1 Introduction and Data
The essence of scope in natural language semantics:

• does the interpretation of an expression $e_1$ affect the interpretation of another expression $e_2$ or not?

(1) Every$^x$ student in my class read a$^y$ paper about scope.

How can we tell whether the interpretation of $Q'y$ (a paper about scope) was affected by $Qx$ (every student)?

• $Q'y$ is in the scope of $Qx$: $y$ may covary with $x$
• $Q'y$ is outside the scope of $Qx$: $y$'s value is fixed relative to $x$

Question 1: Dependence or independence?
Two possible conceptualizations:

(i) dependence-based approach: establish which quantifier(s) $Q'y$ is dependent on

(ii) independence-based approach: establish which quantifier(s) $Q'y$ is independent of

Logic has taken both paths. N(atural) L(anguage) semantics has only followed (i).

Our answer to Question 1
There are advantages to following (ii) and thus to importing the main insight from (in)dependence-friendly logic into NL semantics.

Question 2: Syntax or semantics or both?
NL semantics followed the lead of FOL in treating scope as a syntactic matter; scopal relations are determined at the combinatorial level.

(i) semantic scope is determined by NL syntax if syntax is the only source of constraints on meaning composition

(ii) scope is determined by both NL syntax and NL semantics if semantics contributes its own constraints on meaning composition

Compositional semantics of (1) yields the (equivalent of the) two FOL translations in (2) and (3):

(2) $\forall x[\text{student}(x)] \, (\exists y[\text{paper}(y)] \, (\text{read}(x,y)))$

(3) $\exists y[\text{paper}(y)] \, (\forall x[\text{student}(x)] \, (\text{read}(x,y)))$

Well-known problem 1
Contrast between indefinites and universals (Farkas 1981, Fodor & Sag 1982, Abusch 1994 among many others):

(i) indefinites enjoy free upward scope – disregarding not only clausal but also island boundaries

(4) Every$^x$ student read every$^y$ paper that a$^z$ professor recommended.

• the indefinite may scope over the direct-object universal or over both universals in (4)

• three possible readings depending on the relation between the indefinite and each universal

  – narrowest-scope (NS): for every student $x$, for every paper $y$ such that there is a professor $z$ that recommended $y$, $x$ read $y$

  – intermediate-scope (IS): for every student $x$, there is a professor $z$ such that, for every paper $y$ that $z$ recommended, $x$ read $y$
widest-scope (WS): there is a professor $z$ such that, for every student $x$, for every paper $y$ that $z$ recommended, $x$ read $y$

(ii) the upward scope of universals is clause-bounded

(5) John read a$x$ paper that every$y$ professor recommended.

• the universal cannot scope over the indefinite: no co-variation possible between professors and paper

Lesson 1
The scopal freedom of indefinites is problematic for syntax-based accounts.

• this type of freedom is similar to anaphoric relations – because it is syntactically non-local . . .

• . . . but we should resist the temptation of treating them alike because the constraints on the two phenomena are different – e.g., WCO effects for pronouns, but not for inverse scope

Well-known problem 2
Scope of indefinites is sensitive to syntax:

• an indefinite with a bound variable in its restrictor cannot outscope the binder of that variable (Abusch 1994, Chierchia 2001, Schwarz 2001)

(6) Every$x$ student read every$y$ paper that one$z$ of its$y$ authors recommended.

The indefinite one$z$ of its$y$ authors can have only narrowest scope.

(7) Binder Roof Constraint: an indefinite cannot scope over a quantifier that binds into its restrictor.

Lesson 2
Configurational matters cannot be disregarded altogether even when dealing with the scope of indefinites.

Our answer to Question 2
Both syntax and semantics, in the following ways:

• syntax constrains the order in which bona fide quantifiers are interpreted but the scope of indefinites is determined by their semantics

• the semantics of indefinites says that the semantic scope of the existential quantifier contributed by the indefinite and the semantic scope of its restrictor formula are the same; consequently: binders block upward scope (Binder Roof Constraint)

The issue
How to capture both the exceptional freedom that the scope of indefinites exhibits and the syntactic limits on this freedom.

