Productivity and the role of the global acquisition market

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Abstract

This paper highlights a positive role of foreign ownership in reallocation via acquisitions. Generally, if policy choices reduce the amount of acquisitions that occur, aggregate productivity falls. Provocatively, reciprocal trade liberalization, which has a strong negative effect on acquisition demand from abroad, may reduce aggregate productivity. Further, foreign investment restrictions always reduce aggregate productivity. Overall, the model suggests that both a higher share of inward foreign acquisitions and a higher scale of acquisition activity yield a higher average productivity of target firms - an industry-level productivity gain. Using a large merger database, these predictions are broadly supported.

1 Introduction

Without question, cross-border mergers and acquisitions (M&A) are one of the fastest growing aspects of globalization. In absolute terms, according to the OECD (2001), the value of cross-border M&A increased five-fold over the period 1990-1999. Relatively, the growth of cross-border M&A is also substantial. The share of North American firms that acquire cross-border rather than domestically has increased 133% between 1985 and 2004.1 And, as noted by Navaretti and Venables (2006), cross-border M&A make-up a majority of FDI between developed countries, and are increasing as a share of FDI to developing countries and transition economies.

In most countries, these cross-border transactions are subject to careful review, and sometimes restricted outright. For example, in the United States, even with a fairly open foreign investment policy, all large inward foreign acquisitions are subject to review by the "Committee on Foreign Investment in the United States" (CFIUS). In India, prior to 1991, whole ownership by foreigners was restricted to only minority shares (Chhibber and Majumdar, 1999). In China, acquisition review varies greatly by

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1 Author’s calculation using the Thomson Financial database.
industrial sector, and often involves multiple government agencies with different agendas. Overall, these heterogeneous policies suggest an uncertain understanding of the motivations and consequences of foreign ownership. This point is not lost on the United Nations Conference on Trade and Development; "Indeed, perhaps to a greater extent than many other aspects of globalization, cross-border M&As and the expanding global market for firm ownership and control in which they occur — raise questions about the balance of their benefits and costs for host countries" (UNCTAD, 2000).

However, a fact often lost in policy discussions is that cross-border M&A usually comprise only a minority share of all acquisitions. This is illustrated in Figure 1, which reports the yearly mean and median foreign share of acquisitions over target industry-country pairs for worldwide mergers over the period 1980-2006. Clearly, while the mean prevalence of foreign acquisitions has increased over time, on average (and at the median), foreign acquisitions tend to be less common. Thus, while examining the causes and effects of cross-border M&A is important, it is equally (if not more) important to examine how these causes and effects relate to domestic M&A. Indeed, while firms may consolidate domestically for many of the same reasons they do so internationally, it is the location-dimension that will be most affected by trade and investment policy. Therefore, alongside the role of acquisitions within FDI, three equally appropriate questions arise: (1) why does a firm acquire cross-border rather than acquire domestically, (2) how is this decision affected by policy parameters, and (3) what implications does this have for measures of efficiency and welfare?

Unfortunately, in most acquisition models with heterogeneous firms, the location of acquisitions is restricted for analytical simplicity. For example, in Jovanovic and Rousseau (2002), Breinlich (2007), and Spearot (2008), all acquisitions are homogeneous, assuming an identical location of acquiring and selling firms. Nocke and Yeaple (2007), building on the model of Helpman, Melitz and Yeaple (2004), present a model which is primarily concerned with the choice of foreign investment; greenfield

2For a concise review of Chinese acquisition policy, see http://www.hg.org/articles/article_443.html
or acquisition. In their work, domestic acquisitions are included. However, a simplifying feature is that domestic acquisitions are a function of necessity, where firms only acquire domestically if endowed with capabilities that are insufficient for any production. Put differently, firms do not choose between domestic and foreign acquisitions.

In general, the existing literature fails to model an active decision between domestic and foreign acquisitions. In this paper, I present a tractable investment model with firms choosing between domestic and foreign acquisitions. My theoretical contributions are two-fold. First, I examine how firms sort into acquisition choices by productivity, and how these choices relate to the underlying fundamentals of the model. In particular, I show that a trade-off between production efficiency and market access determines who acquires and who does not. Second, specific to a model with both domestic and foreign acquisitions, I articulate a new channel through which changing costs of trade affect aggregate productivity.

The basic acquisition framework follows Spearot (2008), where firms trade "lumps" of pre-assembled capital after realizing their productivity level. In each country, the market price of these lumps is determined by a perfective competitive acquisition market. By acquiring additional capital, any acquisition can improve the efficiency of variable factors by spreading production over additional units of capital. In addition, foreign acquisitions avoid a per unit trade cost. Since high-productivity firms have the highest incentive to avoid trade costs, firms which acquire abroad will tend to be of higher productivity.

In equilibrium, many standard results arise. In particular, low productivity firms sell their lump of assets to higher productivity firms, and of firms that acquire, those that acquire abroad are more productive. Other equilibrium results are more novel. Similar to Spearot (2008), there exist parameter values such that acquisition demand is made up of both domestic and foreign firms within a mid-range of productivity. However, there also exist parameter values such that the only firms which acquire abroad are at the high-end of productivity. Overall, a number of qualitatively different equilibria are possible.

The focus of this paper, however, is how trade costs and other policy variables affect both the composition of acquisition behavior, and industrial efficiency. The results as they pertain to trade liberalization are provocative. In equilibrium, if foreign acquisitions play a significant role in the acquisition market, trade liberalization will reduce aggregate productivity. What defines a significant role? I find that this result holds unless foreign acquisitions do not occur, or the mass around the most productive acquiring firms (foreign) is small relative to the least productive acquiring firms (domestic).

The intuition for this result is fairly simple. Since the acquisition market dictates a transfer of capital from inefficient firms to more efficient firms, changes to the acquisition market have an effect on aggregate productivity. When all acquisitions are foreign, aggregate productivity worsens with trade

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3 Indeed, this is the critical difference with the traditional heterogeneity literature. In my model, all firms can profitably produce if they choose to do so. The decision to sell and exit is based on the profitability of production relative to the acquisition price. Trade liberalization affects the market through the acquisition mechanism. In Melitz and Ottaviano (2005), for example, the exiting firms (and the effects of trade costs on these firms) are determined by the profitability of production.
liberalization, as foreign firms contract their demand for assets used by inefficient domestic firms. In contrast, when all acquisitions are domestic, aggregate productivity improves with trade liberalization via an expansion of export-driven domestic acquisitions. In between, the effect of trade liberalization on aggregate productivity is analytically ambiguous. This ambiguity results only from the acquisition market clearing condition; the mass of selling firms must equal the mass of buying firms, and the distribution of productivity can always dictate which effect wins out. However, more productive firms, who are more affected by trade costs, are precisely the firms that are more likely to acquire abroad rather than domestically. Thus, the effects of trade costs on these firms tend to dominate.

The case of investment liberalization is more straightforward. By removing foreign investment restrictions, additional demand for domestic assets by foreign firms pushes up the acquisition price, which leads to more inefficient firms selling and exiting. Thus, with a more liberal policy regarding foreign investment, aggregate productivity improves.

A common result in both policy exercises is that a larger share of inward foreign acquisitions may be associated with a higher aggregate productivity of firms that sell. If true empirically, it suggests that, controlling for other factors, foreign acquisitions are beneficial to aggregate efficiency. I examine this and other questions using a worldwide database of acquisitions from Thomson Financial over the period 1985-2006. Since all predictions are related to firm-level productivity, I must find a suitable proxy within the Thomson database. Absent detailed reporting of firm-specific inputs, and given the selected sample of only firms involved in mergers or acquisitions, I assume that productivity is simply approximated by yearly sales and sales per employee.

At the acquisition-level, I find strong support for the model. First, within "standard" acquisitions (those which are not associated with bankruptcy, liquidation, reverse takeover, or restructuring/divestment), I find that relative to the acquiring firm, the target is 5-7 times smaller in terms of overall sales, and 20-26% less productive in terms of sales per employee. While it is hard to gauge the reasonableness of the former result, the latter seems quite sensible. Second, within acquiring country-industry pairs, firms acquiring abroad tend to be 40-50% larger in terms of sales than those acquiring at home, and 7% more productive when using sales per employee.

Finally, examining the relationship between the foreign composition of acquisition demand and the average productivity of target firms, the results strongly support the model. Precisely, within target country-industry pairs, going from no foreign acquisitions to all foreign acquisitions is associated with higher target sales, and higher target labor productivity. Regarding the latter, defining industries at the two-digit level yields a 7-10% increase in productivity, and at the four-digit SIC level, a 3-4% increase in productivity. While quite modest, these results suggest that foreign acquisitions play a small but significant positive role in industry-level efficiency. Further, I provide evidence that acquisitions generally affect aggregate productivity in a positive manner. Consistent with the intuition of the model, there is a significant and positive relationship between the scale of acquisition activity and the average productivity of target firms. Overall, the results from this paper imply that policies which generally restrict acquisition activity, whether domestic or foreign, are likely to be detrimental to aggregate productivity.
Broadly, this paper contributes to a number of different areas. On the theoretical side, it adds to the growing literature examining the role of firm heterogeneity in investment decisions (see for example, Helpman, Melitz and Yeaple, 2004, or Yeaple, 2008). In particular, it adds to the heterogeneity literature by allowing for investment by acquisition (Nocke and Yeaple, 2007). On the empirical side, like Breinlich (2007), the present work examines the relationship between acquisition behavior and industry specific factors. Also, the paper also adds to the growing literature examining the role of productivity in the selection into acquiring or target groups (Breinlich, 2007; Andrade, Mitchell, and Stafford; 2001). Finally, the present work adds to the literature examining the effects of foreign ownership, where in particular, Gopinath and Romalis (2005) show that foreign acquisitions add stock-market value above that of domestic acquisitions, particularly in times of target-country crises.

The rest of the paper is organized as follows. In section two, I develop the theoretical model. In section three, I engage in policy exercises related to trade and investment restrictions. In section four, I test the equilibrium predictions of sections two and three using a large sample of acquisitions. Finally, in section five, I discuss limitations of the model and conclude with areas for future research.

2 Model

The theoretical model used in this paper closely follows Spearot (2008). There are two countries, identical in every respect. The model consists of three stages: an entry/endowment stage, an acquisition stage, and the product market. For simplicity, I focus on the last two stages in this paper. A supplementary technical appendix solves the full-three stage, two-country model.

At the beginning of the acquisition stage, a measure $M$ firms in each country have entered, each endowed with $k$ units of capital located in their home market. At this point, firms realize their productivity, and trade "lumps" of industry-specific capital on a perfectly competitive acquisition market. However, capital from the entry stage is indivisible in the acquisition stage; firms may not buy or sell fractions of capital. Additionally, due to unmodeled organizational factors, it is assumed that a firm only has enough resources to acquire one firm in the acquisition stage. Thus, firms are restricted to three basic options: sell all capital and exit, buy the capital of an exiting firm (at home or abroad), or do nothing.

In the final stage, each active firm supplies its individual variety to the product market. Active firms are monopolists in their own variety, taking other industry variables as given. At this point, any capital accrued during the entry and acquisition stages is fixed, and firms only procure variable factors.

The model is solved by backward induction, and will be introduced in this order.
2.1 Product Market Equilibrium

Consumers

Consumers in each country have preferences over a continuum of varieties and a numeraire good, $x_0$. The numeraire is freely traded at unit cost, and ensures balanced trade. Demand for each variety is assumed to follow a simple linear relationship. The inverse demand function for variety $i$ is written as,

$$p_i = A - bq_i$$  \hspace{1cm} (1)

In (1), $p_i$ is the price of variety $i$, $q_i$ is the quantity of variety $i$, and $A$ and $b$ are demand parameters. To reduce clutter (and many pages), I assume that $A$ is fixed. Under this restriction, there is no substitutability between varieties in the differentiated industry. Thus, entry is not discussed in this paper. In a supplementary technical appendix, I drop this assumption and solve the model with an endogenous value of $A$, pinned down by a free entry condition. Importantly, I discuss conditions such that this value is uniquely determined. Overall, the main results as they pertain to trade liberalization remain unchanged.4

Finally, to capture the standard features of the trade and investment literature as simply as possible, I assume that there are two countries which are identical in every dimension, each with segmented markets for products (varieties) and assets. This will yield a symmetric equilibrium, and facilitate an examination of only one acquisition market. For a model with asymmetric countries and trade costs, see Spearot (2008b).

2.2 Firms

Capital, the industry-specific factor, influences firm decisions through the cost function. Similar to Perry and Porter (1985) and Spearot (2008), the costs of production at any location $j$ will be increasing at an increasing rate. This results from the assumption that capital at location $j$ is fixed at the time of production, yielding diminishing returns to variable factors. Specifically, I assume that the cost of producing $q_j$ subject to $K_j$ is written as:

$$C_j(q_i|\alpha_i, K) = \frac{1}{2} \cdot \frac{q_j^2}{\alpha_i K_j}$$  \hspace{1cm} (2)

In (2), $\alpha_i$ is firm-level productivity. Productivity is continuously distributed according to $g(\alpha)$, defined over $\alpha \in (0, \infty)$. Firm-level productivity is transferrable across all holdings of capital within the firm.

Firms have two markets in which to sell their variety, and two locations at which production can take place. In each, the inverse demand function for each variety is $p_i = A - b \cdot q_i$. A trade cost, $t$, is incurred per unit of exports. For any value of trade cost, each firm solves the following profit

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4This appendix is available at http://people.ucsc.edu/~aspearot.
maximization problem:

\[
\pi(\alpha_i, K^H, K^F) = \max_{q_i, q^e_i, q^F_i} \left\{ \begin{array}{rl}
(A - b \cdot q_i) \cdot q_i + (A - b \cdot (q^e_i + q^F_i)) \cdot (q^e_i + q^F_i) \\
\frac{1}{2} \alpha_i \cdot \left( \frac{q^e_i + q^F_i}{K^H} \right)^2 - \frac{1}{2} \alpha_i \cdot \left( \frac{q^F_i}{K^F} \right)^2 - t \cdot q^F_i 
\end{array} \right. 
\]

such that: \( q_i \geq 0 \), \( q^F_i \geq 0 \), and \( q^e_i \geq 0 \)

In (3), \( q_i \) is the home production of variety \( i \) for home consumption, \( q^e_i \) is home production of variety \( i \) for sale in the foreign country, and \( q^F_i \) is foreign production of variety \( i \) for the foreign market. These production levels are dictated by costs in each country, which as described in (2), are fundamentally dictated by the capital holdings in each country, \( K^H \) at home and \( K^F \) abroad. These capital holdings will be determined in the stage two acquisition market. Lastly, note that export production at home increases the costs of non-export production at home, and vice versa. This will be a particularly important feature when discussing foreign acquisitions.

With this setup, (3) can be solved and profits characterized and compared. Upon entry, all firms are endowed with \( k \) units of capital at home. Thus, firms that do nothing (\( N \)) in the acquisition stage retain their initial capital endowment and earn the following in the product market:

\[
\pi^N(\alpha) = \left\{ \begin{array}{ll}
\frac{A^2 \alpha k}{4 b \alpha k + 2} & \alpha \leq \frac{t}{2 b k (A-t)} \\
\frac{A^2 \alpha k}{2 b \alpha k + 2} - \frac{t (4 b A \alpha k - 2 b k t)}{4 b (2 b \alpha k + 2)} & \alpha > \frac{t}{2 b k (A-t)}
\end{array} \right. 
\]

In (4), when \( \alpha \leq \frac{t}{2 b k (A-t)} \), firms do not export. If a firm’s productivity is too low, the maximum trade-cost adjusted marginal revenue of serving the foreign market is lower than the equilibrium marginal cost of only serving the domestic market. If \( \alpha > \frac{t}{2 b k (A-t)} \), the opposite is the case, where firms have low enough production costs such that additional domestic production intended for exports is optimal.

Firms that purchase domestically (\( B \)) in the acquisition market double their initial capital, holding \( 2k \) at home and nothing abroad. Subject to this capital position, firms earn the following profits from operating in the product market:

\[
\pi^B(\alpha) = \left\{ \begin{array}{ll}
\frac{A^2 \alpha k}{4 b \alpha k + 1} & \alpha \leq \frac{t}{4 b k (A-t)} \\
\frac{A^2 \alpha k}{2 b \alpha k + 1} - \frac{t (8 b A \alpha k - 4 b k t)}{8 b (2 b \alpha k + 1)} & \alpha > \frac{t}{4 b k (A-t)}
\end{array} \right. 
\]

Again, firms of low productivity, \( \alpha \leq \frac{t}{4 b k (A-t)} \), cannot export since their costs are too high after serving the domestic market. For \( \alpha > \frac{t}{4 b k (A-t)} \), firms may optimally serve the both the domestic and foreign market from a domestic source. However, under \( B \), by virtue of lower costs, exporting is more
likely. Precisely, \( \frac{t}{b_k(A-t)} < \frac{t}{2b_k(A-t)} \).

