Donkey Pluralities: Plural Information States vs. Non-Atomic Individuals

Adrian Brasoveanu
Stanford University

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Abstract

The paper argues that two distinct and independent notions of plurality are involved in natural language anaphora and quantification: *plural reference* (the usual non-atomic individuals) and *plural discourse reference*, i.e. reference to a quantificational dependency between sets of objects (e.g. atomic / non-atomic individuals) that is established and subsequently elaborated upon in discourse. Following van den Berg (1996), plural discourse reference is modeled as plural information states (i.e. as sets of variable assignments) in a new dynamic system couched in classical type logic that extends Compositional DRT (Muskens 1996). Given the underlying type logic, compositionality at sub-clausal level follows automatically and standard techniques from Montague semantics become available. The idea that plural info states are semantically necessary (in addition to non-atomic individuals) is motivated by relative-clause donkey sentences with multiple instances of singular donkey anaphora that have mixed (weak and strong) readings. At the same time, allowing for non-atomic individuals in addition to plural info states enables us to capture the intuitive parallels between singular and plural (donkey) anaphora, while deriving the incompatibility between singular (donkey) anaphora and collective predicates. The system also accounts for empirically unrelated phenomena, e.g. the uniqueness effects associated with singular (donkey) anaphora discussed in Kadmon (1990) and Heim (1990) among others.

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1 Introduction: Plural Discourse Reference and Mixed Reading Donkey Anaphora

The main goal of this paper is to systematically distinguish two notions of plurality involved in natural language anaphora and quantification, namely: (*i*) *plural reference*, i.e. the usual reference to non-atomic individuals, e.g. the non-atomic / plural / sum individual *megan*⊕*gabby* in *Megan and Gabby are deskmates* (see Link 1983 and Schwarzschild 1992 among many others); (*ii*) *plural discourse reference*, i.e. reference to a quantificational dependency between sets of objects (e.g. atomic / non-atomic individuals, but also times, eventualities, possible worlds etc.) that is established and subsequently elaborated upon in discourse, e.g. the dependency between gifts and girls introduced in the first conjunct and elaborated upon in the second conjunct of the discourse in (1) below. Antecedents are superscripted with the discourse referent (dref) they introduce, while anaphors are subscripted with the dref they retrieve.

1. Linus bought a^{u} gift for every$^{u'}$ girl in his class and asked their$_{u'}$ deskmates to wrap them$_{u}$.

The first conjunct in (1) introduces a quantificational dependency between the set *u'* of girls in Linus's class and the set *u* of gifts bought by Linus: each *u'*-girl is correlated with the *u*-gift(s) that Linus bought for her. This correlation / dependency is elaborated upon in the second conjunct: for each *u'*-girl, Linus asked her deskmate to wrap the corresponding *u*-gift(s).

However, morphologically plural anaphora of the kind instantiated in (1) does not provide a clear-cut argument for distinguishing plural reference and plural discourse reference: both of them / either of them could be involved in the interpretation of (1). Nor does it provide a forceful argument for a semantic (as opposed to a pragmatic) encoding of discourse-level reference to quantificational dependencies: it might be that the second conjunct in (1) is cumulatively interpreted (in the sense of Scha 1981) and that the correlation between girls and gifts (brought to salience by the first conjunct) is only pragmatically supplied.

I will therefore use sentences with multiple instances of *singular* donkey anaphora like (2) and (3) below to provide independent semantic motivation for plural discourse reference.

2. Every^{u} person who buys a$^{u'}$ book on amazon.com and has a$^{u''}$ credit card uses it$_{u''}$ to pay for it$_{u'}$.
3. Every^{u} boy who bought a$^{u'}$ Christmas gift for a$^{u''}$ girl in his class asked her$_{u''}$ deskmate to wrap it$_{u'}$.

Sentence (2) shows that singular donkey anaphora can refer to non-singleton sets of atomic individuals, while (3) shows that singular donkey anaphora can refer to a dependency between such sets. Let us examine them in turn.

Example (2) is a mixed weak & strong donkey sentence[1]: it asserts that, for *every* book (strong) that any credit-card owner buys on amazon.com, there is *some* credit card (weak) that s/he uses to pay for the book[2].