Well-known problem 3
Intra- and cross-linguistic variety of indefinites

• simple indefinites: English $a(n)$-indefinites and their equivalents in other languages

• special indefinites: special because more complex determiners or special because less complex (bare nominals)

Special indefinites:

(i) special morphology

(ii) special semantics that often involves restrictions on scopal interpretation

(iii) special distribution which, ideally, should follow from the special semantics

Question 3: How do special indefinites connect to simple ones?

Ideal answer to Question 3

• If the special indefinite is more complex than the ordinary one, the semantic contribution of the complex morphology should account for the interpretational properties of the special indefinite.

• If the special indefinite is simpler than the simple one, it should be associated with simpler semantics.

Dependent indefinites

• special complex indefinites that must be interpreted as covarying with another variable introduced by a licensor, and therefore necessarily interpreted within the scope of that licensor (Farkas 1997b)

• associated with:
  – reduplicative morphology (Hungarian, Bengali, Georgian, Pashto)
Aim
Account for the cross-linguistically common semantic denominator of dependent indefinites in a way that isolates the semantic contribution of dependent morphology and makes room for the extra restrictions.

2 Outline of Proposal

• interpret indefinites in situ thereby partially divorcing semantic scope from configurational matters (see Steedman 2007, Farkas 1997a)
• depart from previous linguistic accounts in conceptualizing scope as a matter of independence by marking when the interpretation of an expression must be rigid (invariant) relative to another
• main role of syntactic structure: if a quantifier \( Q_x \) structurally commands a quantifier \( Q'_y \), \( x \) becomes available for \( y \) to potentially covary with or be independent of
• just as in choice / Skolem function approaches, we take the essence of the semantics of indefinites to be choosing a witness
• unlike choice / Skolem function approaches and like independence-friendly (IF) logic: witness choice is part of the interpretation procedure
• indefinites choose a witness at some point in the evaluation and require its non-variation from that point on
• non-variation is ensured by directly constraining the values taken by the first-order variable contributed by the indefinite
• an indefinite is indexed with the set of variables it is dependent on, and requires non-variation relative to all the other variables
• the semantics of scopal (in)dependence is different from the semantics of anaphoric dependencies
  – scopal (in)dependence: the index, i.e., the set of variables, provides parameters relative to which a new value is chosen
  – anaphoric dependencies: the index on pronouns provides a way to retrieve the old entity that the pronoun refers to

An existential:
• accesses the set \( V \) of variables previously introduced by quantifiers taking syntactic scope over the existential
For simplicity, we ignore the ‘non-surface’ IS determined by \( \mathcal{V}' = \{ y \} \).

Comparison with choice / Skolem function approaches

- similarity: witness choice is the essence of existential interpretation (see Schlenker 2006, Steedman 2007 and references therein)
- difference: witness choice is connected to specification of independence relative to some set of variables, which is not encapsulated in a functional variable contributed by the indefinite, but happens during the evaluation of the first-order existential

Example

(16) Every \( x \) professor recommended every \( y \) paper to a \( z \) student.

(17) \( \forall x[\text{PROFESSOR}(x)] (\forall y[\text{PAPER}(y)] (\exists z[\text{STUDENT}(z)] (\text{RECOMMEND-TO}(x,y,z)))) \)

Assume:

- the set of professors \( x \) is \( \{ \alpha_1, \alpha_2 \} \)
- the set of papers \( y \) is \( \{ \beta_1, \beta_2 \} \)

The existential \( \exists z \) chooses a witness that satisfies its restrictor \( \text{STUDENT}(z) \) and its nuclear scope \( \text{RECOMMEND-TO}(x,y,z) \).

The set of variables contributed by previously evaluated quantifiers: \( \mathcal{V} = \{ x, y \} \); possible choices of index on the existential are as in (15).