Finally, firms that purchase a foreign firm \( (B^*) \) hold \( k \) units of capital at home, and \( k \) units of capital abroad. The profits from operating under this setup are written as:

\[
\pi^{B^*}(\alpha) = \frac{A^2\alpha k}{(2b\alpha k + 1)}
\]

Since firms have an equal amount of capital in each country (identical costs), profit maximization will never include exports. Thus, trade costs do not enter into equilibrium profits when buying foreign capital.

### 2.3 Acquisition Incentives

Firms must choose between four options in the acquisition stage: Sell their capital and exit \( (S) \), do nothing \( (N) \), buy domestic capital \( (B) \), or buy foreign capital \( (B^*) \). Respectively, the profits of each option can be written as:

\[
\begin{align*}
\Pi^S(R_a) &= R_a \\
\Pi^N(\alpha) &= \pi^N(\alpha) \\
\Pi^B(\alpha, R_a) &= \pi^B(\alpha) - R_a \\
\Pi^{B^*}(\alpha, R_a) &= \pi^{B^*}(\alpha) - R_a - \delta
\end{align*}
\]

Firms pay a fixed cost, \( R_a \), for each lump of capital. This acquisition price will be endogenously determined, in equilibrium, using a market clearing condition for assets. Given the assumption that countries are symmetric in every dimension, I will impose that the acquisition price is the same in each country. For the moment, since firms are assumed to be small outside their own variety, \( R_a \) will be taken as given. Finally, note that in the last equation, firms acquiring abroad pay an additional fixed cost of serving the foreign market, \( \delta \). This is to embody the additional organizational, legal, or marketing costs associated with serving a foreign market.

A firm of productivity \( \alpha \) chooses the acquisition option which maximizes profits in the acquisition market. Defining \( V(\alpha, R_a) \) as acquisition market profits for a firm of productivity \( \alpha \), this decision is formally written as:

\[
V(\alpha, R_a) = \max \left\{ R_a, \pi^N(\alpha), \pi^B(\alpha) - R_a, \pi^{B^*}(\alpha) - R_a - \delta \right\}
\]

The following normalization of (7) is convenient:

\[
\hat{V}(\alpha, R_a) \equiv V(\alpha, R_a) - \pi^N(\alpha) = \max \left\{ R_a - \pi^N(\alpha), 0, \pi^B(\alpha) - \pi^N(\alpha) - R_a, \pi^{B^*}(\alpha) - \pi^N(\alpha) - R_a - \delta \right\}
\]

In \( \hat{V}(\alpha, R_a) \), the profits of each option are normalized relative to the outside option of doing nothing.
2.3.1 Domestic Acquisitions

Before characterizing $\bar{V}(\alpha, R_a)$ as a function of productivity, it is useful to examine the effects of positive trade costs on the incentives to acquire, both domestically and abroad. Defining $\Delta \Pi(\alpha)$ as the profits of a domestic acquisition relative to doing nothing:

$$
\Delta \Pi(\alpha) = \pi^B(\alpha) - \pi^N(\alpha)
$$

(8)

$$
= \begin{cases} 
\frac{A^2_{\alpha k}}{2(2b\alpha k+1)(4b\alpha k+2)} & \text{if } 0 \leq \alpha \leq \frac{t}{4bk(A-t)} \\
\frac{A^2_{\alpha k}}{2(2b\alpha k+1)(2b\alpha k+1)} - \frac{tk(4A-t)}{8(6b\alpha k+1)} & \frac{t}{4bk(A-t)} \leq \alpha < \frac{t}{2bk(A-t)} \\
\frac{A^2_{\alpha k}}{2(2b\alpha k+1)(2b\alpha k+1)} - \frac{tk(4A-t)}{8(6b\alpha k+1)} & \frac{t}{2bk(A-t)} \leq \alpha 
\end{cases}
$$

In (8), the incentives to acquire a domestic firm are split-up into three regions of productivity. For low values of $\alpha$, firms cannot export before or after an acquisition, and the incentive to acquire a domestic firm is identical to the closed economy model. For firms in a middle range of productivity, the acquisition of additional capital provides the cost-improvement required to make exporting profitable. For high productivity firms, exporting is profitable before and after an acquisition. The incentives available to these firms are a combination of the incentives under free trade, and the negative effect of positive trade costs.

Despite its analytical complexity, the behavior of $\Delta \Pi(\alpha)$ in response to changes in productivity is similar to the closed economy and free trade models in Spearot (2008). Precisely, I can prove the following properties of $\Delta \Pi(\alpha)$:

**Lemma 1** $\Delta \Pi(\alpha)$ has the following properties:

$$
\frac{\partial \Delta \Pi(\alpha)}{\partial \alpha} > 0 \quad \text{if} \quad \alpha < \tilde{\alpha}
$$

$$
\frac{\partial \Delta \Pi(\alpha)}{\partial \alpha} < 0 \quad \text{if} \quad \tilde{\alpha} < \alpha
$$

where,

$$
\tilde{\alpha} = \begin{cases} 
\frac{\sqrt{\pi}}{2bk} & \text{if } t \leq \frac{2}{2+\sqrt{2}}A \\
\frac{\sqrt{\pi}}{4bk} & t > \frac{2}{2+\sqrt{2}}A 
\end{cases}
$$

Furthermore,

$$
\Delta \Pi(0) = 0
$$

$$
\lim_{\alpha \to \infty} \Delta \Pi(\alpha) = 0
$$

$$
\frac{1}{2} \pi^B(\alpha) < \pi^N(\alpha)
$$

**Proof.** See Appendix
These incentives are explained as follows. For low productivity firms, costs are high before and after an acquisition, and thus the absolute returns to an acquisition are quite low. The returns are also quite low for high productivity firms, though the reasons are rooted in demand rather than costs. Precisely high productivity firms operate on a more inelastic portion of the demand curve, which limits the marginal incentive to invest. At a high enough level of productivity, the additional revenues catalyzed by a small reduction in variable costs will not be sufficient to compensate for an endogenous acquisition price. Finally, in contrast with low and high productivity firms, mid-productivity firms are relatively constrained by neither low productivity or bounds on market revenues for each variety. Thus, in Lemma 1, mid-productivity firms earn the highest returns from a cost-lowering acquisition.

2.3.2 Foreign Acquisitions and Trade Costs

Now turning attention to foreign acquisitions, the profits from acquiring foreign capital relative to doing nothing ($\Delta \Pi^* (\alpha)$) are written as:

$$
\Delta \Pi^* (\alpha) = \pi^{B^* (\alpha)} - \pi^{N,STC (\alpha)} - \delta
$$

$$
= \begin{cases}
\frac{A^2 \alpha k}{2(2b\alpha k + 1)} - \delta & \alpha \leq \frac{t}{2b\alpha k(A-t)} \\
\frac{A^2 \alpha k}{2(b\alpha k + 1)(2b\alpha k + 1)} - \delta + \frac{t(4b\alpha k - 2b\alpha k t - t)}{8b(b\alpha k + 1)} & \alpha > \frac{t}{2b\alpha k(A-t)}
\end{cases}
$$

In (9), the incentive to acquire a foreign firm is split into two regions of productivity. The first part is for firms of low enough productivity such that exporting is not profitable given $k$ units of capital at home. Since exporting is irrelevant for these firms, a foreign acquisition simply provides additional market access. Since the market is of the same size as the domestic market, a foreign acquisition doubles total profits (with the net increase equaling the size of the original market). The second part of (9) is relevant for firms that are able to export given $k$ units of capital at home. For these firms, acquiring a foreign firm not only affords additional market access, but also diverts export production to a newly purchased foreign affiliate.

It is this production diversion which is crucial to the relationship of trade costs and foreign acquisition incentives. This can be seen by deriving the relevant properties of $\Delta \Pi^* (\alpha)$.

**Lemma 2** Generally, $\Delta \Pi^* (\alpha)$ has the following properties:

$$
\Delta \Pi^* (0) = -\delta
$$

$$
\lim_{\alpha \to \infty} \Delta \Pi^* (\alpha) = \left(\frac{2A-t}{4b}\right) t - \delta
$$

$$
\frac{1}{2} \pi^{B^* (\alpha)} \leq \pi^{N (\alpha)}
$$
Furthermore, if \( t \leq \frac{2}{2+\sqrt{2}} A \), then
\[
\frac{\partial \Delta \Pi^*(\alpha)}{\partial \alpha} > 0 \quad \text{if} \quad \alpha < \frac{\sqrt{2}(2A+t+\sqrt{2}t)}{2k(2A-2t-\sqrt{2}t)}
\]
\[
\frac{\partial \Delta \Pi^*(\alpha)}{\partial \alpha} < 0 \quad \alpha > \frac{\sqrt{2}(2A+t+\sqrt{2}t)}{2k(2A-2t-\sqrt{2}t)}
\]

Conversely, if \( t > \frac{2A}{2+\sqrt{2}} \), then \( \frac{\partial \Delta \Pi^*(\alpha)}{\partial \alpha} > 0 \) for all \( \alpha \).

**Proof.** See Appendix □

Lemma 2 is a central result in the open economy model. It states that if trade costs are low, mid-productivity firms will have the highest incentive to acquire a foreign firm. In contrast, for high trade costs, high-productivity firms have the highest incentive to purchase a foreign firm. The intuition for this result is best explained by how trade costs affect incentives for market access and variable factor efficiency.

If trade costs are non-zero, purchasing a foreign firm always provides additional market access. Similar to Helpman, Melitz, and Yeaple (2004), the incentive to gain additional market access is increasing in productivity. High productivity firms are the largest exporters, and given that exporting costs are independent of productivity, these firms have the highest incentive to avoid these costs. Thus, absent any other incentives, high-productivity firms always have the highest incentive to invest abroad.

However, in this particular model, there are additional incentives on the cost-side. Not only does purchasing a foreign firm provide additional market access, but it will also under certain conditions divert export production that would otherwise be produced at home. Critically, given the fixedity of capital, higher production in one particular location increases the marginal cost of producing at that location.\(^5\) By spreading production across capital at home and abroad, a firm can improve production costs similar to a domestic acquisition. And, as has been discussed above, mid-productivity firms have the highest incentive to acquire to improve production costs when faced with a bounded market for their variety.

Thus, the central issue is whether market access or production costs considerations dominate. As characterized in Lemma 2, the relative size of trade costs provides a resolution to this issue. This is also illustrated in Figure 2. When trade costs are relatively high (\( t_{\text{high}} \) in Figure 2), exports are minimal to zero, and the incentives to gain additional market access dominate. Thus, high productivity firms have the highest incentive to acquire abroad. In contrast, when trade costs are low (\( t_{\text{low}} \) in Figure 2), foreign sales via exports are already significant and the incentive to gain market additional access is trumped by the incentive to lower production costs. In this case, mid-productivity firms have the highest incentive to acquire abroad.

Essentially, one can think of foreign acquisitions as a skewed version of domestic acquisitions. If trade costs are zero, they function identically to domestic acquisitions, save for the fixed cost \( \delta \).

\(^5\)Indeed, this is the critical difference with other cost assumptions, such as total costs written as \( C(q) = \frac{1}{\eta k} q \). Here, diverting export production would yield no cost-side effects.
However, as trade costs increase, the incentives to acquire a foreign firm become skewed toward firms of higher productivity. Eventually, if trade costs are high enough, the incentive to acquire a foreign firm is a monotone increasing function of productivity, and looks nothing like the incentive to acquire domestic firm.\footnote{One important implication of this characteristic is how target assets will be utilized. Specifically, if target assets are randomly assigned to firms on the demand side, the average productivity gain of each transaction will be larger if the asset is acquired by a foreign firm.}

While these properties are novel, they are a function of (and must be compared with) the aggregate measures \( (A, R_a) \) that are pinned down in general equilibrium. Again, the analysis of \( A \) is relegated to a supplementary technical appendix. However, the acquisition price \( R_a \) is central to the results in this paper, and is addressed below. Crucially, depending on the size of the acquisition price, the non-monotone features of domestic and foreign acquisitions can lead to an array of possible equilibrium firm sortings. Rather than sift and discuss each possible outcome, I will focus on only one outcome to prove that an equilibrium exists, and to analyze the effects of policy parameters on the equilibrium sorting of firms and aggregate efficiency.

### 2.4 Equilibrium

With the incentives for domestic and foreign acquisitions in hand, I now characterize the equilibrium of the model subject to small trade and small foreign investment costs. Precisely, the following lemma characterizes the region over which I am solving the model.

**Lemma 3** There exists a \( \Gamma \) and \( \delta \) such that, over the space \( [0, \Gamma] \times [0, \delta] \), any acquisition stage equilibrium must satisfy the following properties:
1. All active firms can export

2. Domestic and foreign acquisitions are not trivially unprofitable relative to the acquisition price.

3. \( \frac{(2A-\alpha)t}{4b} - \delta < R_a \)

**Proof.** See Appendix ■

Will all details relegated to the appendix, Lemma 3 is proven using free trade as an analytical benchmark. To setup the equilibrium conditions of the model, I now discuss each condition outlined within Lemma 3.

Condition 1 simplifies the model substantially. Without Condition 1, export status conditional on acquisition status would have to be analyzed alongside acquisition choices. This would make the problem much more complex. Functionally, with Condition 1, I can restrict attention to the highest ranges of productivity in (4), (5), (8), and (9). Condition 2 states that at least some firms find a domestic acquisition or foreign acquisition profitable relative to the acquisition price. If this was not the case, then I could simply dispose of one type of acquisition depending on the value of trade costs. Finally, Condition 3 states that over \( [0, \delta] \times [0, \delta] \), the highest productivity firms will not find any acquisition profitable. Simply put, I am proving that there exists a range of trade and investment costs in equilibrium such that high productivity firms do not invest, whether at home or abroad. Functionally, this guarantees the existence of two productivity cutoffs related to foreign acquisitions.

With Lemma 3 in-hand, attention can now turn to the determination of firm-level behavior. Firms are indifferent between doing nothing and selling at \( \alpha_S \). This cutoff is defined by the following:

\[
\pi^N(\alpha_S) = R_a
\]

where,

\[
For \alpha < \alpha_S \, , \, S \succ N
\]

Firms prefer selling if the acquisition price is greater than the return from staying in the market.

At \( \underline{\alpha}_B \) and \( \overline{\alpha}_B \), firms are indifferent between doing nothing and buying domestic capital. Precisely, these cutoffs are defined by the following equations:

\[
\Delta \Pi(\underline{\alpha}_B) = R_a
\]

\[
\Delta \Pi(\overline{\alpha}_B) = R_a
\]

where,

\[
For \alpha \in (\underline{\alpha}_B, \overline{\alpha}_B) \, , \, B \succ N
\]

Here, similar to Spearot (2008), firms within a mid-range of productivity find a domestic acquisition profitable. The existence of \( \underline{\alpha}_B \) and \( \overline{\alpha}_B \) is guaranteed by Condition 2 in Lemma 3.

Similarly, defining \( \underline{\alpha}_B^* \) and \( \overline{\alpha}_B^* \) as the levels of productivity such that firms are indifferent between foreign acquisitions and doing nothing, the indifference between these two options is summarized by
the following:

\[ \Delta \Pi^* (\alpha_B^*) = R_a \]  
(15)

\[ \Delta \Pi^* (\bar{\alpha}_B) = R_a \]  
(16)

where,

\[ For \ \alpha \in (\alpha_B^*, \bar{\alpha}_B), \ \bar{\alpha}_B \succ N \]  
(17)

The existence of two productivity cutoffs, \( \alpha_B^* \) and \( \bar{\alpha}_B \), are guaranteed by Conditions 2 and 3 in Lemma 3.