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[1] To my knowledge, the existence of mixed reading relative-clause donkey sentences was observed for the first time by van der Does (1993). His example is provided in (i) below – and it is accompanied by the observation that "clear intuitions are absent, but a combined reading in which a whip is used to lash all horses seems available" (van der Does 1993: 18). The intuitions seem much clearer with respect to example (2) above; moreover, it is crucial for our purposes that the weak reading of *a credit card* in (2) does not require the set of credit cards to be a singleton set – that is, some people might use different credit cards to buy different (kinds of) books.

(i) Every farmer who has a horse and a whip in his barn uses it to lash him. (van der Does 1993: 18, (26))

The existence of mixed reading conditional donkey sentences has been observed at least since Dekker (1993); his example is provided in (ii) below.

Intuitively, example (2) does not apply only to persons that bought exactly one book on amazon.com or that have exactly one credit card, e.g. (2) is felicitous as a generalization about the behavior of all amazon.com customers over the last year[3]. That is, morphologically singular donkey anaphora is not semantically singular – at least not in the sense in which singular (Russellian) definite descriptions like *the (one) book s/he buys* or *the (one) credit card s/he has* are semantically singular.

Moreover, the credit card can vary from book to book, e.g. I can use my MasterCard to buy set theory books and my Visa to buy detective novels; that is, even the weak indefinite $a^{u''}$ credit card can introduce a non-singleton set of atoms. And, for each buyer, the two sets of atoms, i.e. all the purchased books and some of the credit cards, are correlated and the dependency between these sets (left unspecified in the restrictor) is specified in the nuclear scope: each book is correlated with the credit card that was used to pay for it. The translation of sentence (2) in classical (static) first-order logic, provided in (4) below, summarizes these observations.

4. $\forall x(pers(x) \wedge \exists y(bk(y) \wedge buy(x, y)) \wedge \exists z(card(z) \wedge hv(x, z))$
$\rightarrow \forall y'(bk(y') \wedge buy(x, y') \rightarrow \exists z'(card(z') \wedge hv(x, z') \wedge use\_to\_pay(x, z', y'))))$

Given that (2) is intuitively interpreted as shown in (4) above, a plausible hypothesis is that singular donkey anaphora involves plural reference, i.e. non-atomic individuals (or, if you prefer, sets of atoms), as proposed in Lappin and Francez (1994) for example. That is, (multiple) singular donkey anaphora is analyzed in much the same way as the (multiple) plural anaphora in sentence (5) below, where the two plural pronouns $them_{u'}$ and $them_u$ are anaphoric to the plural individuals obtained by summing the domains (i.e. restrictors) of the quantifier $every^{u'}$ girl in his class and the (narrow scope) indefinite a^u gift, respectively.

5. Linus bought a^u gift for $every^{u'}$ girl in his class and asked $them_{u'}$ / $the_{u'}$ girls to wrap $them_u$ / the_u gifts.

This kind of approach analyzes sentence (2) as follows: the strong donkey anaphora to *u'*-books involves the maximal sum individual *y* containing all and only the books bought by a given *u*-person; at the same time, the weak donkey anaphora to *u''*-credit cards involves a non-maximal individual *z* (possibly non-atomic) containing some of the credit cards that said *u*-person has[4]. Finally, the nuclear scope of (2) is cumulatively interpreted, i.e. given the maximal sum *y* of books and the sum *z* of some credit cards, for any atom $y' \leq y$, there is an atom $z' \leq z$ such that *z'* was used to pay for *y'* and, also, for any atom $z' \leq z$, there is an atom $y' \leq y$ such that *z'* was used to pay for *y'*[5].

(ii) If a man has a dime in his pocket, he throws it in the parking meter. (Dekker 1993: 183, (25))

[2] Note that the same kind of interpretation is associated with non-generic variants of (2), e.g. *Based on last year's statistics, every person who bought a book on amazon.com and had a credit card used it to pay for it*.

[3] Some speakers find the variants in (i) below intuitively more compelling:

(i) Every person who buys a computer / TV and has a credit card uses it to pay for it.

[4] This is basically the E-type approach to weak / strong donkey ambiguities in Lappin and Francez (1994).