Scope depicted by matrices:

\[
\begin{array}{ccc}
\alpha_1 & \beta_1 & \gamma \\
\alpha_1 & \beta_2 & \gamma' \\
\alpha_2 & \beta_1 & \gamma'' \\
\alpha_2 & \beta_2 & \gamma'''
\end{array}
\]

Requirement imposed by dependent indefinites

- witness choice must be dependent on some parameter of evaluation
- the WS matrix in (18) is not possible if the variable \( z \) is introduced by a dependent indefinite.

Role of syntax: syntactic scope of the universals determines the possible parameters of dependency of the indefinite.

Questions to be addressed below:

- how should such matrices be formalized?
- how should we treat the non-variation requirement that may be contributed by the existential?

3 Scope in First-Order Logic with Choice (C-FOL)

Two formal novelties in the semantics

(i) formulas are evaluated relative to sets of assignments \( G, G', \ldots \) – instead of single assignments \( g, g', \ldots \) (see Hodges 1997, Väänänen 2007)

- the interpretation function has the form \( [\cdot]^{\text{NR},G} \)

(ii) the index of evaluation for a quantifier contains the set \( \mathcal{V} \) of variables introduced by the previously evaluated (i.e., syntactically higher) quantifiers

- the interpretation function has the form \( [\cdot]^{\text{NR},G,\mathcal{V}} \)

Use of sets of assignments

A set of total assignments \( G \):

\[
\begin{array}{ccccccc}
G & \ldots & x & y & z & \ldots \\
g_1 & \ldots & \alpha_1 (=g_1(x)) & \beta_1 (=g_1(y)) & \gamma_1 (=g_1(z)) & \ldots \\
g_2 & \ldots & \alpha_2 (=g_2(x)) & \beta_2 (=g_2(y)) & \gamma_2 (=g_2(z)) & \ldots \\
g_3 & \ldots & \alpha_3 (=g_3(x)) & \beta_3 (=g_3(y)) & \gamma_3 (=g_3(z)) & \ldots \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots
\end{array}
\]
• independence of $Q''z$ from $Q'y$: $z$ has to have a fixed value relative to the (possibly) varying values of $y$

(20) Fixed value condition (basic version): for all $g, g' \in G$, $g(z) = g'(z)$.

• if $z$ does not covary with $y$, this means that:
  - $Q''z$ is not in the semantic scope of $Q'y$ (although it may very well be in its syntactic scope)
  - $Qx$ may take both syntactic and semantic scope over $Q''z$ (intermediate scope (IS) in (18)): value of $z$ is fixed relative to $y$, but may covary with $x$

We need to be able to relativize the fixed value condition for $z$ to the variable $x$.

(21) Fixed value condition (relativized version):
for all $g, g' \in G$, if $g(x) = g'(x)$, then $g(z) = g'(z)$.

• the values of $z$ may covary with $x$ but are fixed relative to other variables ($y$ in our example)

Use of a set of variables as indices on existentials

• index of evaluation contains the set of variables $\mathcal{V} = \{x, y, \ldots\}$ introduced by previous quantifiers; these are the variables the existential could covary with

• an existential has a choice: chooses which of the variables that take syntactic scope over the existential also take semantic scope over it

• $\mathcal{V}' \subseteq \mathcal{V} = \{x, y, \ldots\}$: variables in $\mathcal{V}'$ are the variables that the indefinite is possibly dependent on

(22) $x \quad y \quad z$

| $\alpha_1$ | $\beta_1$ | $\gamma$ |
| $\alpha_1$ | $\beta_2$ | $\gamma'$ |
| $\alpha_2$ | $\beta_1$ |

• advantage of sets of assignments over single assignments: we can formulate non-variation / fixed-value conditions relativized to particular variables

• still needed: way of keeping track of the variables introduced by the syntactically higher quantifiers so as to relativize such conditions to particular quantifiers

Empirical predictions

(i) an indefinite may be within the semantic scope of a quantifier $Qx$ only if $Qx$ has syntactic scope over that indefinite (more generally: only if the semantic composition makes it so that the quantifier $Qx$ is evaluated before the indefinite)

(ii) an indefinite may in principle be outside the semantic scope of a quantifier that takes syntactic scope over it