Finally, for \( \frac{(2A-t)}{4b} - \delta > 0 \), the indifference point between foreign acquisitions and domestic acquisitions, \( \alpha_{BB^*} \), is defined as follows:

\[ \Delta \Pi (\alpha_{BB^*}) - \Delta \Pi^* (\alpha_{BB^*}) = 0 \]  
(18)

\[ \alpha_{BB^*} = \frac{(t^2 + 8\delta b)}{4bk((2A-t)t - 4\delta b)} \]

where,

\[ \frac{\partial (\Delta \Pi (\alpha, A) - \Delta \Pi^* (\alpha, A))}{\partial \alpha} = -\frac{tk(4A-t)}{4(2bck+1)^2} < 0 \]

Thus,

\[ For \ \alpha > \alpha_{BB^*}, \ B^* \succ B \]  
(19)

The most productive firms prefer foreign acquisitions to domestic acquisitions, where only the most efficient are able to recover the added fixed cost of foreign investment. For \( \frac{(2A-t)}{4b} - \delta < 0 \), where the maximum market access benefit, \( \frac{(2A-t)}{4b} \), is dominated by the fixed cost, \( \delta \), domestic acquisitions are preferred to foreign acquisitions for all \( \alpha \).

Figure 3 illustrates one possible solution to the acquisition choice problem for a specific pair of \( t \) and \( \delta \) in which both domestic and foreign acquisitions occur, and the three conditions in Lemma 3 are satisfied. In Figure 3, the least productive firms choose to sell and exit. Firms in a mid-range of productivity find some type of acquisition profitable, where the most-efficient firms within this group prefer foreign acquisitions. Of the remaining active firms, the least productive and most productive choose to do nothing in the acquisition market. For the highest productivity firms, the incentives to avoid trade costs are not sufficient to compensate for the acquisition price, which itself has been bid-up by mid-productivity firms.

Given Lemma 3, Proposition 1 summarizes optimal acquisition choice as a function of small \( t \) and small \( \delta \).7

**Proposition 1** Given \( R_a \), and \( (t, \delta) \in [0, \hat{t}] \times [0, \hat{\delta}] \), there exist functions \( \underline{t}(\delta) \) and \( \overline{t}(\delta) \) such that the

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7The curvature in Figure 4 is approximate.
composition of the acquisition market is the following:

For \( t \leq \ell(\delta) \) only domestic acquisitions occur

\[ \ell(\delta) < t < \overline{\ell}(\delta) \] both domestic and foreign acquisitions occur

\[ \overline{\ell}(\delta) \leq t \] only foreign acquisitions occur

Furthermore, for \( \overline{\ell}(\delta) \leq \hat{t} \), optimal acquisition choice can be represented by Figure 4.

Proof. See Appendix

The role of trade costs in Figure 4 is fairly intuitive. In Figure 4, \( \Theta_S \) represents the measure of firms that sell and exit the market. Given that all firms are exporters, higher trade costs diminish the value of staying in the market, leading to a larger measure of selling firms. Also in Figure 4, \( \Theta_B \) and \( \Theta_{B^*} \) represent the measure of firms that buy domestic and foreign capital, respectively. Intuitively, as trade costs increase, domestic acquisitions become less profitable and foreign acquisitions become more profitable. In Figure 4, this is illustrated in the contracting range of domestic acquisitions, \( \Theta_B \), and the expanding range of foreign acquisitions, \( \Theta_{B^*} \). Finally, there exist two regions, \( \Theta_N \) and \( \overline{\Theta}_N \), such that firms do nothing.\(^8\)

An important feature of Figure 4 is that the set of firms that acquire domestically, \( \Theta_B \), and the set of firms that acquire abroad, \( \Theta_{B^*} \), are non-nested. In other words, acquiring domestically is not the next best option every firm that acquires abroad, and vice versa. For some firms, the next best option to a domestic or foreign acquisition is no acquisition. The crucial implication of this feature for the forthcoming policy analyses is that there exist margins through which both domestic and foreign

\(^8\)The formal representation of \( \Theta_S, \Theta_B, \Theta_{B^*}, \Theta_N, \overline{\Theta}_N \) are contained in the appendix, as a function of \( t \).
acquisition demand can influence total acquisition demand. This would not be the case if a CES-type demand function was used subject to the same cost-structure, where since all incentives are increasing in productivity, there exists only one margin of adjustment.

Lastly, Figure 4 illustrates two predictions of the model which can be easily tested given available data. These predictions are the following:

**Equilibrium Prediction 1** Acquiring firms are more productive than target firms

**Equilibrium Prediction 2** Firms acquiring abroad are more productive than those acquiring domestically.

Both predictions are fairly standard, and result from a wide class of models. Thus, before using the data to address the novel implications of the forthcoming policy section, it will be helpful to first confirm (empirically) the above simple equilibrium predictions.

**General Equilibrium**

To close the equilibrium of the model, I now characterize the key general equilibrium parameter, $R_a$. The assumption of fixed $R_a$ restricts the above discussion to a partial equilibrium analysis. Since interactions between buying and selling firms will ultimately dictate changes to industry efficiency, I

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9. One exception in Nocke and Yeaple (2007), where if capabilities are immobile across borders, firms that acquiring domestically are on average more productive than those that acquire abroad.

10. The one prediction missing from the above list is that firms in a middle range of productivity are more likely to acquire. Absent a database of all active firms, this is not testable. However, this prediction is confirmed using a database of North American firms in Spearot (2008).
now characterize how these elements determine the acquisition market clearing price, $R_a$, in general equilibrium.

Subject to firm-level acquisition decisions, the acquisition market must clear. Formally, this condition is written as:

$$\int_{\alpha \in \Theta_B} dG(\alpha) + \int_{\alpha \in \Theta_B^*} dG(\alpha) = \int_{\alpha \in \Theta_S} dG(\alpha)$$  \hspace{1cm} (20)

It is straightforward to show that there exists a unique acquisition market clearing price, $R_a$. Domestic and foreign acquisition demand are both decreasing in the acquisition price, while the supply is increasing in the acquisition price. Thus, there exists an intersection of demand and supply at $R_a$ such that there is a positive level of acquisition activity. Using the parameter space characterized in Lemma 3, and the acquisition behavior summarized in Proposition 1, this is proven in the following proposition.

**Proposition 2** Subject to the parameter space defined in Lemma 3 and optimal firm behavior in Proposition 1, there exists a unique $R_a$ that clears the acquisition market.

**Proof.** See Appendix \[\blacksquare\]

Proposition 2 proves that there exists a unique solution to the model presented in this section. Focusing on the acquisition market itself, higher trade costs shift up foreign acquisition demand and shift down domestic acquisition demand. In general, the acquisition market dictates how changes to the incentives affecting one type of firm (a selling firm, for example) spread throughout the economy affecting other types of firms. Given Lemma 3 and Proposition 1, falling trade costs will directly impact all firms. However, the effects on buying firms will generally be larger than the effects on selling firms. Changes in foreign investment restrictions, embodied by the level of $\delta$, may also affect the relative incentives of each type of acquisition. I now examine the effects of these policy parameters, focusing on changes to firm-level behavior, and implications for aggregate efficiency.

### 3 Policy Implications

#### 3.1 Trade Liberalization

To begin this section, I examine the effects of trade costs on acquisition decisions, and the corresponding effects on aggregate productivity. Specifically, I examine the effects of trade costs when the combination of $t$ and $\delta$ is prohibitive to foreign acquisitions occurring in equilibrium. In Figure 4, this is the region where $t < t^*(\delta)$. These effects are summarized in the following Proposition.\[11\]

\[11\] In the supplementary technical appendix, allowing $A$ to be endogenous, it is shown that $\frac{\partial A}{\partial t} > 0$. That is, higher trade costs lessen competition among varieties. This is consistent with Nocke and Yeaple (2006), and Melitz and Ottaviano (2005).
Proposition 3  For $t < \xi(\delta)$, where no foreign acquisitions occur, higher trade costs diminish acquisition activity. In terms of productivity cutoffs:

$$\frac{\partial \alpha_S}{\partial t} < 0 \quad \frac{\partial \alpha_B}{\partial t} > 0 \quad \frac{\partial \sigma_B}{\partial t} < 0$$

Proof. See Appendix. ■

The effects of trade costs are driven by the interaction of the marginal acquiring and marginal selling firms. At a constant acquisition price, higher trade costs diminish domestic acquisition demand. Since higher trade costs reduce the effective size of the world market, fewer firms have sufficient incentives to increase production via domestic acquisitions. On the supply side, higher trade costs reduce the value of staying in the market, which increases the supply of selling firms. In equilibrium, the effects on the demand side dominate. Since selling-firms are very small exporters, the marginal effect of trade costs on these firms is minimal. Thus, higher trade costs shrink equilibrium acquisition behavior.12

The effect of higher trade costs leads to lower aggregate industry productivity via reallocation in the acquisition market.13 Since Proposition 1 dictates that acquisitions transfer capital from the least efficient firms to more efficient firms, and acquisition activity shrinks with higher trade costs, average industry productivity falls. Put differently, it is possible for less-efficient firms to remain in the market; $\frac{\partial \alpha_S}{\partial t} < 0$.

The response of aggregate productivity to trade costs is similar to that in Melitz and Ottaviano (2005), although their results are via a different exit mechanism. In their work, firms exit if they cannot profitably produce in the product market, where higher trade costs make this outcome more likely. In my model, all firms have the opportunity to profitably sell to the product market, since the marginal cost of the first unit of production is zero. However, firms of low productivity find it in their best interest to sell their lump of capital to firms who are better able to utilize it.

The results of Proposition 3 are qualitatively identical to the results in work by Breinlich (2007). He also tests this prediction on a firm-level sample of US and Canadian firms, where the response of the acquisition market to trade costs is confirmed in the data. That is, trade liberalization expands acquisition activity. However, one important feature within the US-Canada case study is that domestic acquisitions comprise roughly 95% of all acquisitions. Thus, Breinlich’s empirical work is well motivated by a model with only domestic acquisitions. When addressing the policy predictions of this paper, it will be more important to address the effects of foreign acquisitions, as a much larger share of acquisitions will be cross-border in my sample.

Next, the equilibrium effects of trade costs are examined when both domestic and foreign acquisitions occur simultaneously. In Figure 4, this occurs when $\xi(\delta) < t < \bar{t}(\delta)$. The effects of trade costs within this region are summarized in Proposition 4.

Proposition 4  For $\xi(\delta) < t < \bar{t}(\delta)$, where both domestic and foreign acquisitions occur, the effect of

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12 This result would remain if the marginal selling firm did not export.

13 I define industry productivity as the average value of $\alpha$ for active firms. Thus, $\frac{\partial \sigma_B}{\partial t}$ is a sufficient statistic to examine changes in industry productivity.
trade costs on acquisition activity is ambiguous. In terms of productivity cutoffs:

\[ \frac{\partial \alpha_S}{\partial t} \leq 0 \quad \frac{\partial \alpha_B}{\partial t} > 0 \quad \frac{\partial \alpha_B^*}{\partial t} < 0 \quad \frac{\partial \sigma_B}{\partial t} > 0 \]

However, if \( \frac{g(\pi_B^*)}{g(\alpha_B)} \) is sufficiently large, then \( \frac{\partial \alpha_S}{\partial t} > 0 \).

**Proof.** See Appendix. ■

Since the highest productivity firms do not acquire, foreign acquisition activity may expand to higher productivity firms. This is the key source of the ambiguity in Proposition 4. Both domestic and foreign acquisitions have an effect on total acquisition demand. Since foreign acquisitions expand and domestic acquisitions contract in response to higher trade costs, the effect of trade costs on the acquisition market is theoretically ambiguous.14

The refinement of Proposition 4 is more striking. Within the acquisition market clearing condition, the effects on \( \pi_B \) will be dominant over \( \alpha_B \) so long as \( g(\pi_B) \) is not too small relative to \( g(\alpha_B) \). If this is the case, then \( \frac{\partial \alpha_S}{\partial t} > 0 \), and trade liberalization will decrease aggregate productivity. However, it is important to note that the productivity advantage of foreign acquisitions relative to domestic makes this outcome more likely. Firms which acquire abroad are more productive, and thus more concerned with foreign market access. Independent of the productivity distribution, the effect of trade costs on these firms (including \( \pi_B \)) are larger than lower productivity firms (including \( \alpha_B \)). If this was not the case, then \( \frac{g(\pi_B^*)}{g(\alpha_B)} \) would have to be much higher for \( \frac{\partial \alpha_S}{\partial t} > 0 \).

To complete the analysis, I examine the region of trade costs such that \( t > T(\delta) \), where only foreign acquisitions occur. The effects within this region are summarized in the following Proposition:

**Proposition 5** For \( t > T(\delta) \), where only foreign acquisitions occur, higher trade costs increase acquisition activity. In terms of productivity cutoffs:

\[ \frac{\partial \alpha_S}{\partial t} > 0 \quad \frac{\partial \alpha_B^*}{\partial t} \leq 0 \quad \frac{\partial \sigma_B}{\partial t} > 0 \]

**Proof.** See Appendix. ■

In Proposition 5, the qualitative movement of productivity cutoffs is nearly opposite of the movement in Proposition 3. Since foreign acquisitions expand with higher trade costs, the demand for domestic assets increases. Thus, more low-productivity firms exit the market via a sell-off. This tends to increase aggregate productivity.

In Propositions 4 and 5, the effect of trade costs on aggregate productivity is different than in Melitz and Ottaviano (2005). Again, this highlights the assumption of firm-exit via a sell-off. In their work, the marginal exiting firm is determined by the profitability of production. Independent

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14In contrast, a CES demand system subject to the same cost structure would not deliver such a prediction. Since revenues are effectively unbounded, the incentives to acquire another firm are increasing in productivity, and the most efficient firms will choose foreign acquisitions. In equilibrium, total acquisition demand must decrease with higher trade costs, as domestic acquisitions are the only type of acquisition that affect total acquisition demand on the margin. Thus, higher trade costs, which promote foreign acquisitions relative to domestic, unambiguously reduce aggregate productivity.
of the decisions facing other (more productive) firms, higher trade costs decrease market toughness, leading to fewer exiting firms that find production unprofitable.

In my model, the measure of exiting firms is determined by the relative value of the acquisition price, where the relative value of the acquisition price is positively affected by the willingness of other firms to pay for an acquisition. In the case outlined in Proposition 5, foreign firms are the only acquiring firms, and higher trade costs increase their willingness to pay for an acquisition. Thus, a greater share of (inefficient) firms find it in their best interest to sell and exit.

3.2 Investment Liberalization

Continuing, I now examine the effects of foreign investment liberalization on acquisition decisions, and the corresponding effects on aggregate productivity. The proxy for foreign investment liberalization will be $\delta$, with larger values of $\delta$ corresponding with more restrictive investment policies. Naturally, foreign investment liberalization will only affect equilibria if foreign investment occurs. Thus, I will restrict attention to the case in which $t > t(\delta)$. First, focusing on $\underline{t}(\delta) < t < \overline{t}(\delta)$, the effects of $\delta$ are the following:

**Proposition 6** For $\underline{t}(\delta) < t < \overline{t}(\delta)$, where both domestic and foreign acquisitions occur, higher values of $\delta$ decrease acquisition activity. In terms of productivity cutoffs:

$$\frac{\partial \alpha_S}{\partial \delta} < 0 \quad \frac{\partial \alpha_B}{\partial \delta} < 0 \quad \frac{\partial \alpha_{BB}^*}{\partial \delta} > 0 \quad \frac{\partial \alpha_B^*}{\partial \delta} < 0$$

**Proof.** See Appendix. ■

The intuition for Proposition 6 is fairly simple. With higher foreign investment costs, foreign acquisition demand for domestic assets decreases. This depresses the price for assets, which makes domestic acquisitions more profitable, and allows for more inefficient firms to remain in the market. Thus, aggregate productivity falls.

To conclude the analysis of investment liberalization, a similar result obtains when analyzing the range of highest trade costs, $t > \overline{t}(\delta)$.

**Proposition 7** For $t > \overline{t}(\delta)$, where only foreign acquisitions occur, higher values of $\delta$ decrease acquisition activity. In terms of productivity cutoffs:

$$\frac{\partial \alpha_S}{\partial \delta} < 0 \quad \frac{\partial \alpha_B^*}{\partial \delta} > 0 \quad \frac{\partial \alpha_B^*}{\partial \delta} < 0$$

**Proof.** See Appendix. ■

This result is virtually the same as Proposition 6, with the exception that all acquisition activity is comprised of foreign acquisitions. Again, higher foreign investment costs depress foreign acquisition demand, diminish the acquisition price, which yields a higher share of inefficient firms choosing to stay active in the acquisition market.
3.3 Discussion and Empirical Questions

The results of this section can be summarized as a single theme. The effects of policy parameters on aggregate productivity are driven by the demand side of the acquisition market. Whether trade or investment, if the manipulation of a policy results in an overall reduction in demand for industry specific assets, the effects on aggregate productivity will be negative.