[5] Or we can provide a more flexible cumulative analysis based on the notion of cover (see Schwarzschild 1996). That is, the nuclear scope of (2) is cumulatively interpreted relative to some cover of the maximal sum *y* of books and the sum *z* of some credit cards such that, for any part $y' \leq y$ in the cover, there is a part $z' \leq z$ in the cover such that *z'* was used to pay for *y'* and, also, for any part $z' \leq z$ in the cover, there is a part $y' \leq y$ in the cover such that *z'* was used to pay for *y'*.

Such a plural reference approach to weak / strong donkey anaphora faces the following problem, noticed in Kanazawa (2001): if the classical strong donkey sentence $\texttt{Every}^{u}$ $\texttt{farmer who owns a}^{u'}$ $\texttt{donkey beats it}_{u'}$ involves reference to non-atomic individuals, we predict that singular donkey anaphora is compatible with collective predicates (at least in a situation in which all donkey-owning farmers have more than one donkey). This prediction, however, is incorrect, as shown by the infelicitous sentence in (6) below (based on Kanazawa 2001: 396, (56)).

6. $\texttt{\#Every}^{u}$ $\texttt{farmer who owns a}^{u'}$ $\texttt{donkey gathers it}_{u'}$ $\texttt{around the fire at night.}$

One way to maintain the plural reference approach and derive the infelicity of (6) is to assume (following a suggestion in Neale 1990) that singular donkey pronouns always distribute over the non-atomic individual they are anaphoric to. For example, the singular pronoun $\texttt{it}_{u'}$ in (6) contributes a distributive operator and requires each donkey atom in the maximal sum of u'-donkeys to be gathered around the fire at night. The infelicity of (6) follows from the fact that collective predicates do not apply to atomic individuals.

But this domain-level (as opposed to discourse-level) distributivity strategy will not help us with respect to (3) above. Sentence (3) contains two instances of strong donkey anaphora: we are considering *every* Christmas gift and *every* girl. Moreover, the restrictor of the quantification in (3) introduces a dependency between the set of gifts and the set of girls: each gift is correlated with the girl it was bought for. Finally, the nuclear scope retrieves not only the two sets of objects, but also the dependency between (i.e. the structure associated with) them: each gift was wrapped by the deskmate of the girl that the gift was bought for. Thus, we have here donkey anaphora to structure in addition to donkey anaphora to values / objects.

Importantly, the structure associated with the two sets of atoms, i.e. the dependency between gifts and girls that is introduced in the restrictor and elaborated upon in the nuclear scope of the quantification, is *semantically* encoded and not pragmatically inferred. That is, the nuclear scope of the quantification in (3) is not interpreted cumulatively and the correlation between the sets of gifts and girls is not left vague / underspecified and subsequently made precise based on various extra-linguistic factors. This kind of pragmatic approach is what we would expect in view of the interpretation of sentences like (5) above, where the 'buying' correlation / dependency between the gift-atoms
and the girl-atoms introduced in the first conjunct can be different from the 'wrapping' correlation / dependency in the second conjunct.
To see that the structure in (3) is semantically encoded, consider the following situation: suppose that Linus buys two gifts, one for Megan and the other for Gabby; moreover, the two girls are deskmates. Intuitively, sentence (3) is true if Linus asked Megan to wrap Gabby's gift and Gabby to wrap Megan's gift and it is false if Linus asked each girl to wrap her own gift. But if the 'wrapping' relation between gifts and girls were semantically vague / underspecified and only pragmatically supplied (as it is in sentence (5) above), we would predict that sentence (3) would be intuitively true even in the second kind of situation.

In sum, we need to: (*i*) account for singular weak / strong donkey anaphora to structured (non-singleton) sets of individuals (see (2) and (3) above) and (*ii*) derive the incompatibility between singular donkey anaphora and collective predicates (see (6) above).