3.1 Basic C-FOL notions

• the heart of the account – the recursive definition of the interpretation function $[[\cdot]]_{\mathfrak{M}, G, \mathcal{V}}$

• a model $\mathfrak{M}$ for C-FOL has the same structure as the standard models for FOL: $\mathfrak{M}$ is a pair $\langle \mathcal{D}, \mathcal{I} \rangle$
  - $\mathcal{D}$ is the domain of individuals
  - $\mathcal{I}$ the basic interpretation function
  - when we need it, we can add the set of possible worlds $\mathfrak{W}$ and relativize $\mathcal{I}$ to this set in the usual way

• an $\mathfrak{M}$-assignment $g$ for C-FOL is also defined just as in FOL: $g$ is a total function from the set of variables $\mathcal{V}AR$ to $\mathcal{D}$, i.e., $g \in \mathcal{D}^{\mathcal{V}AR}$

The essence of quantification in FOL: pointwise (i.e., variablewise) manipulation of variable assignments.

• $g'[x]g$ abbreviates that the assignments $g'$ and $g$ differ at most with respect to the value they assign to $x$

(24) $g'[x]g :=$ for all variables $v \in \mathcal{V}AR$, if $v \neq x$, then $g'(v) = g(v)$

We work with sets of variable assignments, so we generalize this to a notion of pointwise manipulation of sets of assignments.
• $G'[x]G$ is just the cumulative-quantification style generalization of $g'[x]g$ – any $g' \in G'$ has an $[x]$-predecessor $g \in G$ and any $g \in G$ has an $[x]$-successor $g' \in G'$

\[(25)\] $G'[x]G := \begin{cases} \text{for all } g' \in G', \text{there is a } g \in G \text{ such that } g'[x]g & \text{for all } g \in G, \text{there is a } g' \in G' \text{ such that } g'[x]g \end{cases}$

The interpretation of atomic formulas:

\[(26)\] Atomic formulas: $[R(x_1, \ldots, x_n)]^{\mathcal{M},G,V} = \top$ iff

\[a.\] $\{x_1, \ldots, x_n\} \subseteq V$

\[b.\] $G \neq \emptyset$

\[c.\] $(g(x_1), \ldots, g(x_n)) \in \mathcal{I}(R)$, for all $g \in G$

\[(27)\] $G \vdash \begin{array}{ccccccc} g & \ldots & \alpha_1 (= g(x_1)) & \ldots & \alpha_n (= g(x_n)) & \ldots \\
\end{array}$

\[g' \vdash \begin{array}{ccccccc} \langle \alpha_1, \ldots, \alpha_n \rangle \in \mathcal{I}(R) \\
\end{array}

\[g'' \vdash \begin{array}{ccccccc} \beta_1 (= g'(x_1)) & \ldots & \beta_n (= g'(x_n)) & \ldots \\
\end{array}

\[\langle \beta_1, \ldots, \beta_n \rangle \in \mathcal{I}(R) \]

\[\gamma_1 (= g''(x_1)) & \ldots & \gamma_n (= g''(x_n)) & \ldots \\
\end{array}

\[\langle \gamma_1, \ldots, \gamma_n \rangle \in \mathcal{I}(R) \]

• bans free variables – condition (26a)

– deictic pronouns require the discourse-initial set of variables $V$ to be non-empty – much like the discourse-initial partial assignments in DRT/FCS are required to have a non-empty domain

• distributes over the set $G$ and, in this way, relates the C-FOL notion of set-based satisfaction to the standard FOL notion of single-assignment-based satisfaction – condition (26c) below

• the non-emptiness condition (26b) rules out the case in which the distributive requirement in (26c) is vacuously satisfied

Conjunction – interpreted as usual: we just pass the current index of evaluation down to each conjunct.

\[(28)\] Conjunction: $[\phi \land \psi]^{\mathcal{M},G,V} = \top$ iff $[\phi]^{\mathcal{M},G,V} = \top$ and $[\psi]^{\mathcal{M},G,V} = \top$.

