Midterm 2 – 85 Points

You must show your work to receive full credit.

I have neither given nor received unauthorized aid on this examination, nor have I concealed any similar misconduct by others.

Signature

1. (40 Points) Suppose that you wish to predict a student’s GPA using the following specification:

\[ GPA = \beta_0 + \beta_{\text{study}} \text{studyhours} + u \]

GPA is a continuous variable between 0 and 4, and studyhours is average hours studying per day (minimum 0, maximum 24).

a.) Suppose that you estimate the following:

\[ GPA = 1.4 + 0.15 \text{studyhours} + u \]

What is wrong, if anything, with the way I’ve written this equation? If there is something wrong, please state the issue(s) exactly, and rewrite the equation. (10 Points)

Two mistakes:

1. Equation should read \( \hat{GPA} = 1.4 + 0.15 \text{studyhours} + 6 \)
   1. \( E(u|x) = 0 \), so "\( u \)" goes away + 2
   2. "\( u \)" should be above \( \hat{GPA} \) - It is a predicted value + 2

b.) Please interpret the intercept from ‘a’. (5 Points)

On average, a student studying \( \frac{0}{2} \) hours per day has a \( 1.4 \) GPA, +3

+2
c.) Suppose that $\sigma_{\text{gpa}} = 2$ and $\sigma_{\text{studyhours}} = 3$. What is the correlation between GPA and studyhours? (10 Points)

\[
\beta_{\text{study}} = \frac{\hat{\sigma}_{\text{gpa, study}}}{\sigma_{\text{study}}} \Rightarrow 0.15 = \frac{\hat{\sigma}_{\text{gpa, study}}}{2} \Rightarrow \hat{\beta}_{\text{gpa, study}} = 1.35
\]

\[
\hat{\sigma}_{\text{gpa, study}} = \frac{\hat{\sigma}_{\text{gpa, study}}}{\sigma_{\text{gpa}} \sigma_{\text{study}}} = \frac{1.35}{2 \cdot 3} \Rightarrow \hat{\sigma}_{\text{gpa, study}} = 0.225 + 2
\]

d.) Suppose that for the regression in ‘a’, I calculate an $R^2$ of 0.04. Should I be worried that this $R^2$ is too low? Why or why not? (5 Points)

No. $R^2$ says nothing about causality +1

$R^2$ can naturally be very low +1

e.) Suppose that I forget to include partyhours, which is the average number of hours a student parties during the day. Needless to say, partyhours is negatively correlated with studyhours and GPA. In what direction is the bias? What, if anything, can be said about the sign of the coefficient on studyhours? (5 Points)

\[\text{GPA} = \beta_0 + \beta_{\text{study}} + \epsilon\]

Bias is upward/Positive +2

Nothing can be said about $\beta_{\text{study}}$. All we know is $\beta_{\text{study}} \geq 0$. +3

f.) Instead of the equation at the beginning of Problem 1, I now estimate the following:

\[\log(\text{GPA}) = \beta_0 + \beta_{\text{study}} \log(\text{studyhours}) + \epsilon\]

Suppose that within the sample, I observe values of studyhours between 0 and 12, and GPA between 0.5 and 4.0. Given this information, in what way does the new log-log specification bias the sample? (5 Points)

We cannot include observations for which studyhours = 0 +1

Hence, the sample is biased toward observations of students that actually study +1

All or nothing.
2. (30 Points) Using a sample of US residents, I wish to estimate
\[ \log(Wealth) = \beta_0 + \beta_{educ} \log(educ) + \beta_{exper} \log(exper) + u \]

where \( educ \) and \( exper \) are in years, and \( Wealth \) is measured in dollars.

a.) Suppose that I estimate \( \hat{\beta}_{educ} = 3 \). Please interpret this estimate. (5 Points)

A 1% change in \( educ \) yields a 3% change in wealth, holding experience constant.