2 Outline of the Proposal: Plural Discourse Reference as Plural Information States

The notion of plural discourse reference (i.e. discourse-level plurality) as distinct and independent from plural reference (i.e. domain-level plurality) is the central component of the analysis. Following the proposal in van den Berg (1994, 1996) (which can be traced back to Barwise 1987 and Rooth 1987), I model plural discourse reference as plural information states in a new dynamic system couched in classical (many-sorted) type logic that extends Compositional DRT (CDRT, Muskens 1996). More precisely, I extend CDRT with plural information states that are modeled as *sets of variable assignments* I, J etc. (as opposed to single assignments i, j etc.) and that can be can be represented as matrices with assignments (sequences) as rows, as shown in (7) below.

Discourse-level plurality, i.e. a matrix / plural info state, is two-dimensional and encodes two kinds of discourse information: values and structure. The values are the sets of objects that are stored in the columns of the matrix, e.g. a dref u stores a set of individuals relative to a plural info state, since u is assigned an individual by each assignment (i.e. row). These individuals can be non-atomic, i.e. plural at the domain-level. The structure (quantificational dependency) is *distributively* encoded in the rows of the matrix: for each assignment / row in the plural info state, the individual assigned to a dref u by that assignment is structurally correlated with the individual assigned to some other dref u' by the same assignment. The resulting system is dubbed Plural CDRT (PCDRT).

7. Info State I

Info State I	…	u		u'		…
i_1	…	x_1	(i.e. ui_1)	y_1	(i.e. $u'i_1$)	…
i_2	…	x_2	(i.e. ui_2)	y_2	(i.e. $u'i_2$)	…
i_3	…	x_3	(i.e. ui_3)	y_3	(i.e. $u'i_3$)	…
…	…	…		…		…

Values – sets of objects (e.g. atomic / non-atomic individuals): $\{x_1, x_2, x_3, \dots\}$, $\{y_1, y_2, y_3, \dots\}$ etc.

Structure (plural discourse reference) – *n*-ary relations between objects: $\{<x_1, y_1>, <x_2, y_2>, <x_3, y_3>, \dots\}$ etc.

Plural info states enable us to capture the non-uniqueness intuitions associated with singular donkey anaphora and to give a compositional account of mixed weak & strong donkey sentences like (2) above by locating the weak / strong donkey ambiguity at the level of the indefinite articles. A weak indefinite article stores in a plural info state *some* of the individuals that satisfy its restrictor and nuclear scope, i.e. a non-maximal witness set, while a strong indefinite article stores in a plural info state *all* the individuals that satisfy its restrictor and nuclear scope, i.e. its maximal witness set[6]. Moreover, plural info states enable us to store and pass on anaphoric information about both values and structure, thereby enabling us to account for the simultaneous donkey anaphora to values and structure in sentence (3) above.

The hypothesis that weak / strong donkey readings should be attributable to the fact that indefinite articles are ambiguous or, better yet, underspecified with respect to the presence / absence of maximization enables us to account for an unrelated phenomenon, namely the variable nature of the uniqueness effects associated with singular (donkey) anaphora (as shown in section **5** below).

Finally, we account for the incompatibility between singular donkey anaphora and collective predicates (see (6) above) by taking *singular* donkey anaphora to be: (*i*) distributive at the discourse level, i.e. predicates need to be satisfied relative to each individual assignment i in a plural info state I; (*ii*) singular, i.e. atomic, at the domain level, i.e. for each $i \in I$, ui is atomic. The discourse-level distributivity is contributed partly by the indefinite article and partly by the main generalized determiner, while the domain-level atomicity is contributed by the singular number morphology on the donkey pronoun (and, to a lesser extent, by the singular morphology on the indefinite article).

Collective predicates, however, apply only to non-atomic individuals – that is, they are felicitous if either (*i*) the individuals stored by each variable assignment are non-atomic, i.e.

[6] A witness set for a static quantifier **DET**(A) (where **DET** is a static determiner and A is a set of individuals) is any set of individuals B such that $B \subseteq A$ and **DET**(A)(B). See Barwise & Cooper (1981): 103 (page references to Portner & Partee 2002).

we have domain-level plurality, e.g. for each $i \in I$, ui is non-atomic and ui was gathered around the fire, or (*ii*) they are interpreted collectively at the discourse level, e.g. we sum all the individuals stored in the plural info state $I=\{i_1, \ldots, i_n, \ldots\}$ and require the resulting sum individual $ui_1 \oplus \ldots \oplus ui_n \oplus \ldots$ to be gathered around the fire.

