Economics 217 **Name:**____________________________________

Exam #3

Instructions. Closed book and notes, 180 minutes. Please directly answer on the exam paper. Partial credit will be granted for brief, relevant remarks and for partial results, but not unrelated equations and text from memory. There are 6 questions, 80 points in total.

Problem 1 - AR processes

Suppose that we have the following AR(1) model:

$$y_t = \phi y_{t-1} + u_t$$

Assume that $\phi \in (0, 1)$. Please <u>derive</u> the theoretical correlation between y_t and its second lag, y_{t-2}. Show your work!! (20 points)

$$\begin{aligned}
\text{Cov}\left(y_t, y_{t-2}\right) &= \text{Cov}\left(\phi y_{t-1} + u_t, y_{t-2}\right) \\
&= \text{Cov}\left(\phi(\phi y_{t-2} + u_{t_1}) + u_t, y_{t-2}\right) \\
&= \phi^2 \text{Var}\left[y_{t-2}\right] + \phi \text{Cov}\left(u_{t-1}, y_{t-2}\right) + \text{Cov}\left(u_t, y_{t-2}\right) \\
&= \phi^2 \text{Var}\left[y_t\right]
\end{aligned}$$

Therefore,

$$\text{Corr}\left(y_t, y_{t-2}\right) = \frac{\text{Cov}\left(y_t, y_{t-2}\right)}{\text{Var}\left[y_t\right]} = \phi^2$$

15 for derivation and 5 for answer.

Problem 2 - MA processes

Suppose that we have the following (restricted) MA(2) model:

$$y_t = u_t + \theta u_{t-2}$$

Please derive the variance of y_t. Show your work! (10 points)

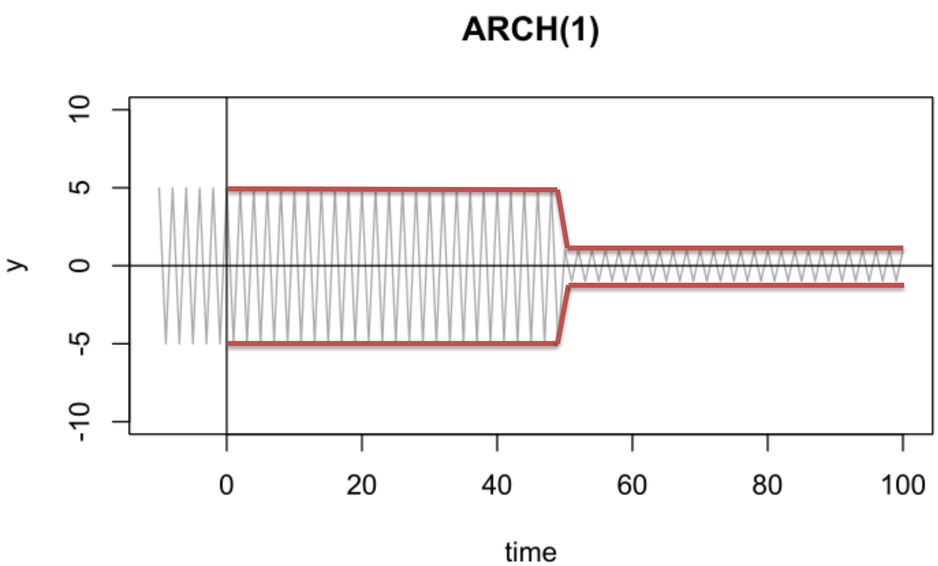
$$\begin{aligned}
\text{Var}\left[y_t\right] &= \text{Var}\left[u_t + \theta u_{t-2}\right] \\
&= \text{Var}\left[u_t\right] + 2\theta \text{Cov}\left(u_t, u_{t-2}\right) + \theta^2 \text{Var}\left[u_{t-2}\right] \\
&= (1+\theta^2)\sigma^2
\end{aligned}$$

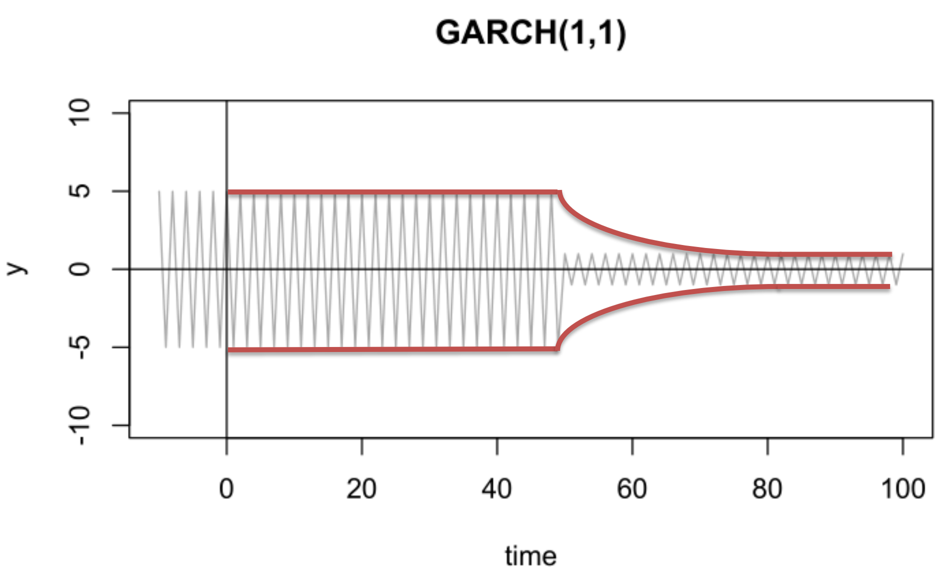
where σ^2 is the variance of the white noise, $\{u_t\}$.

