Midterm 3 – 60 Points

You must answer all questions. Please write your name on every page. The exam is closed book and closed notes. You may use calculators, but they must not be graphing calculators. Do not use your own scratch paper.

**You must show your work to receive full credit**

**Problem 1 (45 Points)**

Suppose that you wish to predict wage outcomes via the following specification:

\[ wage = \beta_0 + \beta_{educ} educ + \beta_{exper} exper + \beta_{IQ} IQ + \beta_{Age} Age + u \]

wage is measured in dollars per month, educ and exper are measured in years. The results from estimating this equation (using the urban subsample of WageData.TXT) are the following:

Call:
\[ \text{lm(formula = wage ~ educ + exper + iq + age, data = subset(x, urban == 1))} \]

Coefficients:

| Estimate | Std. Error | t value | Pr(>|t|) |
|----------|------------|---------|----------|
| (Intercept) | -944.044   | 195.966 | xxxxxxxxxx |
| educ      | 58.218     | 8.524   | xxxxxxxxxx |
| exper     | 11.300     | 4.473   | xxxxxxxxxx |
| iq        | 5.250      | 1.118   | xxxxxxxxxx |
| age       | 15.107     | 5.695   | xxxxxxxxxx |

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Residual standard error: 374.3 on 666 degrees of freedom
Multiple R-squared: 0.1832,  Adjusted R-squared: 0.1783
F-statistic: 37.35 on 4 and 666 DF,  p-value: < 2.2e-16

a.) Please interpret the intercept. Is the value of the intercept meaningful? Why or why not (at least two reasons)? (5 Points)

On average, a person with 0 education, 0 experience, 0 IQ, and 0 age will earn $944 per month. **No, not meaningful**

1. negative
2. Age = 0
3. IQ = 0
b.) Using the 95% confidence level, test whether the coefficient on \( \text{Educ} \), \( \beta_{\text{educ}} \), is significantly different from zero. Please state your null and alternative hypotheses, and briefly interpret the result. (10 Points)

\[ H_0: \beta_{\text{educ}} = 0 \]
\[ H_1: \beta_{\text{educ}} \neq 0 \]

\[ t_{\text{stat}} = \frac{58.7 - 0}{8.52} = 6.83 \]

\[ |t_{\text{stat}}| > t_{\text{crit}} \Rightarrow \text{Reject Null} \]

Education has a positive and statistically significant effect on wages.

\( c. \) Please construct a 92% confidence interval for \( \beta_{\text{iq}} \). Please interpret this confidence interval. (10 Points)

\[ t_{\text{crit}} = 1.75 \]

\[ \hat{\beta}_{\text{iq}} - se(\hat{\beta}_{\text{iq}}) \cdot t_{\text{crit}} < \beta_{\text{iq}} < \hat{\beta}_{\text{iq}} + se(\hat{\beta}_{\text{iq}}) \cdot t_{\text{crit}} \]

\[ 5.25 - 1.12 \cdot 1.75 < \beta_{\text{iq}} < 5.25 + 1.12 \cdot 1.75 \]

\[ 3.325 < \beta_{\text{iq}} < 7.175 \]

IQ has a positive and statistically significant effect on wages.

\( d. \) Suppose I claim that age has no effect on wages. What is the probability that I’m wrong? Please state the null and alternative hypotheses, and show your work! (10 points)

\[ H_0: \beta_{\text{age}} = 0 \]
\[ H_1: \beta_{\text{age}} \neq 0 \]

\[ t_{\text{stat}} = \frac{15.107}{5.695} = 2.65 \]

\[ P_{\text{value}} = P(t > t_{\text{stat}}) \]

\[ = 2 \cdot (1 - \text{Pr}(T < t_{\text{stat}})) \]

\[ = 2 \cdot (1 - 0.996) = 2 \cdot 0.004 = 0.008 \]

0.008 probability of being wrong.
e.) Please derive a new estimating equation that will generate a prediction and standard error for a 50 year old person with 15 years of education, 20 years of experience, and a 140 IQ. Show your work!!! (10 Points)

\[ \theta = \beta_0 + \beta_{\text{educ}} \cdot 15 + \beta_{\text{exp}} \cdot 20 + \beta_{\text{IQ}} \cdot 140 + \beta_{\text{age}} \cdot 50 + 2 \]

\[ \Rightarrow \beta_0 = \theta - \beta_{\text{educ}} \cdot 15 - \beta_{\text{exp}} \cdot 20 - \beta_{\text{IQ}} \cdot 140 - \beta_{\text{age}} \cdot 50 - 2 \]

\[ \text{wage} = \beta_0 + \beta_{\text{educ}} \cdot \text{Ed} + \beta_{\text{exp}} \cdot \text{Exp} + \beta_{\text{IQ}} \cdot \text{IQ} + \beta_{\text{age}} \cdot \text{Age} \]

\[ \text{wage} = \theta + \beta_{\text{educ}} \cdot (\text{Ed} - 15) + \beta_{\text{exp}} \cdot (\text{Exp} - 20) \]

\[ + \beta_{\text{IQ}} \cdot (\text{IQ} - 140) + \beta_{\text{age}} \cdot (\text{Age} - 50) \]
Problem 2 (15 Points)

In economics, it is common to assume that production is “Cobb-Douglas”. Assuming capital and labor are the only inputs, the Cobb-Douglas production function is written as follows:

\[ Y = \exp(\beta_0) \cdot \exp(\varepsilon) \cdot K^{\beta_K} \cdot L^{\beta_L} \]

Here, \( Y \) represents production, \( \beta_0 \) is a constant, \( \varepsilon \) is a random productivity shock, \( K \) is capital used in production, \( L \) is labor used in production, and \( \beta_K \) and \( \beta_L \) are parameters related to capital and labor.

Economists often wish to estimate production, and in particular, the parameters \( \beta_K \) and \( \beta_L \). To do so, we take logs to get:

\[ \log(Y) = \beta_0 + \beta_K \log(K) + \beta_L \log(L) + \varepsilon \]

This production function exhibits “constant returns to scale” if \( \beta_K + \beta_L = 1 \). Please derive an estimating equation that allows you to test whether production exhibits constant returns to scale. Be sure to write down your null and alternative hypotheses. Show your work!!!

Specify hypotheses:

\[ H_0: \Theta = 1 \quad H_1: \Theta \neq 1 \]

Define: \( \Theta = \beta_K + \beta_L \)

\[ \beta_L = \Theta - \beta_K \]

\[ \log(Y) = \beta_0 + \beta_K \log(K) + (\Theta - \beta_K) \log(L) + \varepsilon \]

Please enjoy your weekend.