Interestingly, trade and investment liberalization each may have different effects on the overall utilization of assets. From Propositions 6 and 7, it is clear that foreign investment liberalization will tend to increase average productivity. Trade liberalization may or may not, depending on the relative importance of foreign acquisitions. If the response of foreign demand to tariffs is greater than the response of domestic demand, then reciprocal trade liberalization will tend to reduce aggregate productivity. Practically, these issues are important, as a contentious point of political debate ranges from the treatment of foreign imports to the treatment of foreign firms buying domestic assets. Poorly motivated policy may have unintended consequences for the utilization of assets and growth. Thus, empirical work is necessary to ascertain the sensitivity of the asset (acquisition) market to policy choices.

Unfortunately, examining the effects of trade and investment liberalization is difficult, since these policies are often non-valued and also can be highly colinear. Broad liberalization episodes often include both trade liberalization and foreign investment liberalization, where for example, India engaged in both significant trade and investment liberalization between 1985-1991 (see Chhibber and Majumdar, 1999; Panagariya 2004). Further, although steps have been taken to quantify non-tariff barriers to trade (Kee et al. 2007), the quantification of investment policies is still lacking. Overall, without a precise measure of such policies that is comparable across countries over a long time frame, the analysis of trade and foreign investment restrictions must remain a case-based approach.

Given these issues, rather than looking specifically at each type of policy and the effects on acquisition behavior, I will engage in a broad analysis of the relationship between foreign acquisition behavior and predictions related to aggregate productivity. Guided by the theory, the first relationship is unambiguous:

**Prediction 1** From a given country-industry pair, a higher composition of firms acquiring abroad yields a higher average productivity of acquiring firms.

This prediction can be seen across both policy studies. In Proposition 4, for example, as trade costs increase, this increases the profitability of foreign acquisitions, and decreases the profitability of domestic acquisitions. Since the model dictates that foreign acquiring firms are more productive than domestic acquiring firms, the change in trade costs yields a higher average productivity of firms that acquire. A similar (and simpler) result obtains when looking at the effects of foreign investment costs. Essentially, Prediction 1 is an industry level prediction related to a tariff jumping motive for investment; only higher productivity firms can afford to invest abroad, and anything that makes this choice more likely will tend the skew the set of acquiring firms to those which are more productive.
The second prediction is more of a question, which is whether or not a higher share of inward foreign acquisitions is associated with higher or lower aggregate productivity. This is a relevant question, as countries may control the mix of investment via trade and investment restrictions. Broadly, the relationship between the foreign share of inward investment and aggregate productivity will identify the value of foreign acquisitions relative to domestic. Thus, the second issue to examine within the forthcoming empirical framework is the following:

**Question 1** Within a given country-industry asset market, does a higher foreign composition of acquisition demand yield a higher average productivity of target firms?

Looking at the effects of foreign investment restrictions, the theoretical model unambiguously predicts that a higher share of foreign acquisitions will yield a higher aggregate productivity of active firms. Higher trade costs will also increase the share of foreign acquisitions, but may or may not increase aggregate productivity. This depends on the degree to which a liberalization episode is reciprocal, and the relative response of domestic and foreign acquisition demand. While it is difficult to identify whether or not each individual policy will improve aggregate productivity when liberalized, to primary goal is to ascertain the overall effect of foreign acquisitions on the utilization of industry specific assets. If there exists a significant and non-trivial positive effect of inward foreign acquisitions on industry efficiency, this suggests that countries should be careful when enacting policies which diminish the role of foreign investors, even if the goal is to substitute toward domestic acquisition demand.

4 Empirics

This section tests the predictions/questions detailed in the previous sections. Acquisition data is obtained from the Thomson Financial database, which contains information on firm-level acquisitions, the identity of target and acquiring firms, the location and industry of these firms, and for some firms, detailed balance sheet data. Within the mergers literature, this database has been used by Breinlich (2007) and Gugler et. al. (2003). There are 384311 mergers and acquisitions in the Thomson database over the period 1981-2006.

4.1 Equilibrium Predictions

To begin, I will test the equilibrium predictions from section two. Specifically, at the merger-level, I will examine whether target firms tend to be less productive than their acquiring firm counterparts.
Target - Acquiring firm productivity

The first prediction is that acquiring firms are more productive than selling firms. A simple firm-level specification to test this prediction is the following:

\[
\log(A_{prod,j,t}) - \log(T_{prod,j,t}) = \beta_0 + \beta_{Out} Out_{j,t} + \beta_{Restruct} T_{Restruct,j,t} + \beta_{Divest} Divest_{j,t} + \beta_{Liquid} T_{Liquid,j,t} + \beta_{Bankrupt} T_{Bankrupt,j,t} + \beta_{Reverse} Reverse_{j,t} + \epsilon_{j,t}
\]

In (21), the dependent variable is the difference in the log productivity levels of the acquiring firm and the target firm for acquisition \( j \) in year \( t \). I hypothesize this value to be positive, on average, for each acquisition in the dataset.

Testing this prediction, first and foremost, requires computing the log difference in acquiring firm and target firm productivity levels. To do this, I would ideally have data to compute a true productivity measure, or data to construct a large database of \( Q \) values, as in Spearot (2008). Unfortunately, constructing the former requires a database of all active firms (not just those that merge), which I do not have. Constructing the latter requires market values, which are not reported for most firms in the sample. Thus, I am forced to rely on fairly simple measures of productivity, which are acquiring firm and target sales (in Million nominal US$), and acquiring firm and target sales per employee (million nominal US$ per thousand employees).

Again, controlling for merger type, I expect the average acquisition to involve a buyer that is more productive than the seller. How is this prediction tested? I make the assumption that the difference between buyer and seller productivity is a function of controls which determine the type of merger. The model in section two contained within-industry acquisitions involving firms that could each produce profitably, absent the acquisition. I consider these mergers to be "normal" mergers. Implicit in this assumption/definition is that the target was neither bankrupt, restructuring, or liquidating assets. In (21), \( T_{Restruct,j,t}, T_{Liquid,j,t}, \) and \( T_{Bankrupt,j,t} \) each take on a value of one when the target within acquisition \( j \) in year \( t \), falls in each respective category, and zero otherwise. Further, \( Out_{j,t} \) identifies mergers which occur outside of the acquiring firm’s two-digit SIC industry.

In addition, the model dictates that one firm buys another, and does not assume any sort of sophisticated merger technique. Thus, the target firm should not be a divestment of a larger firm, nor should the acquisition involve a reverse takeover (where a private firm buys up the shares of a public firm and recapitalizes the firm into the private firm). These type of acquisitions are marked as \( Divest_{j,t} \) and \( Reverse_{j,t} \), respectively, taking on a value of 1 if falling into each category, and zero otherwise.

Thus, to test this first prediction using (21), \( \beta_0 \) should be significantly greater than zero. Precisely, this states that for acquisitions that do not fall into one of the aforementioned "special" categories, the average difference in productivity between the buying and selling firms is positive. The results from running the regression in (21) are presented in Table 1.
The results in Table 1 are strongly affirming of the most basic prediction from section two. Looking at the estimates for $\beta_0$, they are broadly positive, and with the exception of the last regression, are quite significant at conventional levels. When using sales as a proxy for productivity, and using the generated estimates, the predicted ratio $\frac{\text{ASales}_{i,c,t}}{\text{TSales}_{i,c,t}}$ for a "normal" acquisition ranges between 5.61 and 7.81. These estimates, or at least interpreting them as productivity differences, are quite unreasonable. The estimates are much more reasonable, however, when using sales per employee as a measure of productivity. Precisely, the predicted ratio $\frac{\text{ASalesEmp}_{i,c,t}}{\text{TSalesEmp}_{i,c,t}}$ falls between 1.2 and 1.26. Thus, acquiring firms are predicted to be between 20% and 26% more productive than target firms.

Other control variables produce estimates which are very sensible. In Table (1), $\beta_{\text{Restruct}}$ is negative. This prediction is significant when using sales, but not when using sales per employee. Thus, if an acquisition involves a large target which is restructuring, perhaps due to a poor productivity shock, it is more likely that the acquire-target sales ratio will be smaller. This same intuition holds when looking at targets that go bankrupt. In Table (1), $\beta_{\text{Bankrupt}}$ is significantly negative.

Finally, acquisitions that involve a reverse takeover are more likely to have a smaller acquiring firm and/or a larger target. Interestingly, the estimated ratios for reverse takeovers are 0.64 when looking at sales, and 0.93 when looking at sales per employee. Thus, for reverse takeovers it is predicted that the seller is actually larger than the target. This may be due to reverse takeovers being more likely when the stock price of the target is low.
Foreign and domestic acquisitions

The second equilibrium prediction is that firms acquiring abroad are more productive than firms acquiring domestically. Since a given acquisition involves only one firm acquiring either domestically or abroad, the approach will be slightly different than in (21). Specifically, compiling a dataset of acquiring firms, I estimate the following specification:

\[
\log(A_{prod,j,t}) = \beta_0 + \beta_{Cross}Cross_{j,t} + \beta_{AMulti}AMulti_{j,t} + \beta_{Out}Out_{j,t} + \beta_{Year}Year_{j,t} + \beta_{Country-SIC}C_{-SIC} + \epsilon_{j,t}
\]  

(22)

In (22), \( A_{prod,j,t} \) is the productivity of acquiring firm \( j \) in year \( t \), \( Cross_{j,t} \) is an indicator specifying whether the acquisition is abroad rather than at home, \( AMulti_{j,t} \) an indicator whether the acquiring firm is a multinational, and \( Out_{j,t} \) an indicator measuring whether the acquiring firm is purchasing a firm outside its primary SIC2 industry.

I hypothesize \( \beta_{Cross} \) to be positive, as specified by the theory. That is, controlling for other factors, I expect firms acquiring abroad to be more productive than those acquiring domestically. Further, I also hypothesize \( \beta_{AMulti} \) to be positive, since multinational firms tend to be more productive than their non-multinational counterparts (for an analysis of US firms, see Yeaple, 2008). I have no specific hypothesis for \( \beta_{Out} \) generated within the present paper, though Qiu (2007) suggests that this should be positive. Finally, I include controls for broad year effects, and acquiring country-industry fixed effects. Since the number of the latter will be very large, I will utilize the within estimator to generate predictions for the parameters in (22). The results from doing so are available in Table 2.

The results in Table 2 are clearly in-line with the theory presented in section two. Regardless of the level of industry refinement, or choice of productivity measure, firms that acquire abroad tend to be more productive than those that acquire domestically. Looking at the range of point estimates, firms acquiring abroad are 42-52% bigger in terms of sales, and roughly 7% more productive in terms
of labor productivity. Looking at the other variables, multinational status seems to have a non-trivial positive effect on the productivity of acquiring firms. While not significant when looking at sales, the multinational acquiring firms tend to be 124% more productive in terms of labor productivity when defining industries at the SIC2 level, and 63% more productive at the SIC4 level. Finally, the estimate of $\beta_{Out}$ tends to suggest that firms looking outside their own industry tend to be less productive than those staying within.

4.2 Policy Predictions

To begin the analysis of the results in section three, I will examine the average productivity of acquiring firms

4.2.1 Outward investment and acquiring firm productivity

I will first examine whether a higher foreign share of outward acquisitions yields a higher average productivity of acquiring firms. On a shallow level, this prediction is very similar to the foreign-domestic productivity comparison outlined above. However, when one digs deeper, it is clearly different. For example, it is empirically possible that foreign acquiring firms are more productive than domestic, but that as the foreign share of outward acquisitions increase, the average productivity of these firms falls. In other words, the ranking of individual acquiring firms is not sufficient to infer industry-specific relationships. Thus, the goal of this section is to test another aspect of the market-access motives which support the theory in sections two and three.

Industries will be defined at two-digit and four-digit SIC level. The dependent variable will be defined generally as $Aprod_{i,c,t}$, which is the average productivity of acquiring firms from industry $i$, country $c$, in year $t$. As above, they are the sales of firm $j$ in year $t$ and the sales per employee for firm $j$ in year $t$. To construct the industry specific measures of productivity, for a given triple of industry $i$, country $c$, and year $t$, I take the average of the two productivity measures described above over all acquiring firms within a given industry, year, and country.

The primary independent variable, $ForShare_{i,c,t}$ will simply report the share of acquiring firms originating from industry $i$, country $c$, in year $t$, that do so abroad. Other explanatory variables which will be included are the share of acquiring firms that are already multinationals, $MultiShare_{i,c,t}$, and the total count of acquisitions that occur, $Count_{i,c,t}$. Again, since the model only deals with first-time multinational formation, I must control for the fact that a large set of acquiring firms will already have multinational status. If these firms tend to be more productive (as they likely are), it is necessary to control for $MultiShare_{i,c,t}$. Since the scale of acquisition activity (due to waves, perhaps) may also affect the composition of acquiring firms, I include $Count_{i,c,t}$.

To test the relationship between outward investment and industry-level productivity, the following
Table 3: Average Productivity of Acquiring firms

<table>
<thead>
<tr>
<th>SIC2</th>
<th>SIC4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^2 )</td>
<td>0.0084</td>
</tr>
<tr>
<td># Obs</td>
<td>11989</td>
</tr>
<tr>
<td>( \beta_{\text{ForShare}} )</td>
<td>0.4118***</td>
</tr>
<tr>
<td>(0.0424)</td>
<td>(0.0429)</td>
</tr>
<tr>
<td>( \beta_{\text{Count}} )</td>
<td>0.0424***</td>
</tr>
<tr>
<td>(0.0131)</td>
<td>(0.0130)</td>
</tr>
<tr>
<td>( \beta_{\text{MultiShare}} )</td>
<td>3.7099</td>
</tr>
<tr>
<td>(2.546)</td>
<td>(2.6127)</td>
</tr>
<tr>
<td>Year Effects?</td>
<td>No</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SIC2</th>
<th>( \log(A\text{Sales}_{i,c,t}) )</th>
<th>SIC4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^2 )</td>
<td>0.0004</td>
<td>0.0033</td>
</tr>
<tr>
<td># Obs</td>
<td>5255</td>
<td>5254</td>
</tr>
<tr>
<td>( \beta_{\text{ForShare}} )</td>
<td>0.0798*</td>
<td>0.1184**</td>
</tr>
<tr>
<td>(0.0458)</td>
<td>(0.0483)</td>
<td>(0.0484)</td>
</tr>
<tr>
<td>( \beta_{\text{Count}} )</td>
<td>0.0528***</td>
<td>0.0525***</td>
</tr>
<tr>
<td>(0.0136)</td>
<td>(0.0136)</td>
<td>(0.0172)</td>
</tr>
<tr>
<td>( \beta_{\text{MultiShare}} )</td>
<td>5.2603</td>
<td>4.8874</td>
</tr>
<tr>
<td>(3.3609)</td>
<td>(3.2276)</td>
<td>(1.9465)</td>
</tr>
<tr>
<td>Year Effects?</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Notes: Parameter estimates in bold. Robust standard errors in parentheses.
***, **, and * denote two-tailed significance at 1%, 5%, and 10%, respectively.

The results from estimating (23) by OLS are presented in Table 3.

Overall, the results in Table 3 suggest that a higher share of foreign acquisitions yields a higher aggregate productivity of firms which acquire abroad. This is consistent with the results from both policy analyses in the previous section. Further, the estimates of \( \beta_{\text{ForShare}} \) using both log(\( A\text{Sales}_{i,c,t} \)) and log(\( A\text{SalesEmp}_{i,c,t} \)) the results are fairly precise at conventional levels, for almost all permutations of industry definitions and year effects.

In terms of interpreting the estimates, the results vary substantially, though in a way that is intuitive. For example, using log(\( A\text{Sales} \)) as the dependent variable, and defining industries at the two-digit level, the predicted effect in going from zero foreign acquisitions to all foreign acquisitions is a 40% increase in the average sales of acquiring firms. Defining industries at the four-digit level, the relationship increases to 60%. The results using sales per employee are more modest. Using log(\( A\text{SalesEmp}_{i,c,t} \)), the effect is around 8-11% when assuming that industries are classified at the two-digit level, and 8-10% at the four digit level.