### 3.2 Existentials in C-FOL

Choice in existential quantification: superscript $V'$ that determines which variables the existential is in the semantic scope of:

• such a superscript can only occur on existentials – and not on bona fide quantifiers

• this is because these superscripts constrain witness choice – and the semantics of bona fide quantifiers cannot be given in terms of single witnesses

\[(29)\] Existential quantification:

$[\exists^V x[\phi(x)]]^{\mathcal{M},G,V} = \top$ iff $V' \subseteq V$ and $[\phi]^{\mathcal{M},G,V \cup \{x\}} = \top$, for some $G'$ such that

\[a.\] $G'[x]G$

\[b.\] $[\phi]^{\mathcal{M},G',V' \cup \{x\}} = \top$

\[c.\] if $V' = \emptyset$: $g(x) = g'(x)$, for all $g, g' \in G'$

\[c.\] if $V' \neq \emptyset$: $g(x) = g'(x)$, for all $g, g' \in G'$ that are $V'$-identical

\[(30)\] Two assignments $g$ and $g'$ are $V'$-identical iff for all variables $v \in V'$, $g(v) = g'(v)$.

• the nuclear scope formula $\psi$ is interpreted relative to $V \cup \{x\}$

• the restrictor formula $\phi$ is interpreted relative to $V' \cup \{x\}$; consequently the restrictor formula has the same semantic scope as the existential

• if $V' = \emptyset$, the value of the existential is fixed, and the indefinite has widest scope and it is given values in terms of (20)

• if $V' \neq \emptyset$, the fixed value condition is relativized to the variables in $V'$ and the indefinite is given values in terms of the relativized fixed value condition in (21)

### 3.3 Deriving the Binder Roof Constraint

**Universals in C-FOL**

The basic idea:

• the nuclear scope is evaluated relative to the set of all $g'$ that satisfy the restrictor

• hence: collect in $G'$ all $g'$ that satisfy the restrictor $\phi$ and evaluate the nuclear scope $\psi$ relative to $G'$

Good enough for formulas without existentials in the restrictor:
(31) Universal quantification (preliminary): \[\forall x[\phi(\psi)]_{\mathcal{M},G,V} = T\] iff \[\forall x[\phi]_{\mathcal{M},G',\forall\cup{x}} = T\], where \(G'\) is the maximal set of assignments that satisfies \(\phi\) relative to \(x\), \(G\) and \(V\).

(32) \(G'\) is the maximal set of assignments that satisfies \(\phi\) relative to the variable \(x\), the set of assignments \(G\) and the set of variables \(V\) iff \(G' = \bigcup_{g \in G} \{g' : g'[x]g\} \in \mathcal{M},\{\phi\}_{\forall\cup(x)} = T\} \).

Existentials in the scope of universals – an example

(33) Every\(^x\) student read a\(^y\) paper.
(34) \(\forall x[\text{student}(x)] (\exists^y y[\text{paper}(y)] (\text{read}(x, y)))\)
(35) \(\forall x[\text{student}(x)] (\exists^{(x)} y[\text{paper}(y)]) (\text{read}(x, y)))\)

- if superscript is \(\emptyset\), as in (34): the existential has wide-scope interpretation
- if superscript is \(\{x\}\), as in (35): the existential has narrow-scope interpretation relative to the universal

The syntax of the two readings is the same; the difference is purely in the semantics and involves the way the witness is chosen.

- the presence of the universal makes possible two ways of choosing the witness: as dependent or as independent of the variable bound by the universal

Interpretation relative to an arbitrary \(G\) and the empty set of variables \(\emptyset\):

(36) Truth: a formula \(\phi\) is true in model \(\mathcal{M}\) iff \([\phi]_{\mathcal{M},G,\emptyset} = T\) for any non-empty set of assignments \(G\), where \(\emptyset\) is the empty set of variables.

Assume that the set of students in \(\mathcal{M}\) is \(\{\text{stud}_1, \text{stud}_2, \text{stud}_3\}\).

