Economics 217 **Name:**__________________________________
Exam #3

Instructions. Closed book and notes, 180 minutes. Please directly answer on the exam paper. Partial credit will be granted for brief, relevant remarks and for partial results, but not unrelated equations and text from memory. There are 6 questions, 80 points in total.

Problem 1 - AR processes

Suppose that we have the following AR(1) model:

$$y_t = \phi y_{t-1} + u_t$$

Assume that $\phi \in (0, 1)$. Please <u>derive</u> the theoretical correlation between y_t and its second lag, y_{t-2}. Show your work!! (20 points)

Problem 1 (cont)

Problem 2 - MA processes

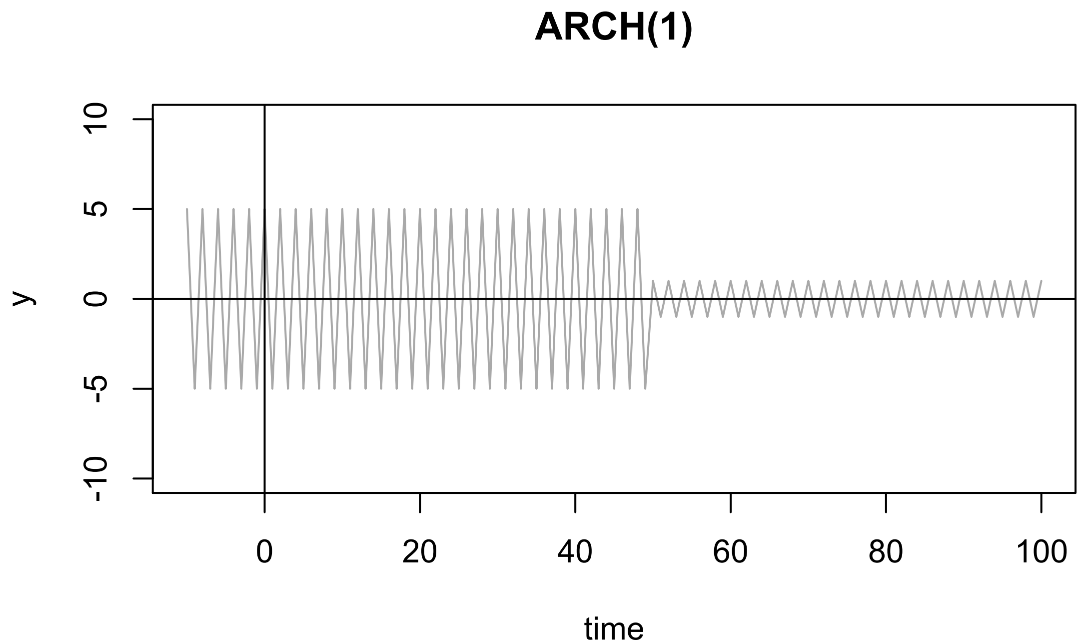
Suppose that we have the following (restricted) MA(2) model:

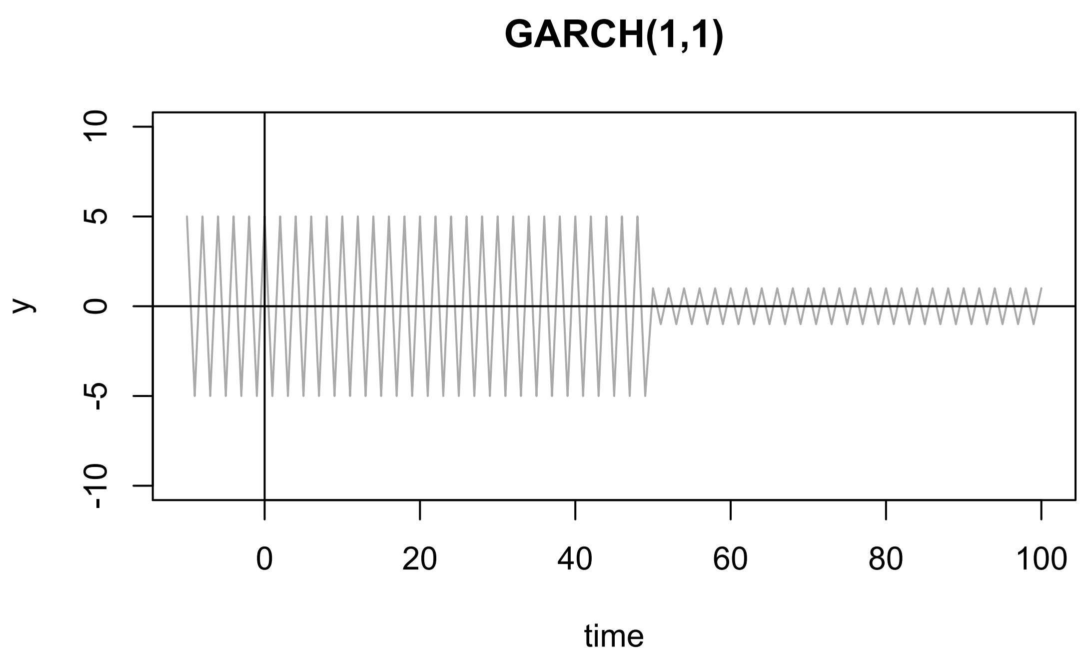
$$y_t = u_t + \theta u_{t-2}$$

Please derive the variance of y_t. Show your work! (10 points)

Problem 3 - ARCH/GARCH

Below, I've plotted the same time series twice. In the first, I would like you to (qualitatively) draw the prediction bounds if this were modeled as an ARCH(1) process. In the second, I would like you to draw the prediction bounds if this were modeled as a GARCH(1,1) process with equal weights on each component of the process. (10 points)





Problem 4 - VAR

For the question below, x is a time series data frame consisting of two variables for the US, log real GDP ($lrgdp$) and log real capital stock ($lcap$), for the period 1950-2012. Log real gdp is the first variable in the data frame, and log real capital stock is the second.

```
==========================================
Reduced Form VAR
==========================================
Number of observations :     61
Degrees of freedom per equation :     58
------------------------------------------
Autoregressive Matrices:
B(1)
         [,1]     [,2]
[1,] 0.898745 0.086479
[2,] 0.094809 0.895774

------------------------------------------
Constants :
0.022314  0.426633
------------------------------------------
```

Please interpret the results in this table. From what set of regressions do these estimates originate? From this table, do we have sufficient information to test for Granger causality? Why or why not? (10 points)

Problem 4 (cont)

Problem 5 - Analysis

Using the same dataset from Problem 5, I have run five ADF tests using the following code. Please summarize the results for each test, and propose a strategy to rigorously evaluate the relationship between *lrgdp* and *lcap*. (20 points)

```
> adf.test(x[,1],k=0)

	Augmented Dickey-Fuller Test

data:  x[, 1]
Dickey-Fuller = -0.95067, Lag order = 0, p-value = 0.9384
alternative hypothesis: stationary

> adf.test(diff(x[,1]),k=0)

	Augmented Dickey-Fuller Test

data:  diff(x[, 1])
Dickey-Fuller = -6.837, Lag order = 0, p-value = 0.01
alternative hypothesis: stationary

> adf.test(x[,2],k=0)

	Augmented Dickey-Fuller Test

data:  x[, 2]
Dickey-Fuller = -0.17455, Lag order = 0, p-value = 0.99
alternative hypothesis: stationary

> adf.test(diff(x[,2]),k=0)

	Augmented Dickey-Fuller Test

data:  diff(x[, 2])
Dickey-Fuller = -5.9414, Lag order = 0, p-value = 0.01
alternative hypothesis: stationary

> adf.test(residuals(lm(x[,1]~x[,2])),k=0)

	Augmented Dickey-Fuller Test

data:  residuals(lm(x[, 1] ~ x[, 2]))
Dickey-Fuller = -2.6438, Lag order = 0, p-value = 0.3143
alternative hypothesis: stationary
```

Problem 5 (cont)

Problem 6 - Testing Models

Suppose I run the following optimization problem:

$$\begin{aligned} \min_{\beta_p} \sum_{i=1}^{N} & \left(y_t - \sum_{p=1}^{10} \beta_p y_{t-p} \right)^2 \\ s.t. \qquad & \sum_{p=1}^{10} \left| \beta_p \right| < \lambda \end{aligned}$$

When λ becomes large, what happens to this optimization problem? What process is modeled? When λ is small enough, what happens to some of the β_p's, if anything? (10 points)