Looking at \( \beta_{\text{Count}} \), it seems clear that when the scale of acquisitions is larger, the productivity of acquiring firms tends to increase. This is effect is significant for all measures of productivity. Jointly
analyzing the results of $\beta_{\text{ForShare}}$ and $\beta_{\text{Count}}$ provides few alternate explanations. Holding $\text{Count}_{i,c,t}$ fixed, the average productivity of acquiring firms tends to increase. This may be due to the measure of acquiring firms shifting to a region of higher productivity, as the model would predict. However, holding $\text{ForShare}_{i,c,t}$ fixed, it appears that the additional demand resulting from merger waves is being driven by firms of higher productivity. While there is no specific prediction of this sort in this paper, this is an interesting relationship to pursue in further studies.

**Inward investment and target firm productivity**

The next issue to address is whether within a given target country-industry pair, a higher share of inward foreign acquisitions relative to domestic corresponds with a higher average productivity of firms that sell. In the theoretical model, this selection effect implies that industry is more productive, on average.

Again, industries will be defined at two-digit and four-digit SIC level. The dependent variable will be defined generally as $T_{\text{prod}}_{i,c,t}$, which is the average productivity of target firms in industry $i$, country $c$, and year $t$. Again, I am forced to rely on fairly simple measures of productivity, which are $\text{Sales}_{i,t}$ and $\text{SalesEmp}_{i,t}$. To construct the industry-country-year specific measures of productivity, for all firms within a given triple of target industry $i$, country $c$, and year $t$, I take the average of each productivity measure.

The primary independent variable, $\text{ForShare}_{i,c,t}$ is defined as the share of acquiring firms in target industry $i$, country $c$, and year $t$ that come from abroad. Other explanatory variables which will be included are the share of target firms that are already multinationals, $\text{MultiShare}_{i,c,t}$, and the total count of acquisitions that occur within target industry $i$, country $c$, and year $t$, $\text{Count}_{i,c,t}$. Both are included for similar reasons as before, except that the object of interest is whether or not these factors affect the average productivity of selling firms.

To see if there is a relationship between the composition of inward acquisitions and target firm productivity, the following specification is estimated by OLS:

$$\log(T_{\text{prod}}_{i,c,t}) = \beta_{\text{ForShare}_{i,c,t}} \text{ForShare}_{i,c,t} + \beta_{\text{Count}_{i,c,t}} \log(\text{Count}_{i,c,t})$$

$$+ \beta_{\text{MultiShare}_{i,c,t}} \text{MultiShare}_{i,c,t} + \beta_{\text{C\_SIC\_T\_C\_SIC}} \text{C\_SIC} + \beta_{\text{Year}_{i,c,t}} \text{Year} + \epsilon_{i,c,t} \tag{24}$$

In (24), $\beta_{\text{C\_SIC\_T\_C\_SIC}}$ is a vector of country-industry fixed effects and the associated vector of coefficients. Similarly, $\beta_{\text{Year}_{i,c,t}}$ is a vector of year fixed effects and the associated coefficients.

The results from estimating (24) are presented in Table 4. The results in Table 4 support the assertion that, on average, a larger share of inward foreign acquisitions yields a higher aggregate productivity of target firms. While the results using $\text{Sales}_{i,c,t}$ are very imprecise at the two-digit industry definition, the results at the four-digit level are quite precise. Using the latter, the results suggest that going from no foreign acquisitions to all foreign acquisitions is associated with a 17% increase in the sales level of selling firms. The results using $\text{SalesEmp}_{i,c,t}$ are qualitatively similar, though quantitatively more modest. Indeed, moving from no foreign acquisitions to all foreign acqui-
Table 4: Average Productivity of Acquiring firms

<table>
<thead>
<tr>
<th></th>
<th>log(Sales(_{i,c,t}))</th>
<th>SIC4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(R^2)</td>
<td></td>
</tr>
<tr>
<td>SIC2</td>
<td></td>
<td></td>
</tr>
<tr>
<td># Obs</td>
<td>16106</td>
<td>16105</td>
</tr>
<tr>
<td>(\beta_{ForShareIn})</td>
<td>0.0206</td>
<td>0.0449</td>
</tr>
<tr>
<td></td>
<td>(0.0422)</td>
<td>(0.0426)</td>
</tr>
<tr>
<td>(\beta_{CountIn})</td>
<td>0.0564***</td>
<td>0.0565***</td>
</tr>
<tr>
<td></td>
<td>(0.0135)</td>
<td>(0.0135)</td>
</tr>
<tr>
<td>(\beta_{MShareT})</td>
<td>-1.3811</td>
<td>-1.0782</td>
</tr>
<tr>
<td></td>
<td>(2.4056)</td>
<td>(2.3917)</td>
</tr>
<tr>
<td>Year effects?</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Notes</td>
<td>Parameter estimates in bold. Robust standard errors in parentheses</td>
<td></td>
</tr>
</tbody>
</table>

|                | log(SalesEmp\(_{i,c,t}\)) |      |
| SIC2           |                         |      |
| # Obs          | 5605                    | 5604 | 5603      | 5580      | 11142     | 11141     | 11140     | 11117     |
| \(\beta_{ForShareIn}\) | 0.0755*                 | 0.0932**| 0.0932**| 0.0781*| 0.0328*| 0.0374*| 0.0371*| 0.0353*|
|                | (0.0435)                | (0.0445) | (0.0445) | (0.0445) | (0.0203) | (0.0204) | (0.0204) | (0.0203) |
| \(\beta_{CountIn}\) | 0.0272*                 | 0.0273*| -0.0006  | 0.0296***| 0.0294***| 0.0254**|
|                | (0.014)                 | (0.014) | (0.018)  | (0.0105) | (0.0105) | (0.0117) |
| \(\beta_{MShareT}\) | -0.7065                | -0.5073| 1.0907    | 1.0363    |
|                | (2.3179)                | (2.1412) | (1.6281) | (1.57)    |
| Year effects?  | No                      | No    | Yes       | No        | No        | Yes       |

Positions will increase the average productivity of target firms by 7% using two-digit industry definitions, and 3.2-3.7% at the four-digit level of aggregation. Overall, a higher foreign composition of inward acquisitions seems to yield a higher average size/productivity of target firms.

Further, this result is robust when accounting for \(Count_{i,c,t}\), which itself has a positive effect on the average productivity of target firms. Precisely, using Sales, the estimates suggest that doubling the number of completed acquisitions yields a 5-7% increase in target firms productivity. Using sales per employee, the results suggest a more modest relationship around 2-3%.

Interpreting the effects of \(Count_{i,c,t}\) and \(ForShare_{i,c,t}\) suggest that there are characteristics of the data which are not addressed by the model. According to the theory, all that really matters for the effect of trade liberalization on target productivity is that the scale of acquisition demand increases more so than supply decreases (if at all). To the extent that \(Count_{i,c,t}\) controls for scale, the results in Table 4 state that beyond the scale effects, there is still a benefit of having a higher share of foreign acquisitions. This may be highlighting unmodeled matching effects, where the presence of additional foreign acquisition demand may entice certain firms to sell that otherwise wouldn’t if acquisition demand was comprised of solely domestic firms. However, this may be a feature best addressed by looking at additional country-specific heterogeneity, where this result would clearly depend on the intrinsic productivity of the target and acquiring nations.

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15 There is an obvious endogeneity problem in these estimates, where unobserved shocks which decrease the sales level of target firms may make acquisitions more affordable, and induce a larger number of acquisitions. However, this endogeneity issue is likely produce a downward bias in the estimates, and thus the estimates in Table 4 can be viewed as conservative.
5 Discussion and Conclusion

The results in Table 3 and especially Table 4 provide evidence for the key policy implications of the theoretical model. Overall, foreign acquisitions are an important source of demand in the reallocation process, and enacting policies which limit them in favor of domestic acquisitions may be detrimental to aggregate industry efficiency. Further, the broad intuition of the acquisition framework, and the accompanying empirical results, suggest that policies which limit acquisition behavior, generally, are likely to achieve the same, negative result.

I would like to close the paper with a discussion of model and empirical limitations, and plans to address these limitations. The presented theoretical model is one with reciprocal markets, with absolute symmetry in preferences, costs, trade costs, etc. When dropping the assumption of symmetry, the results in section three warrant a more careful analysis.

Consider for the moment the result in Proposition 4; the ambiguous effect of trade costs on the acquisition market and aggregate productivity. Any changes in the acquisition market resulting from trade liberalization depend on the degree to which foreign acquisitions play a significant role. In Proposition 4, this was defined as a significant weight in the acquisition market clearing condition. Equally important is an incentive to avoid trade costs which can sufficiently influence the acquisition market. Of course, this will depend on the origin of acquiring firms, and their intrinsic productivity level.

For example, consider a case in which a number of acquiring firms originate from a foreign country with a relatively low productivity distribution. While acquiring firms from this country will be an upgrade over domestic selling firms (they must be in equilibrium), they may be relatively less efficient compared to domestic acquiring firms. Foreign acquiring firms may not be of sufficient productivity to influence the acquisition market clearing condition in a country with a more favorable productivity distribution. In this case, the reallocation resulting from trade liberalization would be similar to Proposition 3, supporting higher aggregate productivity. Conversely, if inward acquisitions are from relatively productive countries, the effects of reallocation would lead to lower aggregate productivity, as in Proposition 4. On this level, the assumption of symmetry hides a dichotomy which is relevant to policy discussions.

Also, the assumption of symmetric trade costs and liberalization is fairly restrictive, where any liberalization episode that is asymmetric will require a more general analysis of tariff changes. Consider the case of a trade agreement. If tariff concessions are symmetric within industries, then section four would be a reasonable way in which their effects on the acquisition market could be analyzed. However, since large importers tend to offer larger concessions within industries (Bagwell and Staiger, 2007), the assumption of symmetry is not realistic. The effects on the acquisition market within a given industry would be more favorable for exporting countries. Not only would this expand the level of export-driven domestic acquisitions, it would also prevent many domestic firms from transferring production directly to a large importer’s market. Thus, the assumption of symmetric tariff changes, while analytically convenient, masks effects of tariffs which may have practical relevance.
Motivated by these and other examples, future research looking at the effects of country and industry-specific asymmetries on firm reallocation is likely to be an insightful venture. Indeed, in Spearot (2008b), I analyze the effects of asymmetric tariff changes, using New Zealand as a case study. Aspects of New Zealand’s tariff policy make it a prime candidate for addressing the effects of inbound and outbound tariffs. Specific to tariffs, I find evidence that is consistent with the predictions of this model, where higher inbound tariffs encourage more foreign investment relative to domestic, but seems to reduce the size of target firms. While additional data is still necessary to ascertain the true effects of tariffs on productivity through the acquisition market, this effect is certainly motivating for future work.
References


[18] Qui, Larry D. and Wen Zhou (2008), "Globalization and Acquisitions", mimeo The University of Hong Kong

[19] Spearot, Alan (2008), "Firm Heterogeneity and Acquisition Incentives", mimeo University of California - Santa Cruz


A  Free Trade Proofs

A.1 Proof of Lemma 1

To begin, I will focus on $\alpha \leq \frac{t}{4bk(A-t)}$. Differentiating $\frac{A^2ok}{2(4akb+1)(2akb+1)}$ with respect to $\alpha$ yields:

$$\frac{\partial \Delta \Pi}{\partial \alpha} = \frac{Aok (1 - 8b^2\alpha^2k^2)}{2(4akb + 1)^2(2akb + 1)^2}$$

Clearly, the positive root of $\Delta \Pi (\alpha)$ is $\frac{\sqrt{2}}{4bk}$. However, this maximum is irrelevant if $\frac{t}{4bk(A-t)} < \frac{\sqrt{2}}{4bk}$. This condition simplifies to $\frac{\sqrt{2}}{4bk} (\frac{2A - (\sqrt{2} + 1)}{A - t}) > 0$. This is satisfied if $t < \frac{2}{2 + \sqrt{2}} A$. Thus, $\frac{\partial \Delta \Pi(\alpha)}{\partial \alpha} > 0$ for $\alpha \leq \frac{t}{4bk(A-t)}$

Now turning to $\frac{t}{4bk(A-t)} < \alpha \leq \frac{t}{20k(A-t)}$, over this range, $\Delta \Pi (\alpha)$ has two roots (in $\alpha$). They are written as:

$$\alpha_1 = \frac{\sqrt{2} (-2A + (2 + \sqrt{2}) t)}{4bk (-A + (A - t) (\sqrt{2} + 1))}$$

$$\alpha_2 = -\frac{\sqrt{2} (2 (A - t) + \sqrt{2} t)}{4bk (A + (A - t) (\sqrt{2} - 1))} < 0$$

Clearly, $\alpha_2$ is not relevant. The root $\alpha_1$ is relevant if the following condition holds.

$$\alpha_1 - \frac{t}{4bk(A-t)} = \frac{\sqrt{2} (2A - t) \left(\frac{2 + \sqrt{2}}{2} - 2A\right)}{8bk (A - t) \left(-A + (A - t) (\sqrt{2} + 1)\right)} < 0$$

(-) if $t < \frac{2}{2 + \sqrt{2}} A$

(+) if $t < \frac{2}{2 + \sqrt{2}} A$ (see below)

To see that $-A + (A - t) (\sqrt{2} + 1) > 0$ if $t < \frac{2}{2 + \sqrt{2}} A$, rearrange the first expression to read $t < \left(1 - \frac{1}{1 + \sqrt{2}}\right) A$. Simplifying, we get $t < \frac{2}{2 + \sqrt{2}} A$. Thus, since the numerator and denominator always have an opposite sign, it must always be the case that $\alpha_1 < \frac{t}{4bk(A-t)}$, and thus not relevant in the mid range of productivity.

Both $\alpha_1 < \frac{t}{4bk(A-t)}$ and $\alpha_2 < 0$ imply that if $t < \frac{2}{2 + \sqrt{2}} A$, the roots of $\Delta \Pi (\alpha)$ do not occur over the range $\frac{t}{4bk(A-t)} < \alpha \leq \frac{t}{20k(A-t)}$. Thus, the sign of $\frac{\partial \Delta \Pi(\alpha)}{\partial \alpha}$ is constant over this range. To see when $\frac{\partial \Delta \Pi(\alpha)}{\partial \alpha} > 0$, note that:

$$\frac{\partial \Delta \Pi}{\partial \alpha} (\frac{t}{4bk(A-t)}) = \frac{k(A - t)^2 \left(2A^2 - 4At + t^2\right)}{2(2A - t)^2}$$

Clearly, the sign of $\frac{\partial \Delta \Pi(\alpha)}{\partial \alpha}$ over the mid range of productivity is dependent on the size of trade costs relative to the market, and is positive if $t < \frac{2}{2 + \sqrt{2}} A$. 

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Finally, for \( \frac{t}{2bk(A-t)} < \alpha \), I can write:

\[
\frac{\partial \Delta \Pi}{\partial \alpha} (\alpha) = \frac{k(2A-t)(4-8b^2\alpha^2k^2)}{2(4akb+2k^2)(4akb+2)^2}.
\]

The positive solution to \( \frac{\partial \Delta \Pi}{\partial \alpha} = 0 \) is \( \sqrt{\frac{T}{2bk}} \). It is also clear that for \( \alpha < \frac{\sqrt{T}}{2bk} \), \( \frac{\partial \Delta \Pi}{\partial \alpha} > 0 \), and for \( \alpha > \frac{\sqrt{T}}{2bk} \), \( \frac{\partial \Delta \Pi}{\partial \alpha} < 0 \).

To finish proving the properties in Lemma 1, I must show that \( \frac{\pi_k}{2} \pi_k^B (\alpha) < \pi^N (\alpha) < \pi_k^B (\alpha) \) for \( \alpha > 0 \). The condition \( \pi^N (\alpha) < \pi_k^B (\alpha) \) is trivial through lower costs. The condition \( \frac{1}{2} \pi_k^B (\alpha) < \pi^N (\alpha) \) is true if \( \Delta \Pi (\alpha) < \pi^N (\alpha) \). Expanding \( \Delta \Pi (\alpha) \), I get \( \pi_k^B (\alpha) - \pi^N (\alpha) < \pi^N (\alpha) \), which simplifies to \( \frac{1}{2} \pi_k^B (\alpha) < \pi^N (\alpha) \). Thus, I will now show that \( \Delta \Pi (\alpha) < \pi^N (\alpha) \).