7 for derivation and 3 for answer.

Problem 3 - ARCH/GARCH

Below, I've plotted the same time series twice. In the first, I would like you to (qualitatively) draw the prediction bounds if this were modeled as an ARCH(1) process. In the second, I would like you to draw the prediction bounds if this were modeled as a GARCH(1,1) process with equal weights on each component of the process. (10 points)





5 for each. The key point is that ARCH prediction bound changes immediately while GARCH changes gradually.

Problem 4 - VAR

For the question below, x is a time series data frame consisting of two variables for the US, log real GDP ($lrgdp$) and log real capital stock ($lcap$), for the period 1950-2012. Log real gdp is the first variable in the data frame, and log real capital stock is the second.

```
===========================================
Reduced Form VAR
===========================================
Number of observations :     61
Degrees of freedom per equation :     58
-------------------------------------------
Autoregressive Matrices:
B(1)
         [,1]     [,2]
[1,] 0.898745 0.086479
[2,] 0.094809 0.895774

-------------------------------------------
Constants :
0.022314  0.426633
-------------------------------------------
```

Please interpret the results in this table. From what set of regressions do these estimates originate? From this table, do we have sufficient information to test for Granger causality? Why or why not? (10 points)

The result gives the following set of regressions:

$$lrgdp_t = 0.898745 lrgdp_{t-1} + 0.086479 lcap_{t-1} + 0.022314$$
$$lcap_t = 0.094809 lrgdp_{t-1} + 0.895774 lcap_{t-1} + 0.426633$$

One percentage increase in current real GDP will increase the next period's real GDP by 0.899 percentage, and increase the next period's capital stock by 0.086 percentage. One percentage increase in current capital stock will increase the next period's real GDP by 0.095 percentage, and increase the next period's capital stock by 0.896 percentage.

From this table we do not have sufficient information to test for Granger causality, because we are lack of standard errors.

4 for equations, 3 for interpretation and 3 for showing not able to test for Granger causality.

Problem 5 - Analysis

Using the same dataset from Problem 5, I have run five ADF tests using the following code. Please summarize the results for each test, and propose a strategy to rigorously evaluate the relationship between *lrgdp* and *lcap*. (20 points)

```
> adf.test(x[,1],k=0)

	Augmented Dickey-Fuller Test

	data:  x[, 1]
	Dickey-Fuller = -0.95067, Lag order = 0, p-value = 0.9384
	alternative hypothesis: stationary

> adf.test(diff(x[,1]),k=0)

	Augmented Dickey-Fuller Test

	data:  diff(x[, 1])
	Dickey-Fuller = -6.837, Lag order = 0, p-value = 0.01
	alternative hypothesis: stationary

> adf.test(x[,2],k=0)

	Augmented Dickey-Fuller Test

	data:  x[, 2]
	Dickey-Fuller = -0.17455, Lag order = 0, p-value = 0.99
	alternative hypothesis: stationary

> adf.test(diff(x[,2]),k=0)

	Augmented Dickey-Fuller Test

	data:  diff(x[, 2])
	Dickey-Fuller = -5.9414, Lag order = 0, p-value = 0.01
	alternative hypothesis: stationary

> adf.test(residuals(lm(x[,1]~x[,2])),k=0)

	Augmented Dickey-Fuller Test

	data:  residuals(lm(x[, 1] ~ x[, 2]))
	Dickey-Fuller = -2.6438, Lag order = 0, p-value = 0.3143
	alternative hypothesis: stationary
```

The test result shows that we cannot reject the existence of unit root in log real GDP ($lrgdp$) or log capital stock ($lcap$), but we reject the unit root in $\Delta(lrgdp)$ (i.e., difference in $lrgdp$) and $\Delta(lcap)$. Therefore, $lrgdp$ and $lcap$ are both $I(1)$ processes.

Further, we cannot reject the unit root in the residual obtained from regressing $\Delta(lrgdp)$ on $\Delta(lcap)$, that is to say, $lrgdp$ and $lcap$ are NOT cointegrated, so we cannot use error correction model.

Since $\Delta(lrgdp)$ and $\Delta(lcap)$ are both $I(0)$, we can use the first differences to understand the short-run relationship.

10 for specifying I(1), 5 for no cointegration and 5 for using first difference to do short-run analysis.

Problem 6 - Testing Models

Suppose I run the following optimization problem:

$$\min_{\beta_p} \sum_{i=1}^{N} \left(y_t - \sum_{p=1}^{10} \beta_p y_{t-p} \right)^2$$
$$s.t. \qquad \sum_{p=1}^{10} |\beta_p| < \lambda$$

When λ becomes large, what happens to this optimization problem? What process is modeled? When λ is small enough, what happens to some of the β_p's, if anything? (10 points)

When λ becomes sufficiently large, the sum of coefficients is unbounded, thus we are estimating an AR(10) process.[1]

When λ is small enough, some of the coefficients will become zero.
5 for each part.

[1]More precisely, we are estimating an AR(10) process, *conditional on* the first 10 outcomes.