Holding experience constant, the elasticity of wealth with respect to education is 3.

b.) The sample variance of the error term is written as:
\[ \text{Var}(\hat{u}) = \frac{1}{n-3} \sum_{i=1}^{n} \hat{u}_i^2 \]

What happened to the mean of \( \hat{u} \)? (5 Points)

\[ \hat{u} = \text{E}(\hat{u}|x) = 0 \]
\( \text{All or nothing} + 5 \)

c.) Building on 'b', I estimate the following:
\[ \text{Var}(\hat{u}) = \hat{\delta}_0 + \hat{\delta}_{educ} \log(educ) + \hat{\delta}_{exper} \log(exper) \]

If I find a relationship between \( educ \) and/or \( exper \) and \( \text{Var}(\hat{u}) \), what is this evidence of? What assumption would this violate? (5 points)

- Evidence of heteroskedasticity
- This would violate the 5th assumption, homoskedasticity
d.) In the same vein as 'c', suppose that I estimate the following:

\[ \hat{u} = \alpha_0 + \alpha_{educ} \log(\text{educ}) + \alpha_{exper} \log(\text{exper}) + \varepsilon \]

What values should I get for all three \( \alpha \)'s? Why? (5 points)

\[ 0 \text{ 's } \log\text{ all three } + 3 \]

\[ \hat{u} = E(u|I\times) = E(u|\log(\text{educ}),\log(\text{exper})} = 0 + 2 \]

e.) The Dude is a US resident, and I am interested in estimating the impact of education and experience on The Dude's earnings. To accomplish this goal, I follow The Dude for 40 years, tracking his wealth, education level, and experience. What kind of data is this? (5 points)

Time-series data + 5

f.) Suppose that I estimate \( \hat{\beta}_0 = 3 \). Please interpret this estimate. (5 Points)

(answers in terms of \( \log(\text{Wealth}) \) are fine. Hint, remember that \( \log(1) = 0 \))

On average, a person with 1 year of education and 4 years of experience earns \( \log(\text{wealth}) = 3 \)

+ 2

\[ + 1 \]
3. (15 Points) There are economists at UCSC that are interested in the effects of business taxes on offshoring (movement of work to another country). Specifically, these economists generally estimate:

\[ \text{OffshoringShare} = \beta_0 + \beta_{\text{tax}} \log(\text{tax}) + u \]

where \( \text{OffshoringShare} \) is the share of businesses that offshore production of goods and services to a foreign location, taking on values from zero to one. The variable \( \text{tax} \) is the US business tax, and assumed to be positive.

a.) Please use a derivation to interpret the coefficient on \( \hat{\beta}_{\text{tax}} \), and precisely state this interpretation. (10 Points)

\[ \text{Offshore} = \beta_0 + \beta_{\text{tax}} \frac{\text{tax}}{\text{tax}} + 6 \]
\[ \text{Offshore} = \frac{\beta_{\text{tax}}}{100} \left( \% \text{ of tax} \right) \]

A 1% change in tax yields a \( \frac{\beta_{\text{tax}}}{100} \) change in the offshoring share.

b.) Business conditions in the Foreign Market can be characterized by the variable \text{conditions}, where higher values imply better business conditions in the Foreign market. Empirically, suppose that when business conditions abroad improve, the US responds by reducing its own business tax. Further, suppose that when foreign business conditions improve, more businesses have incentive to move production to the foreign market. Suppose we estimate that \( \beta_{\text{tax}} = 0.8 \). In which direction is our estimate \( \hat{\beta}_{\text{tax}} \) biased? Does this affect the qualitative conclusion that higher US taxes increase offshoring? (5 Points)

\[ \text{Offshore} = \beta_1 + \beta_{\text{tax}} \log(\text{tax}) + u + 2 \]

Bias is downward/negative

Since \( \beta_{\text{tax}} > \hat{\beta}_{\text{tax}} > 0 \), \( \beta_{\text{tax}} \) is positive

\[ + 3 \]