(i) \(\forall x[\text{student}(x)]\) introduces the set of all students relative to the variable \(x\) and stores them one by one in the assignments \(g \in G\)

(ii) \(\exists^{(x)} y[\text{paper}(y)]\) introduces a paper and chooses whether it is the same for every student (if the superscript is \(\emptyset\)) or whether it is possibly different from student to student (if the superscript is \(\{x\}\))

(iii) each assignment in the resulting set should be such that the \(x\)-student in that assignment read the \(y\)-paper in that assignment.

Capturing the Binder Roof Constraint (7)

- by (29), the restrictor \(\phi\) of an indefinite is interpreted only relative to the variables in \(V^y\)
- the Binder Roof Constraint follows because the semantic scope of the restrictor \(\phi\) is always the same as the semantic scope of the existential \(\exists^y\)
- if the indefinite \(\text{one}^y\) of \(\text{its}_y\) authors in (6) is independent from the universal every\(^y\) paper, the variable \(y\) contributed by \(\text{its}_y\) is free and this is ruled out by the interpretation clause for atomic formulas in (26)

3.4 Deriving Exceptional Scope

(38) Every\(^x\) student read every\(^y\) paper that a\(^z\) professor recommended.
(39) \(\forall x[\text{student}(x)] \left(\forall y[\text{paper}(y)] \land \exists^{(x)} z[\text{professor}(z)] (\text{recommend}(z, y)) \right) \) (\text{read}(x, y))

WS, IS and NS: choice of superscript fixed to \(\emptyset\), \(\{x\}\) or \(\{x, y\}\), given that \(V = \{x, y\}\)

- IS reading (superscript = \(\{x\}\)): for each student \(x\), we choose a professor \(z\) and require \(x\) to have read every paper that \(z\) recommended; professors may covary with the students but not with the papers

Problem: according to (31), when we evaluate the restrictor of the universal \(\forall y\), i.e., \(\text{paper}(y) \land \exists^{(x)} z[\text{professor}(z)]\ldots\), we do not have access to the entire previous set of assignments \(G\) that stores all the \(x\)-students; we cannot impose the requirement that the paper co-varies with the students but not with the papers: assignments that give the same value for \(x\) have to give the same value for \(z\)
• we only examine one assignment \( g \in G \) at a time:
\[
G' = \bigcup_{g \in G} \left\{ g' : g'[y]g \text{ and } \left[ \text{PAPER}(y) \land \exists(z) \text{PROFESSOR}(z) \right] \cdots \right\} \text{ for } \{ g' \}, \{ x, y \} = \top
\]
• hence, the relativized fixed-value condition contributed by the existential is satisfied one assignment \( g' \) at a time: for every singleton set of assignments \( \{ g' \} \), the existential outputs a new set of assignments \( G'' \) such that:
  - \( G''[z] \{ g' \} : G'' \) differs from \( \{ g' \} \) at most with respect to the values of \( z \); in particular, \( h(x) = g'(x) \) for all \( h \in G'' \)
  - for all \( h, h' \in G'' \), if \( h(x) = h'(x) \), then \( h(z) = h'(z) \)
  - since \( h(x) = g'(x) \) for all \( h \in G'' \), we have that \( h(z) = h'(z) \) for all \( h, h' \in G'' \) – so \( G'' \) is also a singleton set of the form \( \{ g'' \} \)
• thus: for every assignment \( g' \), the single witness chosen by the existential is the professor \( g''(z) \)
• the problem: we have no way to ensure that only one \( z \)-professor is chosen for each \( x \)-student because we never have access to the entire set assignments \( G \) (or \( G' \)) that stores all the students
• so, all scopes of the indefinite are conflated to narrowest scope

Solution: interpret the restrictor of a universal relative to the set of assignments \( G' \) taken collectively, as a whole (rather than distributively, one by one).

• that is: we are already interpreting the nuclear scope of a universal relative to the set of assignments \( G' \) taken collectively, we just need to also do this for the restrictor

(40) Universal quantification (final version): \( [\forall x[\phi] \ (\psi)]^{\text{dep}}_{G', \mathcal{V}} = \top \) iff \( [\psi]^{\text{dep}}_{G', \mathcal{V}, \cup \{ x \}} = \top \), for some \( G' \) that is a maximal set of assignments relative to \( x, \phi, G \) and \( \mathcal{V} \).

