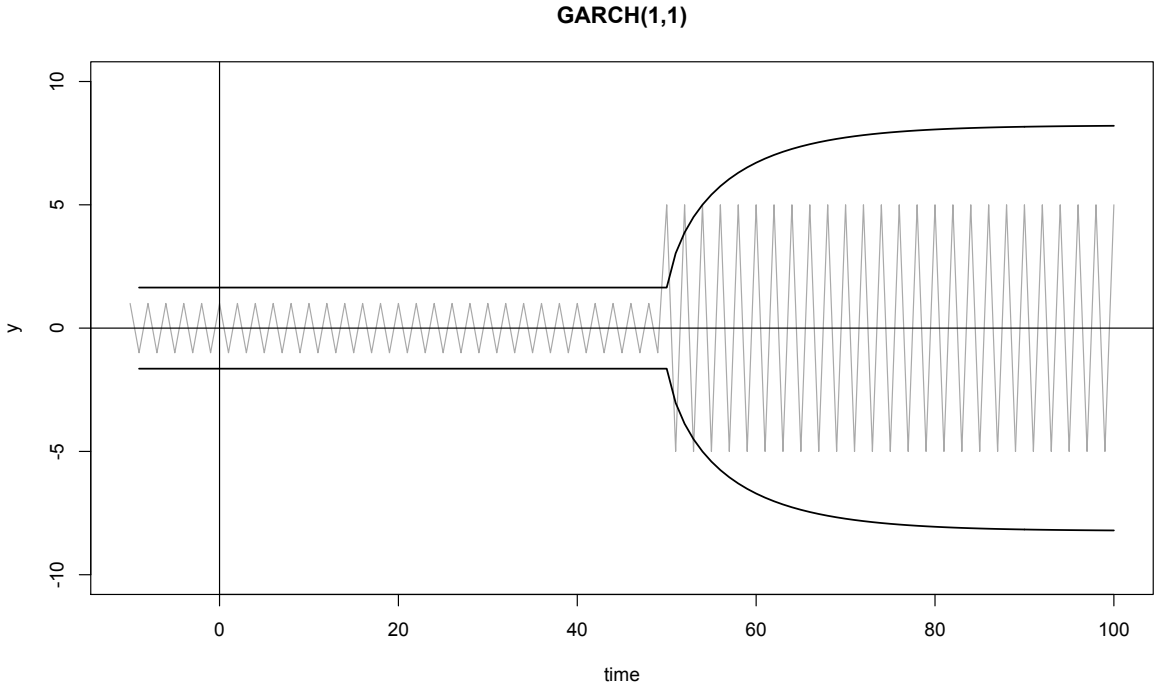
$$y_t = u_t + \theta_2 u_{t-2}$$

Please derive the theoretical correlation between y_t and the first three lags, y_{t-1}, y_{t-2} and y_{t-3}. (20 points)

Problem 4 - ARCH/GARCH

Below, I've plotted the same time series twice. In the first, I would like you to (qualitatively) draw the prediction bounds if this were modeled as an ARCH(1) process. In the second, I would like you to draw the prediction bounds if this were modeled as a GARCH(1,1) process with equal weights on each component of the process. (10 points)





Problem 5 - VAR

Prices, quantities, and wages are all linked through equilibrium conditions. Below is a 3 equation VAR for quantity, q_t, price, p_t, and wages, w_t.

$$\begin{aligned} q_t &= \beta_0 + \beta_1 q_{t-1} + \beta_2 p_{t-1} + \beta_3 p_t + \beta_4 w_t + u_t \\ p_t &= \alpha_0 + \alpha_1 p_{t-1} + \alpha_2 q_{t-1} + \alpha_3 q_t + \alpha_4 w_t + e_t \\ w_t &= \delta_0 + \delta_1 w_{t-1} + \delta_2 p_t + \delta_3 q_t + \epsilon_t \end{aligned}$$

Please derive the reduced-form VAR. (10 Points - you may leave matrix inverses in general terms: eg. B^{-1})

Problem 6 - Granger Tests

For the question below, x is a time series data frame consisting of two variables for the US, real GDP (rgdp) and real capital stock (cap), for the period 1950-2012.

```
> granger.test(x,p=2)

            F-statistic  p-value
cap -> rgdp      2.22       0.118
rgdp -> cap     14.55       0.000
```

Please interpret the results in this table. Further, please write down the equations estimated for Granger test in the second row, and precisely indicate the hypothesis being tested. (20 points)

Problem 7 - Co-integrated Series

Suppose that you have a series of price and quantity, p_t and q_t, each of which have been determined to be I(1). Please detail the next steps you would take, including the code required for those steps, to determine whether it is sensible to evaluate a relationship between these variables, and how you would go about doing so if estimating the relationship were sensible. (20 points)

Problem 8 - Correcting my mistakes

Consider the following AR(1) model with drift.

$$y_t = \delta + \phi y_{t-1} + u_t$$

In my notes, I mistakenly labeled this as "non-stationary", where under certain conditions it is stationary in the long-run. Assuming $\phi \in (0, 1)$ and $\delta > 0$, starting from an initial value of $y_0 = 0$, please derive the expected long-run value of y_t (10 points. Hint: iterate and take expectations. This might remind you of finance).