To do this, I will first show that \( \Delta \Pi (0) = \pi^N (0) \), and then \( \frac{\partial \Delta \Pi}{\partial \alpha} < \frac{\partial \pi^N}{\partial \alpha} \) for all finite \( \alpha \). Showing \( \Delta \Pi (0) = \pi^N (0) \) is straightforward, given the multiplicative nature of \( \alpha \). For \( \alpha \leq \frac{t}{2bk(A-t)} \), I can write \( \frac{\partial \Delta \Pi (\alpha)}{\partial \alpha} - \pi^N (\alpha) \) as:

\[
\frac{\partial \Delta \Pi (\alpha)}{\partial \alpha} - \pi^N (\alpha) = -\frac{2A^2\alpha^2k^2b}{(4akb+1)(2akb+1)} < 0
\]

For \( \frac{t}{4bk(A-t)} < \alpha \leq \frac{t}{2bk(A-t)} \), I can write \( \frac{\partial \Delta \Pi (\alpha)}{\partial \alpha} - \pi^N (\alpha) \) as:

\[
\frac{\partial \Delta \Pi (\alpha)}{\partial \alpha} - \pi^N (\alpha) = -\frac{kt(2A-t)}{(4akb+2)^2} < 0
\]

Finally, for \( \frac{t}{2bk(A-t)} < \alpha \), I can write \( \frac{\partial \Delta \Pi (\alpha)}{\partial \alpha} - \pi^N (\alpha) \) as:

\[
\frac{\partial \Delta \Pi (\alpha)}{\partial \alpha} - \pi^N (\alpha) = -\frac{4bk^2\alpha(2A-t)^2(3bak+2)}{(4akb+2)^2(2akb+2)^2} < 0
\]

**Proof of Lemma 2**

Clearly, \( \Delta \Pi^* (\alpha) = -\delta \). In addition, it is straightforward to show that \( \lim_{\alpha \to \infty} \Delta \Pi^* (\alpha) = \frac{(2A-t)t}{4b} - \delta \).

To derive the slope properties of \( \Delta \Pi^* (\alpha) \), note that for \( \alpha \leq \frac{t}{2bk(A-t)} \), \( \frac{t^24bk}{2(2bk+1)} \) is equal to the no acquisition profit function in the closed economy, \( \pi^N (\alpha) \). Thus, for \( \alpha \leq \frac{t}{2bk(A-t)} \), \( \frac{\partial \Delta \Pi^* (\alpha)}{\partial \alpha} > 0 \). For \( \alpha > \frac{t}{2bk(A-t)} \), with some work, one can show that the only positive root of \( \Delta \Pi^* (\alpha) \) is \( \bar{\alpha} = \frac{\sqrt{T(2A-t+\sqrt{2T})}}{2bk(2A-2t-\sqrt{2T})} \). Differentiating \( \Delta \Pi^* (\alpha) \) with respect to \( \alpha \), setting equal to zero, and solving for \( \alpha \) we
Clearly, $\hat{a}_2$ is irrelevant. Regarding $\hat{a}_1$, it is only relevant if $t < \frac{2}{2+\sqrt{2}} A$. To see this, note that $\hat{a}_1 > \frac{t}{2bk(A-t)}$ if the following holds:

$$\hat{a}_1 - \frac{t}{2bk(A-t)} = \frac{(\sqrt{2} - 1)(2A - t)A}{2bk(A-t) \left( (2 - \sqrt{2}) A - t \right)} > 0$$

This clearly holds if $t < (2 - \sqrt{2}) A$. Note that $t < (2 - \sqrt{2}) A$ can be written as $t < (2 - \sqrt{2}) \frac{2+\sqrt{2}}{2+\sqrt{2}} A = \frac{4 - 2 + 2\sqrt{2} - 2\sqrt{2}}{2 + \sqrt{2}} A = \frac{2}{2 + \sqrt{2}} A$.

Now, all that is left is identifying the sign of $\frac{\partial \Delta \Pi^*(\alpha)}{\partial \alpha}$ on either side of $\hat{a}_1$. At $\alpha = \frac{t}{2bk(A-t)}$, $\frac{\partial \Delta \Pi^*(\alpha)}{\partial \alpha} = \frac{k(A-t)^2}{2} > 0$. The second derivative at $\hat{a}_1$ gives us guidance to the shape at the peak. Precisely:

$$\frac{\partial^2 \Delta \Pi^*(\hat{a}_1)}{\partial \alpha^2} = -\frac{+}{4 (2A-t) A} \left( 17\sqrt{2} - 24 \right) bk^2 (2A - 2t - \sqrt{2}t)^4$$

Clearly, $\frac{\partial^2 \Delta \Pi^*(\hat{a}_1)}{\partial \alpha^2} < 0$ only if $t \neq \frac{2}{2+\sqrt{2}} A$. Thus, if $t \neq \frac{2}{2+\sqrt{2}} A$, $\hat{a}_1$ is a maximum, and $\frac{\partial \Delta \Pi^*(\hat{a}_1)}{\partial \alpha} < 0$ for finite $\alpha > \hat{a}_1$. As $t \rightarrow \frac{2}{2+\sqrt{2}} A$, $\hat{a}_1 \rightarrow \infty$, where $\frac{\partial \Delta \Pi^*(\hat{a}_1)}{\partial \alpha} \rightarrow 0$.

To show that $\frac{1}{2} \pi^{B^*}(\alpha) \leq \pi^N(\alpha) < \pi^{B^*}(\alpha)$, note that for low productivity ($\alpha \leq \frac{t}{2bk(A-t)}$), $\frac{1}{2} \pi^{B^*}(\alpha) = \pi^N(\alpha)$. For $\alpha > \frac{t}{2bk(A-t)}$, firms that do nothing can now export and $\pi^N(\alpha)$ is larger relative to $\pi^{B^*}(\alpha)$. Thus, $\frac{1}{2} \pi^{B^*}(\alpha) \leq \pi^N(\alpha)$.

### A.2 Proof of Lemma 3

The existence of $R_a$ is straightforward. Acquisition demand is decreasing (from positive demand) starting from $R_a = 0$. Eventually, acquisition demand is 0 with $R_a$ high enough. This is since the incentives derived in $\Delta \Pi(\alpha)$ and $\Delta \Pi^*(\alpha)$ are bounded above. The opposite is the case with acquisition supply. For $R_a = 0$, firms have no incentive to sell. For $R_a$ high enough, all firms sell. Thus, using the intermediate value theorem, there must exist a $R_a > 0$ such that the acquisition market clears.

Continuing, it will be convenient to use the case of free trade as an analytical benchmark. Setting
$t = 0$, we find that acquisition incentives may be written as:

$$\Delta \Pi (\alpha | t = 0) = \frac{A^2 \alpha k}{2 (b b + 1) (2 b a k + 1)}$$

$$\Delta \Pi ^* (\alpha | t = 0) = \frac{A^2 \alpha k}{2 (b b + 1) (2 b a k + 1)} - \delta$$

Clearly, these incentives are identical outside of the fixed cost of foreign investment, $\delta$. The intuition is straightforward. With $t = 0$, we have an integrated world market that, holding $A$ fixed, is twice the size of each market. Both domestic and foreign acquisitions provide two lumps of capital, and by the equalization of marginal costs across production locations (lumps of capital), both domestic and foreign acquisitions will yield the same amount of output to the integrated world market.

Taking into account the fixed costs of foreign investment, we see that foreign acquisitions will not occur in free trade. This case is discussed in Spearot (2008), where it is shown that the equilibrium value of $R_a$ is such that $R_a < \frac{A^2 \alpha k}{2 (b b + 1) (2 b a k + 1)}$.

Moving forward, writing the equilibrium value of $R_a$ as $R_a (t, \delta)$, I now formally define $[0, \bar{t}] \times [0, \bar{\delta}]$. First, I will define the space $[0, \bar{t}]$ for any $\delta$. Then, over the space $[0, \bar{t}]$, I will define $[0, \bar{\delta}]$.

**Step 1:** Fix $\delta$. To satisfy $\frac{(3 - 2 \sqrt{2})}{8 b} (2 A - t)^2 > R_a (t, \delta)$, it is sufficient that $t$ must satisfy $\frac{(3 - 2 \sqrt{2})}{8 b} (2 A - t)^2 < R_a (t, \delta)$. Note that $\lim_0 \left(\frac{(3 - 2 \sqrt{2})}{8 b} (2 A - t)^2 \right) = 0$. Since $R_a (t, \delta) > 0$, then there must exist a $t_1 (\delta) > 0$ such that $\frac{(3 - 2 \sqrt{2})}{8 b} (2 A - t)^2 < R_a (t, \delta)$ for $t \in [0, t_1 (\delta)]$. This also satisfies the exporter condition for all active firms, where by manipulating the exporting cutoffs and profit functions, I can show that all active firms are exporters if $\frac{A^2 \alpha k}{2 b} < R_a$. For $A > t$, it is the case that $\frac{A^2 \alpha k}{2 b} < \frac{(3 - 2 \sqrt{2})}{8 b} (2 A - t)^2$.

Next, I must ensure that each acquisition option is profitable relative to the acquisition price for at least some range of firms. For domestic acquisitions, this is satisfied if $\max \Delta \Pi (\alpha) = \frac{(3 - 2 \sqrt{2})}{8 b} (2 A - t)^2 > R_a (t, \delta)$. Since $\frac{\partial \Delta \Pi (\alpha)}{\partial t} \leq 0$, it is possible that for some $t$, $\frac{(3 - 2 \sqrt{2})}{8 b} (2 A - t)^2 < R_a (t, \delta)$. However, the free trade equilibrium developed above dictates that $\frac{(3 - 2 \sqrt{2})}{8 b} A^2 > R_a (0, \delta)$, in equilibrium. This is convenient, since $\lim_0 \left(\frac{(3 - 2 \sqrt{2})}{8 b} (2 A - t)^2 \right) = \frac{(3 - 2 \sqrt{2})}{8 b} A^2$. Thus, there must exist a $t_2 (\delta) > 0$ such that $\frac{(3 - 2 \sqrt{2})}{8 b} (2 A - t)^2 > R_a (t, \delta)$ holds for $t \in [0, t_2 (\delta)]$. Summarizing so far, the first two conditions are jointly satisfied for $t \in [0, \bar{t} (\delta)]$, where $\bar{t} (\delta) = \min \{t_1 (\delta), t_2 (\delta)\} > 0$.

The above a paragraph pins down an upper bound $\bar{t} (\delta) > 0$ for each value of $\delta$. Thus, the minimum upper bound of $t$ can be characterized as follows:

$$\bar{t} = \min_\delta \left\{ \bar{t} (\delta) \right\} > 0$$

Before moving to step two, first note that $\max_\alpha \Delta \Pi^* (\alpha | t = 0) = \frac{(3 - 2 \sqrt{2})}{8 b} A^2 - \delta$. Since $\frac{\partial \Delta \Pi^* (\alpha)}{\partial t} > 0$ for firms that can export, this implies that $\max_\alpha \Delta \Pi^* (\alpha) \geq \max_\alpha \Delta \Pi (\alpha | t = 0)$. This is will prove to be a useful relationship to use below.

**Step 2:** Fix $t \in [0, \bar{t}]$. Over this region of trade costs, $\frac{(3 - 2 \sqrt{2})}{8 b} (2 A - t)^2 > R_a (t, \delta)$. Since $\frac{(3 - 2 \sqrt{2})}{8 b} A^2 > \frac{(3 - 2 \sqrt{2})}{8 b} (2 A - t)^2$, this implies that $\frac{(3 - 2 \sqrt{2})}{8 b} A^2 > R_a (t, \delta)$ over this same region. Thus, there must exist a $\delta (t) = \frac{(3 - 2 \sqrt{2})}{8 b} (2 A - t)^2 - R_a (t, \delta) > 0$ such that for $\delta \in [0, \delta (t)]$, $\frac{(3 - 2 \sqrt{2})}{8 b} A^2 - \delta (t) > R_a (t, \delta)$, and foreign acquisitions are not trivially unprofitable. Thus, the minimum upper bound of
can be characterized as follows:

\[ \hat{\delta} = \min_{t \in [0, \hat{t}]} \{ \hat{\delta}(t) \} > 0 \]

Thus, in equilibrium, there exists a subspace \([0, \hat{t}] \times [0, \hat{t}]\) such that all active firms are exporters, no acquisition choice is trivially unprofitable, and the limit of \(\Delta \Pi^*(\alpha)\) is lower than the acquisition price.

### A.3 Proof of Proposition 1

To prove Proposition 1, I must first establish that \(\alpha_S < \alpha_B < \overline{\alpha}_B\). To show \(\alpha_S < \alpha_B\), first note that from (10) and (12) it must be the case that:

\[ \pi^B(\alpha_B) - \pi^N(\alpha_B) = \pi^N(\alpha_S) \]

Rearranging,

\[ \frac{1}{2} \pi^B(\alpha_B) - \pi^N(\alpha_B) = \pi^N(\alpha_S) - \frac{1}{2} \pi^B(\alpha_B) \]

Since \(\frac{1}{2} \pi^B(\alpha_B) < \pi^N(\alpha_B)\), the RHS must also be negative in equilibrium. This is only possible if \(\alpha_S < \alpha_B\). By definition, \(\alpha_B < \overline{\alpha}_B\). Using this result and \(\alpha_S < \alpha_B\), it is clear that \(\alpha_S < \alpha_B < \overline{\alpha}_B\).

To show that \(\alpha_S < \alpha_B < \overline{\alpha}_B\), the analysis is similar. Equilibrium conditions dictate that:

\[ \pi^B*(\alpha_B^*) - \pi^N(\alpha_B^*) - \delta = \pi^N(\alpha_S) \]

Subtracting \(\frac{1}{2} \pi^B^*(\alpha_B^*)\), we get:

\[ \frac{1}{2} \pi^B^*(\alpha_B^*) - \pi^N(\alpha_B^*) - \delta = \pi^N(\alpha_S) - \frac{1}{2} \pi^B^*(\alpha_B^*) \]

Since all firms are exporters, using the result in Lemma 6, it must be the case that \(\frac{1}{2} \pi^B^*(\alpha_B^*) - \pi^N(\alpha_B^*) < 0\). Thus, the RHS, \(\pi^N(\alpha_S) - \frac{1}{2} \pi^B^*(\alpha_B^*)\) must also be negative. This can only be the case if \(\alpha_B^* > \alpha_S\). Finally, \(\overline{\alpha}_B > \alpha_B^*\) follows from the shape of \(\Delta \Pi^*(\alpha)\). Thus, \(\alpha_S < \alpha_B^* < \overline{\alpha}_B\).

Using \(\alpha_S < \alpha_B < \overline{\alpha}_B\) and \(\alpha_S < \alpha_B^* < \overline{\alpha}_B\), for \(\alpha < \alpha_S\), firms prefer selling. Formally, defining \(\Theta_s\) as the measure of firms that sell, \(\Theta_s = [0, \alpha_S]\).

The relationship \(\alpha_S < \alpha_B^* < \overline{\alpha}_B\) summarizes the decision between doing nothing and buying a foreign firm. The relationship \(\alpha_S < \alpha_B < \overline{\alpha}_B\) summarizes the decision between doing nothing and buying a domestic firm. The productivity cutoff \(\alpha_{BB^*}\), defined by (18), and the corresponding preference condition in (19) summarize the decision between buying a foreign firm and buying a domestic firm. I now jointly analyze these three decisions, completing the proof of Proposition 2.

Moving forward, a few straightforward properties that will be used in the proof are the following:

\[ \frac{\partial \alpha_{BB^*}}{\partial t} = - \left( \frac{\partial \Delta \Pi^*(\alpha_{BB^*})}{\partial t} - \frac{\partial \Delta \Pi(\alpha_{BB^*})}{\partial t} \right) < 0 \]

\[ \frac{\partial \overline{\alpha}_B}{\partial t} = - \left( \frac{\partial \Delta \Pi^*(\overline{\alpha}_B)}{\partial t} \right) > 0 , \quad \frac{\partial \alpha_B^*}{\partial t} = - \left( \frac{\partial \Delta \Pi^*(\alpha_B^*)}{\partial t} \right) < 0 \]

\[ \frac{\partial \overline{\alpha}_B}{\partial t} = - \left( \frac{\partial \Delta \Pi(\overline{\alpha}_B)}{\partial t} \right) < 0 , \quad \frac{\partial \alpha_B^*}{\partial t} = - \left( \frac{\partial \Delta \Pi(\alpha_B^*)}{\partial t} \right) > 0 \]
The strategy of analyzing this decision is to start at free trade and show how the composition of the acquisition market changes with gradually increasing trade costs. To begin, fix $\delta < \bar{\delta}$ and $t = 0$. According to (18), $\alpha_{BB^*}$ is not defined for $t = 0$ and $\delta > 0$, and thus $B > B^*$. Since $B > B^*$ for all $\alpha$, then $\Delta \Pi^*(\alpha) < \Delta \Pi(\alpha)$ for all $\alpha$. More generally, $\alpha_{BB^*}$ is not defined for $t$ such that $(2A - t)t/4b - \delta < 0$. Since $(2A - t)t/4b$ is increasing in $t$ for $t < A$, there exists a range of very low $t$ such that $\Delta \Pi^*(\alpha) < \Delta \Pi(\alpha)$ for all $\alpha$, and thus $B > B^*$ for all $\alpha$.

