Midterm Exam # 1 – 65 Points

The exam is closed book and closed notes. Please show your work step by step. Simple calculators may be used (no graphing calculators and no smart phones or iPods)

You must show your work to receive full credit

I have neither given nor received unauthorized aid on this examination, nor have I concealed any similar misconduct by others.

Signature

Problem 1  (20 Points)
For the working population, suppose that hours worked is distributed according to a normal distribution with mean 40 and standard deviation 8.

a. The Bureau of Labor Statistics (BLS) defines “part-time” workers as those that work 34 hours a week or less. The Affordable Care Act (ACA) defines part-time workers as those that work fewer than 30 hours per week. What is the probability of being defined as a part-time worker by the BLS, but not the ACA? (10 Points)

\[
\Pr(30 < H < 34) = \Pr(H < 34) - \Pr(H < 30) + 2
\]

\[
= \Pr \left( z < \frac{34 - 40}{8} \right) - \Pr \left( z < \frac{30 - 40}{8} \right) + 2
\]

\[
= \Pr(z < -0.75) - \Pr(z < -1.25) + 2
\]

\[
= \left(1 - \Pr(z > 0.75)\right) - \left(1 - \Pr(z > 1.25)\right) + 2
\]

\[
= \Pr(z > -1.25) - \Pr(z > -0.75) + 2
\]

\[
= \Pr(z < 1.25) - \Pr(z < 0.75) + 2
\]

\[
= 0.8944 - 0.7734 + 2
\]
b. Working with only the ACA definition of part-time work, if you are a part-time worker, the probability of having employer-provided insurance is 0.2. If you are a full-time worker, the probability of having employer-provided insurance is 0.9. Given that you have received employer-provided health insurance, what is the probability that you are a full-time worker? (10 points, note, these probabilities are made-up, but we will soon know what they are!)

\[
P_r(H > 30) = P_r(Z > -1.25) = P_r(Z < 1.25) = 0.8944
\]

\[
P_r(H < 30) = 0.1056
\]

\[
P_r(I) = 0.081
\]

\[
P_r(I | H < 30) = 0.2
\]

\[
P_r(I | H > 30) = 0.1
\]

\[
P_r(H > 30 | I) = \frac{P_r(H > 30 \cap I)}{P_r(I)} = \frac{P_r(H > 30) \cdot P_r(I | H > 30)}{P_r(H > 30) \cdot P_r(I | H > 30) + P_r(H < 30) \cdot P_r(I | H < 30)}
\]

\[
= \frac{0.8944 \cdot 0.9}{0.8944 \cdot 0.9 + 0.1056 \cdot 0.2}
\]

\[
P_r(H > 30 | I) = 0.9744
\]
Problem 2 (45 Points)

Suppose that we are interested in examining the effects of earnings on the level of health care coverage. To do so, we run the following regression:

\[ \log(\text{Coverage}) = \beta_0 + \beta_1 \text{Earnings} + u \]

Here, \textit{Earnings} is monthly earnings in thousands of dollars, and \textit{Coverage} is the monthly cost of coverage in dollars.

a. Suppose that we estimate that \( \hat{\beta}_1 = 0.05 \). Please derive using derivatives the interpretation for \( \hat{\beta}_1 \). Please interpret this estimate. (10 Points)

\[ \frac{d \text{Coverage}}{\text{Coverage}} = \beta_1 \frac{d \text{Earnings}}{\text{Earnings}} \left( + \frac{1}{\text{Coverage}} \right) \]

\[ 100 \times \left( \frac{d \text{Coverage}}{\text{Coverage}} \right) = (100 \times \beta_1) \frac{d \text{Earnings}}{\text{Earnings}} \left( + \frac{2}{\text{Coverage}} \right) \]

A one thousand dollar increase in monthly earnings yields a 5\% increase in coverage.
b. “Adverse selection” is a big problem in health insurance markets, where those most likely to get sick are more likely to get coverage. That is, some variable measuring propensity for sickness, Sick, is positively associated with Coverage. Suppose further that those that get sick are less productive. Precisely, there is a negative association between Sick and Earnings. In what direction, if any, is there a bias in our estimate \( \hat{\beta}_1 \)? Can we still say anything about the sign of the parameter we are trying to estimate? And, if there is a bias, what assumption is violated? (10 Points)

\[
\log(\text{Coverage}) = \hat{\beta}_0 + \hat{\beta}_1 \text{Earnings} + u + \text{Downward Bias}
\]

Since the estimate is positive and biased downward, we can still be reasonably sure that it is positive. +4

\[
E(u|x) = 0 \text{ is violated}
\]

---

c. We now adjust the model to account for some non-linear effects of Earnings, and a big aspect of the insurance decision: kids. Specifically, we now run the following regression:

\[
\log(\text{Coverage}) = \beta_0 + \beta_1 \log(\text{Earnings}) + \beta_2 \text{Kids} + u
\]

where Kids is the number of children of the respondent.

I would like to understand when individuals choose no coverage. Does this regression help with that task? Why or why not? (5 points)

No, since Coverage = 0 cannot be estimated.

\[
\log(\text{Coverage}) \text{ is undefined.}
\]

(All or nothing) +5
d. Running the regression in 'c', we estimate the following

\[ \log(\text{Coverage}) = 1.5 + 0.8 \log(\text{Earnings}) + 0.2 \text{Kids} \]

Please derive using derivatives the interpretation for the coefficient on \( \log(\text{Earnings}) \). Please interpret this estimate. (10 Points)

\[
\frac{d\text{Coverage}}{\text{Coverage}} = 0.8 \frac{d\text{Coverage}}{\text{Earnings}} \frac{d\text{Earnings}}{\text{Earnings}} + 1
\]

\[
(100 \times \frac{d\text{Coverage}}{\text{Coverage}}) = 0.8 (100 \times \frac{d\text{Earnings}}{\text{Earnings}}) + 2
\]

A 1\% increase in earnings yields a

0.8\% increase in coverage.

Or

The elasticity of coverage with respect to earnings is 0.8.
Finally, we adjust the coverage regression to remove all logs, and estimate the following:

\[
Coverage = -20 + 100 \text{Earnings} + 200 \text{Kids}
\]

Suppose we have two families. Family A has no kids and earns $10,000 per month, and Family B has three kids and earns $5,000 per month. Remembering that Earnings is measured in thousands, which family buys more coverage and by how much? (10 points)

\[
\Delta \text{Coverage} = 100 \Delta \text{Earnings} + 200 \Delta \text{Kids}
\]

where \( \Delta \) is Family A - Family B

\[
\Delta \text{Coverage} = 100(5) + 200(0 - 3)
\]

\[
= 500 - 600 = -100
\]

Family B buys $100 more coverage.
Normal Distribution
from -oo to Z

\[ z \]

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