EART 254 Problem Set #2
Due: Friday May 6

You are encouraged to work together but, obviously, you must be an active participant in the process and submit your own work in the end.

Question 1. Condensation alone can’t make rain [10 pts]

We discussed in class how condensation alone cannot produce rain drops. We’ll look at this statement more carefully. Let’s assume that an air parcel with volume of 1 cm³ has a dewpoint temperature of 15°C. It travels upwards through a large cloud, whose cloud top is at -5°C (assume no ice crystals form, and SVP of supercooled water at -5°C is 0.40 kPa). Answer the following:

The saturation vapor pressure of water is describes the partial pressure of water vapor at equilibrium with liquid. The equation can be given by this empirical fit:

\[ p_w^* = A \exp\left(-\frac{B}{T}\right) \]

where \( A = 1.222 \times 10^8 \) kPa and \( B = 5223 \) K and \( T \) is temperature in Kelvin.

(a) What is the partial pressure of water vapor in this parcel at cloud base? For this volume of air, what is the mass of water that this corresponds to?

(b) What is the partial pressure of water vapor in this parcel at cloud top? For this volume of air, what is the mass of water that this corresponds to?

(c) Assuming that there are 300 condensation nuclei in this air parcel (a typical number), calculate the diameter (in micrometers) of each cloud drop. Assume that each drop is the same size and ignore the fact that there is a condensation nucleus present (i.e. assume the drop is made entirely of water). Compare this value to a typical rain drop.

Question 2. Aerosol indirect effect [10 pts]

The Earth’s average albedo is currently about 0.30. Of this 0.30, 70% is due to clouds, with the remainder of the sunlight reflected directly by aerosols and the Earth’s surface.

(a) Describe in words what the aerosol indirect effect on climate is.

(b) Currently, what is the albedo of Earth considering only the clouds?

(c) Consider a cloud containing cloud drops with number concentration \( N \) (units are number of drops per cm³ of air). Assume all drops are the same
size, radius \( r \). Write expressions for (i) the total surface area \( A \) of these drops, and (ii) the total volume \( V \) of these drops, both in terms of \( N \) and \( r \).

(d) Now we want to consider a situation where \( N \) increases due to the addition of anthropogenic particles. We want to determine an expression (no numbers!) for \( \frac{dA}{dN} \) assuming \( V \) is constant (i.e. assuming the total volume of cloud drops remains the same).

(e) Using the expression in (d), estimate the change in surface area, \( \Delta A \), given \( \Delta N = 20 \text{ cm}^{-3} \) and if we start with a cloud with \( N = 100 \text{ cm}^{-3} \) and \( r = 10 \mu\text{m} \).

(f) If cloud albedo is determined by surface area (because light is reflected by these drops in proportion to their surface area), what is the new cloud albedo? What, then, is the globally-averaged radiative forcing in W/m\(^2\), assuming that the mean incoming solar radiation is 343 W/m\(^2\)?

Question 3: Mid-latitude cyclones come from potential energy [10 pts]

Assume that the initial state for a mid-latitude cyclone can be idealized as two equal volumes of air side by side (see attached picture). Answer the following:

(a) This is not the lowest energy state for this system. Describe in words what is the lowest energy state.

(b) Assuming the two air masses A and B have constant densities \( \rho_a \) and \( \rho_b \) as shown, calculate the net potential energy released per unit mass of the entire system when the system returns to its lowest energy state. Assume no mixing of the two air masses.

(c) Let’s assume that the volume of air that rises results in condensation, with an average liquid water concentration \( 0.4 \text{ g/kg} \), where the units are grams of liquid water per kg of air. Calculate the additional release of latent heat energy per unit mass of the entire system.

(d) Let’s assume the sum of the energy that is released in (b) and (c) leads to motion of the entire system. Calculate the average wind speed of the entire system.
**Question 4. Rain from collision-coalescence [20 pts]**

The terminal velocity of a drop with radius \( r \) can be given by: \[ u_r = \frac{2r^2 \rho_w g}{9\mu} \]
where \( \mu \) is the viscosity of air, \( 1.8 \times 10^{-5} \) kg m\(^{-1}\) s\(^{-1}\).

Consider a collector drop of radius \( R \) and mass \( M \) falling through a field of small cloud drops with radius \( r \), mass \( m \) and constant concentration \( n \). Assume that \( r \ll R \).

The collision rate of the collector drop with the small cloud drops can be shown to be: \[ K(R, r) = \pi (R + r)^2 [u_r(R) - u_r(r)] \cdot n \]
where \( K \) is the rate of collisions experienced by the collector drop in units of collisions per second.

Our eventual goal is to determine the size of the collector drop as a function of time, i.e. \( R(t) \), given that it will grow as it collides with and collects the small cloud drops.

**No substituting numbers for symbols until (d)**

(a) Each time the collector drop collides and joins with a small cloud drop, it increases in mass. Derive how the mass of the collector drop changes with time through collision with the cloud drops, using the collision rate equation as your starting point. This will yield an expression \( dM/dt = [\text{stuff}] \). Since our goal is to derive \( R(t) \), make sure [stuff] only has \( R \) and \( t \) as variables, all else constants.

(b) But instead of talking about collector drop mass, we usually talk about size (i.e. radius \( R \) rather than \( M \)). Transform the LHS of the equation in (a) to yield an equation \( dR/dt = [\text{stuff}] \).

(c) Integrate (b) between limits to determine an expression \( R(t) = [\text{different stuff}] \). Assume that initially, the collector drop has a size \( R_i \).

(d) Using the expression in (c), determine how much time is needed for a collector drop with \( R_i = 50 \mu m \) to grow to \( R_f = 100 \mu m \) by collision with cloud drops of size \( r = 10 \mu m \) which are present at a concentration of 300 drops/cm\(^3\). Using these same parameters, plot \( R(t) \) for the same initial and final \( R \).