Allowing for non-atomic individuals in the domain, i.e. allowing for plural reference in addition to plural discourse reference, enables us to give an account of multiple (i.e. structured) plural donkey anaphora that is parallel to the account of singular donkey anaphora. For example, the PCDRT analysis of the plural donkey sentence in (8) below is parallel to the analysis of sentence (3) above. Note that the collective predicate *fight (each other)* in (8) is felicitous because, in contrast to example (3), we have domain-level non-atomicity introduced by the plural cardinal indefinite *two$^{u''}$ boys*.

8. Every^{u} parent who gives a$^{u'}$ balloon / three$^{u'}$ balloons to two$^{u''}$ boys expects them$_{u''}$ to end up fighting (each other) for *it$_{u'}$ / them$_{u'}$*.[7]

Allowing for plural reference also enables us to give a parallel account of singular and plural sage plant examples like the ones in (9) (see Heim 1982: 89, (12)) and (10) below. The only difference between the PCDRT analyses of these two examples is that, after we process the restrictor, each assignment in the output plural info state stores a sage plant atom for (9) and a non-atomic individual with two sage-plant atoms for (10). In both cases, we are able to derive the entailment that each customer bought nine sage plants.

9. Everybody^{u} who bought a$^{u'}$ sage plant here bought eight$^{u''}$ others along with it$_{u'}$.
10. Everybody^{u} who bought two$^{u'}$ sage plants here bought seven$^{u''}$ others along with them$_{u'}$.[8]

Finally, the PCDRT account of weak / strong plural donkey readings is parallel to the account of weak / strong singular donkey readings. For example, cardinal indefinites like *two$^{u'}$* can be either (*i*) strong, e.g. *two$^{u''}$ boys* in (8) above, or (*ii*) weak, e.g. *two$^{u''}$ dimes* in (12) below, where (12) is a minimal variation on the classical example of weak donkey readings provided in (11) (see Pelletier & Schubert 1989)[9].

[7] Based on an example due to Maria Bittner (p.c.).

[8] Based on example (49) in Kanazawa (2001): 393, which, in its turn, is adapted from Lapin & Francez (1994).

[9] In contrast to cardinal indefinites, *some*-based plural donkey anaphora seems to always be maximal, as shown by the intuitive interpretation of (i) below: every driver put *every* dime s/he had in the meter. Thus, the difference in interpretation between (12) and (i) indicates that the maximality associated with *some* anaphora (also instantiated by the Evans example *Harry bought some^{u} sheep. Bill vaccinated them$_{u}$*) is not a consequence of the fact that the anaphora is plural, but it should be attributed to the determiner *some*. That is, contrary to what seems to be the received wisdom, plural (donkey) anaphora is not necessarily maximal (at least, not necessarily maximal at the discourse level). The two independent notions of plurality argued for in PCDRT open a way to account for this observation: I think that *some* anaphora (and, perhaps, plural anaphora in general) involves a form of (local, maxima-based) *domain-level* maximality (*a* maximal sum individual such that… – see (ii) below), while the weak / strong donkey ambiguity is captured in terms of (global, supremum-based) *discourse-level* maximality (*the* maximal plural info state such that… – see (41) below). Throughout this paper, I will ignore domain-level maximality, which might in fact prove to be part and parcel of both *some*-based and cardinal-based plural (donkey) anaphora. See section **3** of the paper for the notation used in (ii) and (iii).

(i) Every^{u} driver who had some$^{u'}$ dimes put them$_{u'}$ in the meter.

(ii) $\mathbf{max\_individual}_u(D) := \lambda I_{st}.\lambda J_{st}.\ DIJ \wedge \neg\exists K_{st}(([u];\ D)IK \wedge \oplus uJ \leq \oplus uK \wedge \oplus uJ \neq \oplus uK)$,
where u is of type $\mathbf{e} := se$ and D is of type $\mathbf{t} := (st)((st)t)$.

(iii) $some^{\mathbf{wk}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [u];\ \mathbf{dist}(\mathbf{max\_individual}_u(P(u);\ P'(u)))$
$some^{\mathbf{str}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^u(\mathbf{dist}(\mathbf{max\_individual}_u(P(u);\ P'(u))))$

11. Every^{u} driver who had a$^{u'}$ dime put it$_{u'}$ in the meter.
12. Every^{u} driver who had two$^{u'}$ dimes put them$_{u'}$ in the meter.