(41) \( G' \) is a maximal set of assignments relative to a variable \( x \), a formula \( \phi \), a set of assignments \( G \) and a set of variables \( \mathcal{V} \) iff
a. \( G'[x]G \) and \( [\phi]^{\text{dep}}_{G', \mathcal{V}, \cup \{ x \}} = \top \)

b. there is no \( G'' \neq G' \) such that \( G' \subseteq G'' \) and:
\( G''[x]G \) and \( [\phi]^{\text{dep}}_{G'', \mathcal{V}, \cup \{ x \}} = \top \)

3.5 Dependent Indefinites in C-FOL

Ingredients of analysis of ordinary indefinites:

• the superscript on the existential that stores the set of parameters relative to which the indefinite may covary

• the fixed-value constraint that makes use of this superscript and that constrains the values of the indefinite stored in the resulting matrix

Expectation: existence of special indefinites that target the same superscript and possibly enforce further constraints on the values stored in the matrix.

Essence of dependent indefinites

• dependent indefinites add a non-fixed / plural value condition relativized to their superscript

Interpretation rule for dependent indefinites:

(42) \( [\text{dep} \exists^z x[\phi] \ (\psi)]^{\text{dep}}_{G', \mathcal{V}} = \top \) iff \( \mathcal{V}' \subseteq \mathcal{V} \) and \( [\psi]^{\text{dep}}_{G', \mathcal{V}, \cup \{ x \}} = \top \), for some \( G' \) such that
a. \( G'[x]G \)
b. \( [\phi]^{\text{dep}}_{G', \mathcal{V}, \cup \{ x \}} = \top \)
c. \( \begin{cases} & \text{if } \mathcal{V}' = \emptyset : g(x) = g'(x), \text{ for all } g, g' \in G' \\ & \text{if } \mathcal{V}' \neq \emptyset : g(x) = g'(x), \text{ for all } g, g' \in G' \text{ that are } \mathcal{V}'\text{-identical} \\ & g(x) \neq g'(x), \text{ for at least two } g, g' \in G' \text{ that are not } \mathcal{V}'\text{-identical} \end{cases} \)
d. \( g(x) \neq g'(x) \), for at least two \( g, g' \in G' \) that are not \( \mathcal{V}'\)-identical

(43) Two assignments \( g \) and \( g' \) are \( \mathcal{V}'\)-identical iff for all variables \( u \in \mathcal{V}' \), \( g(u) = g'(u) \).

Effect of (42d) is to impose covariation:

• \( \mathcal{V}' \) must be non-empty because there must be at least two assignments \( g, g' \in G' \) that are not \( \mathcal{V}'\)-identical
  - that is, there must be a variable \( u \in \mathcal{V}' \) such that \( g(v) \neq g'(v) \)
• hence, the fixed-value condition is reduced to the second case in which \( \mathcal{V}' \neq \emptyset \)
• licensor must be a bona fide quantifier because only quantifiers can introduce multiple values for the same variable and we need assignments that are \( \mathcal{V}'\)-non-identical

Example

(44) Minden diák olvasott egy-egy cikket.
  every student read a-a paper
  ‘Every student read a paper.’