Within this range of very low $t$, it is straightforward to show that $\alpha_B < \alpha^*_B < \bar{\alpha}_B < \bar{\alpha}_B$. To see this, first suppose to the contrary that $\bar{\alpha}_B > \alpha_B$. Since by definition, $\Delta \Pi(\bar{\alpha}_B) < 0$ for $\alpha \geq \alpha_B$, this would imply that $\Delta \Pi(\alpha) < R_a$. Given the equilibrium condition $\Delta \Pi^*(\bar{\alpha}_B) = R_a$, this is contradiction of $\Delta \Pi^*(\alpha) < \Delta \Pi(\alpha)$ for all $\alpha$. A similar argument applies to $\alpha_B < \alpha^*_B$. Thus, for $t$ such that $(2A - t)t/4b - \delta < 0$, it must be that $\alpha_B < \alpha^*_B < \bar{\alpha}_B < \bar{\alpha}_B$.

Now consider the range of $t$ such that $(2A - t)t/4b - \delta > 0$. At $t$ close to $\hat{t}$ such that $(2A - \hat{t})t/4b - \delta = 0$, $\alpha_{BB^*}$ is very high (approaches $\infty$ from below with lower $t$) and decreases with higher $t$. Note that since $\frac{\partial \bar{\alpha}_B}{\partial t} > 0$, $\frac{\partial \alpha_B}{\partial t} < 0$, and $\bar{\alpha}_B > \alpha_B$, it is possible that there exists a unique value of $t$ such that $\alpha_B = \bar{\alpha}_B$. Supposing that this $t$ exists and is labeled $t(\delta)$, at this $t$ it must be the case that $\Delta \Pi(\bar{\alpha}_B) = \Delta \Pi^*(\bar{\alpha}_B)$. This implies that at $t(\delta)$, $\alpha_{BB^*} = \bar{\alpha}_B = \bar{\alpha}_B$.

Thus, for $t < \alpha(\delta)$, $\alpha_B < \alpha^*_B < \bar{\alpha}_B < \alpha_{BB^*}$. Using (17) and (14), foreign acquisitions are not the preferred acquisition choice for any value of $\alpha$. If foreign acquisitions are profitable for some region of productivity, domestic acquisitions are more profitable over this same region. If $\hat{t} > \alpha(\delta)$, the above ranking still holds. Defining $\alpha(\delta) = \min \{\hat{t}, \alpha(\delta)\}$, we have the following:

$$\text{For } t < \alpha(\delta), \quad \{ \Theta_S = (0, \alpha_s) \quad \Theta_N = (\alpha_s, \alpha_B) \quad \Theta_B = (\alpha_B, \bar{\alpha}_B) \quad \Theta_{BB^*} = (\bar{\alpha}_B, \bar{\alpha}_B) \}$$

As an aside, note that $\frac{\partial \alpha(\delta)}{\partial \delta} > 0$. This is a straightforward calculation obtained by differentiating $\Delta \Pi(\bar{\alpha}_B, \alpha(\delta)) = \Delta \Pi^*(\bar{\alpha}_B, \alpha(\delta))$ with respect to $t$, and substituting in the derivative of $\Delta \Pi(\bar{\alpha}_B, \alpha(\delta)) = R_a$ with respect to $t$. This yields $$\frac{\partial \Delta \Pi^*(\alpha_B)}{\partial \alpha} \frac{\partial \alpha_B}{\partial t} + \frac{\partial \Delta \Pi(\alpha_B)}{\partial t} \frac{\partial \alpha(\delta)}{\partial \delta} = -\frac{\partial \alpha(\delta)}{\partial \delta}.$$

As trade costs increase, a firm remains indifferent between domestic and foreign acquisitions if there is a concurrent increase in foreign investment costs. Also, $\alpha(0) = 0$. Intuitively, if the cost of foreign investment is zero, a firm is indifferent between a foreign and domestic acquisition only if the relative incentive to acquire abroad is also zero. Functionally, this shows that $t(\delta)$ will exist for $\delta$ sufficiently low.

Moving forward, using the simple results above, for $t > \alpha(\delta)$, $\alpha_B < \bar{\alpha}_B$. In addition, the following shows that for $t > \alpha(\delta)$, $\alpha_{BB^*} < \bar{\alpha}_B$. Evaluating $\frac{\partial \alpha_{BB^*}}{\partial t} - \frac{\partial \alpha_B}{\partial t}$ at $\alpha_{BB^*} = \bar{\alpha}_B$ yields $\frac{\partial \alpha_{BB^*}}{\partial t} - \frac{\partial \alpha_B}{\partial t} < 0$. This can be seen by writing:

$$\frac{\partial \alpha_{BB^*}}{\partial t} - \frac{\partial \alpha_B}{\partial t} = \left( \frac{\partial \alpha_{BB^*}}{\partial \alpha} \frac{\partial \alpha_B}{\partial t} - \frac{\partial \alpha_{BB^*}}{\partial \alpha_B} \frac{\partial \alpha_B}{\partial t} \right) - \frac{\partial \alpha_B}{\partial \alpha_B} \frac{\partial \alpha_B}{\partial t} - \frac{\partial \alpha_{BB^*}}{\partial \alpha_{BB^*}} \frac{\partial \alpha_{BB^*}}{\partial t}.$$

Imposing $\alpha_{BB^*} = \bar{\alpha}_B$, I can simply the above expression as:
Thus, since \( \alpha_{BB'} > \overline{\alpha}_B \) for \( t < \bar{t}'(\delta) \), \( \alpha_{BB'} \) must be less than \( \overline{\alpha}_B \) for \( t > \bar{t}'(\delta) \).

Further, note that for \( t \) slightly above \( \bar{t}'(\delta) \), where \( \alpha_{BB'} \) is sufficiently close to \( \overline{\alpha}_B \) and \( \alpha_B \), \( \alpha_B \) and \( \alpha_B^* \) are both less than \( \alpha_{BB'} \). This is since \( \alpha_B < \overline{\alpha}_B \) and \( \alpha_B^* < \overline{\alpha}_B \). Since \( \alpha_B \) and \( \alpha_B^* \) are both be less than \( \alpha_{BB'} \), this implies that \( \alpha_B < \alpha_B^* \). To see this, suppose to the contrary that \( \alpha_B > \alpha_B^* \). Since by definition, \( \frac{\partial \Delta \Pi (\alpha_B)}{\partial \alpha} > 0 \) for \( \alpha < \overline{\alpha}_B \), this would imply that \( \Delta \Pi (\alpha_B^*) < \Delta \Pi (\alpha_B) \). Given the equilibrium condition \( \Delta \Pi^* (\overline{\alpha}_B) = R_\alpha \), this is contradiction of \( \Delta \Pi^* (\alpha) < \Delta \Pi (\alpha) \) for \( \alpha < \alpha_{BB'} \). In summary, for \( t \) greater than \( \bar{t}'(\delta) \), \( \alpha_{BB'} < \overline{\alpha}_B < \alpha_B \), and for \( t \) slightly greater than \( \bar{t}'(\delta) \), \( \alpha_B < \alpha_B^* < \alpha_{BB'} \).

Continuing, increase \( t \) and note that \( \frac{\partial \alpha_B}{\partial t} < 0 \) and \( \frac{\partial \alpha_B}{\partial \delta} > 0 \). Since \( \alpha_B < \alpha_B^* \) for \( t \) close to \( \bar{t}'(\delta) \), there may exist a unique value of \( t \) such that \( \alpha_B = \alpha_B^* \). Supposing that this \( t \) exists and is labeled \( \bar{t}(\delta) \), at this \( t \) it must be the case that \( \Delta \Pi (\alpha_B) = \Delta \Pi^* (\alpha_B^*) \). This implies that at \( \bar{t}(\delta) \), \( \alpha_{BB'} = \alpha_B = \alpha_B^* \). Thus, for \( \bar{t}(\delta) < t < \bar{t}'(\delta) \), \( \alpha_B < \alpha_B^* < \alpha_{BB'} < \overline{\alpha}_B < \alpha_B^* \). If \( \bar{t}(\delta) < \bar{t} < \bar{t}'(\delta) \), the same ranking holds. Thus, defining \( \bar{t}(\delta) = \min \{ \bar{t}, \bar{t}'(\delta) \} \), for \( \bar{t}(\delta) < t < \bar{t}'(\delta) \):

\[
\begin{align*}
\{ \Theta_S = (0, \alpha_s), \quad \Theta_N = (\alpha_s, \alpha_B) \} \\
\Theta_N = (\alpha_B, \infty), \quad \Theta_B = (\alpha_B, \alpha_B) \\
\Theta_B = (\alpha_B, \alpha_B)
\end{align*}
\]

By similar derivations to above, note that \( \frac{\partial \bar{t}(\delta)}{\partial \delta} > 0 \), \( \bar{t}(0) = 0 \). Thus, for low enough \( \delta \), the value \( \bar{t}(\delta) \) will exist.

Moving forward, using \( \frac{\partial \alpha_B}{\partial t} < 0 \) and \( \frac{\partial \alpha_B}{\partial \delta} > 0 \), \( t > \bar{t}'(\delta) \), \( \alpha_B < \alpha_B^* \). Further, the following shows that for \( t > \bar{t}'(\delta) \), \( \alpha_{BB'} < \alpha_B^* \). Evaluating \( \frac{\partial \alpha_{BB'}}{\partial t} - \frac{\partial \alpha_B^*}{\partial t} \) at \( \alpha_{BB'} = \alpha_B^*(= \alpha_B) \) yields \( \frac{\partial \alpha_{BB'}}{\partial t} - \frac{\partial \alpha_B^*}{\partial t} < 0 \). This can be seen by writing:

\[
\frac{\partial \alpha_{BB'}}{\partial t} - \frac{\partial \alpha_B^*}{\partial t} = - \left( \frac{\partial \Delta \Pi^* (\alpha_{BB'})}{\partial \alpha} - \frac{\partial \Delta \Pi (\alpha_{BB'})}{\partial \alpha} \right) - \left( \frac{\partial \Delta \Pi^* (\alpha_B^*)}{\partial \alpha} \right)
\]

Imposing \( \alpha_{BB'} = \alpha_B^*(= \alpha_B) \), I can simply the above expression as:

\[
\frac{\partial \alpha_{BB'}}{\partial t} - \frac{\partial \alpha_B^*}{\partial t} = - \left( \frac{\partial \Delta \Pi^* (\alpha_B^*)}{\partial \alpha} \right) - \left( \frac{\partial \Delta \Pi (\alpha_{BB'})}{\partial \alpha} \right) < 0
\]

Since \( \alpha_{BB'} > \alpha_B^* \) for \( t < \bar{t}'(\delta) \), \( \alpha_{BB'} < \alpha_B^* \) for \( t > \bar{t}'(\delta) \). In summary, I have shown that \( \alpha_{BB'} < \alpha_B^* < \overline{\alpha}_B < \alpha_B < \alpha_B^* < \alpha_{BB'} \).
\[ \alpha^*_B \leq \alpha_B < \alpha^*_B < \alpha_{B^*}. \] Thus, for \( t > \bar{t}(\delta) \), I can state the following:

For \( \bar{t}(\delta) < t \), \[ \{ \Theta_S = (0, \alpha_s) \quad \Theta_N = (\alpha_s, \alpha^*_B) \quad \Theta_B = \{ \emptyset \} \quad \Theta_{B^*} = (\alpha^*_B, \alpha^*_B) \} \]

Domestic acquisitions are never preferred over this region.

To summarize the results of Proposition 1, the following figure (with assumed linear curvature) illustrates the composition of the acquisition market for \([0, \delta] \times [0, \hat{\delta}]\) under two different relative positions of \( t'(\delta) \) and \( \bar{t}(\delta) \).

Independent of the relative position of \( \hat{\delta}, \bar{t}, t'(\delta) \) and \( \bar{t}(\delta) \), the two-panels in the above figure show that for a given value of \( \delta \), the higher are trade costs, the more likely a firm will acquire abroad.

### A.4 Proof of Proposition 2

It is straightforward to show how the productivity cutoffs are affected by \( R_a \). Precisely, \( \frac{\partial \alpha_S}{\partial R_a} > 0 \), \( \frac{\partial \alpha^*_B}{\partial R_a} > 0 \), \( \frac{\partial \alpha_{B^*}}{\partial R_a} < 0 \), \( \frac{\partial \alpha^*_B}{\partial R_a} > 0 \), and \( \frac{\partial \alpha^*_B}{\partial R_a} = 0 \). In the below analysis, define \( K_D (R_a) \) and \( K^*_D (R_a) \) as domestic and foreign acquisition demand, respectively. Define \( K_S (R_a) \) as acquisition supply.

If \((2A-t)t/4b - \delta < 0\), then \( \alpha_{B^*} \) does not exist. This implies that domestic acquisitions are always preferred to foreign acquisitions. Thus, \( K_D^*_S (R_a) = 0 \). In this case, the proof a unique market clearing price \( R^*_a \) is identical to the closed economy.

Now, suppose that \((2A-t)t/4b - \delta > 0\). Foreign acquisitions are now profitable at some point along the range of \( \alpha \). Begin at \( R_a = 0 \). Here, \( K^*_D (R_a) = (G(\infty) - G(\alpha_{B^*}))kM_E = (1 - G(\alpha_{B^*}))kM_E, K_D (R_a) = (G(\alpha_{B^*}) - G(0)) = G(\alpha_{B^*})kM_E \) and \( K_S (R_a) = 0 \). Acquisition demand is greater than acquisition supply and cannot be an equilibrium.

Increasing \( R_a \), it is straightforward to show that \( \frac{\partial K_D^*_S (R_a)}{\partial R_a} \leq 0 \), \( \frac{\partial K_D^*_S (R_a)}{\partial R_a} \leq 0 \), \( \frac{\partial K_S (R_a)}{\partial R_a} \geq 0 \). When \( R_a = \frac{A^2}{4b} + \frac{(A-t)^2}{4b} \), \( K_D (R_a) = 0, K_D^*_S (R_a) = 0, K_S (R_a) = 1 \). This also cannot be an equilibrium since supply is greater than demand. However, by intermediate value theorem, this proves that there is a unique intersection of \( K_D (R_a) + K_D^*_S (R_a) \) and \( K_S (R_a) \) at \( R^*_a > 0 \). This completes the proof.
A.5 Proof of Proposition 3

Differentiating the equilibrium conditions in (10), (12), (13), and (20) for $t < \bar{t}(\delta)$, we get:

$$
\begin{bmatrix}
-\frac{\partial \pi^N(\alpha_S)}{\partial g(\alpha_S)} & \frac{\partial \pi^N(\alpha_S)}{\partial \alpha} & 0 \\
-\frac{\partial \pi^N(\alpha_S)}{\partial g(\alpha_B)} & \frac{\partial \pi^N(\alpha_S)}{\partial \alpha} & g(\alpha_B) \\
g(\alpha_B) & -g(\alpha_B) & 0
\end{bmatrix}
\begin{bmatrix}
\frac{\partial \alpha}{\partial t} \\
\frac{\partial \alpha}{\partial t} \\
\frac{\partial \alpha}{\partial t}
\end{bmatrix}
= 
\begin{bmatrix}
\frac{\partial \pi^N(\alpha_S)}{\partial g(\alpha_S)} & \frac{\partial \pi^N(\alpha_S)}{\partial \alpha} & 0 \\
-\frac{\partial \pi^N(\alpha_S)}{\partial g(\alpha_B)} & \frac{\partial \pi^N(\alpha_S)}{\partial \alpha} & g(\alpha_B) \\
g(\alpha_B) & -g(\alpha_B) & 0
\end{bmatrix}
\begin{bmatrix}
\frac{\partial \pi^N(\alpha_S)}{\partial \alpha} \\
\frac{\partial \pi^N(\alpha_S)}{\partial \alpha} \\
\frac{\partial \pi^N(\alpha_S)}{\partial \alpha}
\end{bmatrix}
$$

Using the equilibrium conditions in (10), (12) and (13), note that:

$$
\frac{\partial \pi^N(\alpha_S)}{\partial t} - \frac{\partial \Delta \Pi(\alpha_B)}{\partial t} = \frac{\partial \pi^N(\alpha_S)}{\partial t} - \frac{\partial \Delta \Pi(\alpha_B)}{\partial t} > 0
$$

we can solve the system of equations and get (unless otherwise labeled, individual terms are positive):

$$
\frac{\partial \alpha_S}{\partial t} = \frac{1}{D} \left( -\frac{\partial \Delta \Pi(\alpha_B)}{\partial \alpha} g(\alpha_S) + \frac{\partial \Delta \Pi(\alpha_B)}{\partial \alpha} g(\alpha_B) \right) \left( \frac{At}{2 \cdot 2k(2A - t)} \right)
$$

$$
< 0
$$

$$
\frac{\partial \alpha_B}{\partial t} = \frac{1}{D} \left( \frac{\partial \Delta \Pi(\alpha_B)}{\partial \alpha} g(\alpha_S) \right) \left( \frac{At}{2 \cdot 2k(2A - t)} \right)
$$

$$
> 0
$$

$$
\frac{\partial \sigma_B}{\partial t} = \frac{1}{D} \left( \frac{\partial \Delta \Pi(\alpha_B)}{\partial \alpha} g(\alpha_S) \right) \left( \frac{At}{2 \cdot 2k(2A - t)} \right)
$$

$$
< 0
$$

where:

$$
D = \frac{\partial \Delta \Pi(\alpha_B)}{\partial \alpha} \frac{\partial \Delta \Pi(\alpha_B)}{\partial \alpha} g(\alpha_S) - \frac{\partial \Delta \Pi(\alpha_B)}{\partial \alpha} \frac{\partial \Delta \Pi(\alpha_B)}{\partial \alpha} g(\alpha_B)
$$

$$
< 0
$$

A.6 Proof of Proposition 4

Differentiating the equilibrium conditions in (10), (12), (16), and (20), for $\bar{t}(\delta) < t < \bar{t}(\delta)$, we get:

$$
\begin{bmatrix}
\frac{\partial \pi^N(\alpha_S)}{\partial g(\alpha_S)} & \frac{\partial \pi^N(\alpha_S)}{\partial \alpha} & 0 \\
-\frac{\partial \pi^N(\alpha_S)}{\partial g(\alpha_B)} & \frac{\partial \pi^N(\alpha_S)}{\partial \alpha} & g(\alpha_B) \\
g(\alpha_B) & -g(\alpha_B) & 0
\end{bmatrix}
\begin{bmatrix}
\frac{\partial \alpha}{\partial t} \\
\frac{\partial \alpha}{\partial t} \\
\frac{\partial \alpha}{\partial t}
\end{bmatrix}
= 
\begin{bmatrix}
\frac{\partial \pi^N(\alpha_S)}{\partial g(\alpha_S)} & \frac{\partial \pi^N(\alpha_S)}{\partial \alpha} & 0 \\
-\frac{\partial \pi^N(\alpha_S)}{\partial g(\alpha_B)} & \frac{\partial \pi^N(\alpha_S)}{\partial \alpha} & g(\alpha_B) \\
g(\alpha_B) & -g(\alpha_B) & 0
\end{bmatrix}
\begin{bmatrix}
\frac{\partial \pi^N(\alpha_S)}{\partial \alpha} \\
\frac{\partial \pi^N(\alpha_S)}{\partial \alpha} \\
\frac{\partial \pi^N(\alpha_S)}{\partial \alpha}
\end{bmatrix}
$$

In addition to the work in the previous proposition, the following result follows directly from differentiating profit functions:

$$
\frac{\partial \Delta \Pi^+(\sigma_B)}{\partial t} - \frac{\partial \pi^N(\alpha_S)}{\partial t} > 0
$$
Continuing, I can write:

\[
\frac{\partial \alpha_S}{\partial t} = -\frac{1}{D} \left( \partial \Delta \Pi (\alpha_B) \frac{\partial \Delta \Pi^* (\alpha_B)}{\partial \alpha} \frac{\partial \Delta \Pi^* (\alpha_B)}{\partial t} - \frac{\partial \pi^N (\alpha_S)}{\partial t} \right) + \frac{1}{D} \left( \partial \Delta \Pi^* (\alpha_B) \frac{\partial \pi^N (\alpha_S)}{\partial \alpha} \right) - \frac{\partial \pi^N (\alpha_S)}{\partial \alpha} \frac{\partial \Delta \Pi (\alpha_B)}{\partial t} \\
\leq 0
\]

\[
\frac{\partial \alpha_B}{\partial t} = -\frac{1}{D} \left( \partial \pi^N (\alpha_S) \frac{\partial \Delta \Pi (\alpha_B)}{\partial \alpha} + \partial \pi^N (\alpha_S) \frac{\partial \Delta \Pi (\alpha_B)}{\partial \alpha} \right) - \frac{\partial \pi^N (\alpha_S)}{\partial \alpha} \frac{\partial \Delta \Pi (\alpha_B)}{\partial t} \\
> 0
\]

\[
\frac{\partial \bar{\alpha}_B}{\partial t} = -\frac{1}{D} \left( \partial \pi^N (\alpha_S) \frac{\partial \Delta \Pi (\alpha_B)}{\partial \alpha} + \partial \pi^N (\alpha_S) \frac{\partial \Delta \Pi (\alpha_B)}{\partial \alpha} \right) - \frac{\partial \pi^N (\alpha_S)}{\partial \alpha} \frac{\partial \Delta \Pi (\alpha_B)}{\partial t} \\
> 0
\]

where:

\[
D = \frac{\partial \Delta \Pi (\alpha_B)}{\partial \alpha} \frac{\partial \Delta \Pi^* (\alpha_B)}{\partial \alpha} g(\alpha_S) - \frac{\partial \Delta \Pi (\alpha_B)}{\partial \alpha} \frac{\partial \pi^N (\alpha_S)}{\partial \alpha} g(\alpha_B) + \frac{\partial \Delta \Pi^* (\alpha_B)}{\partial \alpha} \frac{\partial \pi^N (\alpha_S)}{\partial \alpha} g(\alpha_B)
\]

< 0

and,

\[
\frac{\partial \pi^N (\alpha_S)}{\partial t} - \frac{\partial \Delta \Pi (\alpha_B)}{\partial t} = \frac{At}{2bk(2A-t)}
\]

Finally, differentiating (18),

\[
\frac{\partial \alpha_{BB^*}}{\partial t} = \left( \frac{\partial \Delta \Pi^* (\alpha_{BB^*})}{\partial \alpha} - \frac{\partial \Delta \Pi (\alpha_{BB^*})}{\partial \alpha} \right) - \left( \frac{\partial \Delta \Pi^* (\alpha_{BB^*})}{\partial t} - \frac{\partial \Delta \Pi (\alpha_{BB^*})}{\partial t} \right) < 0
\]

To conclude, note that \(\frac{\partial \alpha_S}{\partial t} > 0\) if:
First note that $S(t)$ is finite. Further, note that $\frac{\partial \Delta \Pi^* (\pi_B)}{\partial t} \bigg|_{\pi_B=\pi_*} > 0$. Using the fact that $\frac{\partial^2 \Delta \Pi^* (\alpha)}{\partial t^2} = \frac{(2A-t)k}{4(bak+c^2)} > 0$, I can show that

\[
\frac{\partial \Delta \Pi^* (\pi_B)}{\partial t} \bigg|_{\pi_B=\pi_*} > 0 \quad \text{for exporters}
\]

Thus, $\frac{\partial \Delta \Pi^* (\pi_B)}{\partial t} > 0$. That is, the marginal effect on foreign acquiring firms is larger than the marginal effect on domestic acquiring firms.

Clearly, there must exist at ratio of $\frac{g(\pi_B)}{g(\alpha_B)}$ such that $\frac{\partial \alpha_B}{\partial t} > 0$. Ideally, I would also be able to show that this ratio can be quite small. Unfortunately, since I cannot show generally that $\frac{\partial \Delta \Pi (\alpha_B)}{\partial t} \bigg|_{\alpha_B=\alpha_*} > 0$, this is difficult. However, I can show that this condition is satisfied in a neighborhood of $t = \bar{t}(\delta)$. First note that $\frac{\partial \Delta \Pi (\alpha_B)}{\partial t} \bigg|_{\alpha_B=\alpha_*} > 0$. Precisely, $\frac{\partial \Delta \Pi (\alpha_B)}{\partial t} = \frac{k(2A-t)^2(\pi_B^{\alpha_B})k^2-v^2)^2}{8v(\pi_B^{\alpha_B}k+v)(\pi_B^{\alpha_B}k+v)^2} > 0$. Next, note that from (18), $\frac{\partial \Delta \Pi (\pi_B)}{\partial t} > \frac{\partial \Delta \Pi (\pi_B)}{\partial \alpha}$, or $-\frac{\partial \Delta \Pi (\pi_B)}{\partial \alpha} < -\frac{\partial \Delta \Pi (\pi_B)}{\partial t}$. Observing that at $t = \bar{t}(\delta)$, $\pi_B = \pi_* B$, we have the desired result: $\frac{\partial \Delta \Pi (\alpha_B)}{\partial t} > 0$. Thus, there exists a range of $t$ such that foreign acquisitions dominate even if $\frac{g(\pi_B)}{g(\alpha_B)}$ is less than one.

### A.7 Proof of Proposition 5

Differentiating the equilibrium conditions in (10), (15), (16), and (20), for $\bar{t}(\delta) < t$, we get:

\[
\begin{bmatrix}
-\partial \pi_N (\alpha_S) \\
-\partial \pi^* (\alpha_B) \\
\frac{\partial \alpha_S}{\partial t} \\
\frac{\partial \alpha_B}{\partial t}
\end{bmatrix}
- \begin{bmatrix}
\frac{\partial \pi_N (\alpha_S)}{\partial t} \\
\frac{\partial \pi^* (\alpha_B)}{\partial t} \\
\frac{\partial \alpha_S}{\partial t} \\
\frac{\partial \alpha_B}{\partial t}
\end{bmatrix}
= \begin{bmatrix}
\frac{\partial \pi^* (\pi_B)}{\partial t} \\
\frac{\partial \pi^* (\alpha_B)}{\partial t} \\
\frac{\partial \alpha_S}{\partial t} \\
\frac{\partial \alpha_B}{\partial t}
\end{bmatrix}
\]

Continuing, we can write:
\[ \frac{\partial \alpha_S}{\partial t} = -\frac{1}{D} \left( -\frac{\partial \Delta \Pi^* (\alpha_B)}{\partial \alpha} g(\alpha_B^*) \left( \frac{\partial \Delta \Pi^* (\alpha_B^*)}{\partial t} - \frac{\partial \pi^N (\alpha_S)}{\partial t} \right) \right) \]

\[ > 0 \]

\[ \frac{\partial \alpha_B^*}{\partial t} = \frac{1}{D} \left( -\frac{\partial \Delta \Pi (\alpha_B)}{\partial \alpha} g(\alpha_B) \left( \frac{\partial \Delta \Pi (\alpha_B)}{\partial t} - \frac{\partial \pi^N (\alpha_S)}{\partial t} \right) \right) \]

\[ > 0 \]

\[ \frac{\partial \pi_B}{\partial t} = -\frac{1}{D} \frac{\partial \Delta \Pi (\alpha_B)}{\partial \alpha} g(\alpha_S) \left( \frac{\partial \Delta \Pi (\alpha_B)}{\partial t} - \frac{\partial \pi^N (\alpha_S)}{\partial t} \right) \]

\[ > 0 \]

where:

\[ D = \frac{\partial \Delta \Pi (\alpha_B)}{\partial \alpha} \frac{\partial \Delta \Pi (\alpha_B)}{\partial \alpha} g(\alpha_S) - \frac{\partial \Delta \Pi (\alpha_B)}{\partial \alpha} \frac{\partial \pi^N (\alpha_S)}{\partial \alpha} g(\alpha_B^*) + \frac{\partial \Delta \Pi (\alpha_B)}{\partial \alpha} \frac{\partial \pi^N (\alpha_S)}{\partial \alpha} g(\alpha_B) \]

\[ < 0 \]

**A.8 Proof of Proposition 6**

Differentiating the equilibrium conditions in (10), (12), (16), and (20) with respect to \( \delta \), for \( t(\delta) < t < \bar{t}(\delta) \), we get:

\[
\begin{bmatrix}
-\frac{\partial \pi^N (\alpha_S)}{\partial t} & \frac{\partial \Delta \Pi (\alpha_B)}{\partial t} & 0 \\
\frac{\partial \Delta \Pi (\alpha_B^*)}{\partial t} & 0 & \frac{\partial \Delta \Pi (\alpha_B)}{\partial t} \\
g(\alpha_S) & g(\alpha_B^*) & -g(\alpha_B)
\end{bmatrix}
\begin{bmatrix}
\frac{\partial \alpha_S}{\partial \alpha} \\
\frac{\partial \alpha_B^*}{\partial \alpha} \\
\frac{\partial \pi_B}{\partial \alpha}
\end{bmatrix}
= \begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix}
\]
Continuing, I can write:

\[
\frac{\partial \alpha_S}{\partial \delta} = -\left( \frac{\partial \Pi^N(\alpha_S)}{\partial \alpha} \right) g(\alpha^*_B) + \frac{\partial \Pi^N(\alpha_B)}{\partial \alpha} g(\bar{\alpha}_B) < 0
\]

\[
\frac{\partial \alpha_B}{\partial \delta} = -\left( \frac{\partial \Pi^N(\alpha_S)}{\partial \alpha} \right) g(\alpha^*_B) + \frac{\partial \Pi^N(\alpha_B)}{\partial \alpha} g(\bar{\alpha}_B) < 0
\]

\[
\frac{\partial \bar{\alpha}_B}{\partial \delta} = -\left( \frac{\partial \Pi^N(\alpha_S)}{\partial \alpha} \right) g(\alpha^*_B) + \frac{\partial \Pi^N(\alpha_B)}{\partial \alpha} g(\bar{\alpha}_B) < 0
\]

## A.9 Proof of Proposition 7

Differentiating the equilibrium conditions in (10), (15), (16), and (20) with respect to $\delta$, for $\bar{t}(\delta) < t$, we get:

\[
\begin{bmatrix}
-\frac{\partial \Pi^N(\alpha_S)}{\partial \alpha} & \frac{\partial \Pi^N(\alpha_B)}{\partial \alpha} & 0 \\
-\frac{\partial \Pi^N(\alpha_S)}{\partial \alpha} & 0 & \frac{\partial \Pi^N(\alpha_B)}{\partial \alpha} \\
\end{bmatrix}
\begin{bmatrix}
\frac{\partial \alpha_B}{\partial \delta} \\
\frac{\partial \bar{\alpha}_B}{\partial \delta} \\
\end{bmatrix}
= \begin{bmatrix}
1 \\
0 \\
\end{bmatrix}
\]

Continuing, I can write:

\[
\frac{\partial \alpha_S}{\partial \delta} = -\left( \frac{\partial \Pi^N(\alpha_S)}{\partial \alpha} \right) g(\alpha^*_B) + \frac{\partial \Pi^N(\alpha_B)}{\partial \alpha} g(\bar{\alpha}_B) < 0
\]

\[
\frac{\partial \alpha_B}{\partial \delta} = -\left( \frac{\partial \Pi^N(\alpha_S)}{\partial \alpha} \right) g(\alpha^*_B) + \frac{\partial \Pi^N(\alpha_B)}{\partial \alpha} g(\bar{\alpha}_B) < 0
\]

\[
\frac{\partial \bar{\alpha}_B}{\partial \delta} = -\left( \frac{\partial \Pi^N(\alpha_S)}{\partial \alpha} \right) g(\alpha^*_B) + \frac{\partial \Pi^N(\alpha_B)}{\partial \alpha} g(\bar{\alpha}_B) < 0
\]