(45) \( \forall x[\text{STUDENT}(x)] \ (\text{dep} \exists^z x \ / / (x) y[\text{PAPER}(y)] (\text{READ}(x, y))) \)
(46) \[
\begin{array}{c}
\forall x \text{student}(x) \\
\text{stud}_1 \\
\text{stud}_2 \\
\text{stud}_3 \\
\end{array}
\]
\[
\begin{array}{c}
\text{paper} \\
\text{paper} \\
\text{paper} \\
\end{array}
\]
\[
\begin{array}{c}
\text{dep-} \exists^\# y~\text{paper}(y) \\
\text{stud}_1 \\
\text{stud}_2 \\
\text{stud}_3 \\
\end{array}
\]
\[
\begin{array}{c}
x \\
y \\
\end{array}
\]
\[
\begin{array}{c}
\text{dep-}\exists^\gamma x~y~\text{paper}(y) \\
\text{stud}_1 \\
\text{stud}_2 \\
\text{stud}_3 \\
\end{array}
\]
\[
\begin{array}{c}
x \\
y \\
\end{array}
\]
\[
\begin{array}{c}
\text{read}(x,y) \\
\text{stud}_1 \text{ read} \text{ paper} \\
\text{stud}_2 \text{ read} \text{ paper'} \\
\text{stud}_3 \text{ read} \text{ paper''} \\
\end{array}
\]

Details of the evaluation of (44):

(i) the restrictor of $\forall x$ introduces the set of all students in column $x$

(ii) moving to the existential, if $\mathcal{V}' = \emptyset$, variation condition (42d) is not satisfied because $y$ has a single value

(iii) existential must have the superscript \{x\}, which makes covariation of papers with students possible

(iv) condition (42d) requires actual rather than merely possible covariation

(v) the nuclear scope of the existential checks that each $x$-student read the corresponding y-paper

Some consequences

- we have isolated the contribution of dependent morphology making dependent indefinites ordinary indefinites + a special condition
- dependent indefinites end up being evaluation plurals in the sense of Brasoveanu (2008) (as opposed to the usual non-atomic individuals, which are ontologically plural): requirement that there be more than one witness in the $y$ column
- this can be generalized to pluractionality (more than one event)

(47) A gyerek fel-fel ébredt. 
the child up-up woke
'The child kept waking up.'

- in (47): covariation of events of waking up with time-points/intervals
- special sortal conditions on the licensing variable can be imposed
- Hungarian reduplicated indefinites cannot be licensed by modals (or by an ordinary indefinite within the scope of a modal)
- Russian dependent indefinites can (see Pereltsvaig 2008)
- the semantic rule in (42) above is correct for Russian, but needs to be further refined for Hungarian

(48) $[\text{dep-} \exists^\gamma x \phi(x)]^{\mathcal{G}, \mathcal{V}} = \top$ iff $\mathcal{V}' \subseteq \mathcal{V}$ and $[\psi]^{\mathcal{G}, \mathcal{V}^\cup \{x\}} = \top$, for some $G'$ such that

a. $G'[x]G$

b. $[\phi]^{\mathcal{G}, \mathcal{V}^\cup \{x\}} = \top$

c. $\{ \text{if } \mathcal{V}' = \emptyset : g(x) = g'(x), \text{ for all } g, g' \in G' \}$

d. $\{ \text{if } \mathcal{V}' \neq \emptyset : g(x) = g'(x), \text{ for all } g, g' \in G' \text{ that are } \mathcal{V}' \text{-identical but not } \mathcal{V}_D \text{-identical } \}$

(49) For any set of variables $\mathcal{V}', \mathcal{V}'_D$ is the set of variables over individuals in $\mathcal{V}'$ and $\mathcal{V}'_D$ is the set of variables over worlds in $\mathcal{V}'$.

(50) Two assignments $g$ and $g'$ are $\mathcal{V}'$-identical iff for all variables $v \in \mathcal{V}'$, $g(v) = g'(v)$.

4 Conclusion

(i) formulas are evaluated relative to a set of assignments and a set of variables

(ii) existentials are interpreted in situ

(iii) the semantic scope of existentials is a matter of choice relative to their index of evaluation (set of assignments + set of variables)

(iv) more specifically: semantic scope of existential: choice of subset of previously introduced variables

(v) this index determines which variables the existential must be independent of and which variables the existential may covary with

(vi) existentials have free upward scope limited by the Binder Roof Constraint
(vii) dependent indefinites are analyzed as indefinites that are special because of an extra condition that imposes covariation, i.e., evaluation plurality
(viii) we expect other special indefinites to be amenable to an analysis in terms of covariation/lack of covariation requirements

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References
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