3 Plural Compositional DRT (PCDRT): Compositional DRT with Plural Info States and Non-Atomic Individuals

We work with a Dynamic Ty2 logic, i.e., basically, with Muskens' Logic of Change (Muskens 1996), which is based on Gallin's Ty2 (Gallin 1975). There are three basic types:

- type t (truth-values);
- type e (atomic and non-atomic individuals); constants of type e: *dobby*, *megan* etc.; variables of type e: x, x' etc.;
- type s (modeling variable assignments as they are used in Dynamic Predicate Logic[10]); variables of type s: i, j etc.

A suitable set of axioms ensures that the entities of type s actually behave as variable assignments[11].

Following Link (1983) and Schwarzschild (1992) (among others), I take the domain of type e to be the power set of a given non-empty set $\mathbf{IN}$ of entities. More precisely, the domain of type e is $\wp^{+}(\mathbf{IN}) := \wp(\mathbf{IN})\backslash\{\varnothing\}$. The sum of two individuals $x_e \oplus y_e$ (subscripts on terms indicate their type) is the union of the sets x and y, e.g. $\{\mathbf{megan}\}\oplus\{\mathbf{gabby}\}=\{\mathbf{megan}, \mathbf{gabby}\}$. For a set of atomic and/or non-atomic individuals X_{et}, the sum of the individuals in X (i.e. their union) is $\oplus X$, e.g. $\oplus\{\{\mathbf{megan}, \mathbf{gabby}\}, \{\mathbf{gabby}\}, \{\mathbf{linus}\}\}=\{\mathbf{megan}, \mathbf{gabby}, \mathbf{linus}\}$. The part-of relation over individuals $x \leq y$ (x is a part of y) is the partial order induced by inclusion $\subseteq$ over the set $\wp^{+}(\mathbf{IN})$. The atomic individuals are the singleton subsets of $\mathbf{IN}$, identified by means of the predicate $\mathbf{atom}(x) := \forall y \leq x(y=x)$.

A dref for individuals u is a function of type se from assignments i_s to individuals x_e. Intuitively, the individual $u_{se}i_s$ is the individual that the assignment i assigns to the dref u. Dynamic info states I, J etc. are plural: they are sets of variable assignments, i.e. they are terms of type st. As shown in matrix (7) above, an individual dref u stores a set of atomic and/or non-atomic individuals with respect to a plural info state I, abbreviated as $uI := \{u_{se}i_s: i_s \in I_{st}\}$, i.e. uI is the image of the set of assignments I under the function u.

Thus, dref's are modeled like individual concepts in Montague semantics: just as the sense of the definite description *the chair of the Stanford linguistics department* (where, following Frege, sense is no more and no less than a way of giving the reference) is modeled as an individual concept, i.e. as a function from indices of evaluation to individuals, the meaning of a pronoun is basically a dref, i.e. a discourse-relative invidual concept, which is modeled as a

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Alternatively, *some* might be treated as a generalized determiner (we need to provide an externally dynamic definition of generalized determiners – see the discussion in section **3.4** below; for the externally dynamic definition of generalized determiners in PCDRT, see chapter **6** in Brasoveanu (2007), which would make *some* maximal because maximality is a necessary component of the dynamic definition of generalized determiners (this is due to right downward entailing determiners like *few* and *no*, for which we need both a maximal restrictor and a maximal nuclear scope). However, bare plurals (which are non-quantificational) could provide an independent argument for domain-level maximality – in view of examples like (iv) and (v) below, where the plural donkey anaphora receives a maximal interpretation. (I am grateful to Pranav Anand and Donka Farkas for discussion of this point).

(iv) Every^{u} farmer who bought donkeys$^{u'}$ vaccinated them$_{u'}$.

(v) Every^{u} driver who had dimes$^{u'}$ put them$_{u'}$ in the meter.

[10] See Groenendijk & Stokhof (1991).

[11] See the **Appendix** for more details.

function from discourse salience states to individuals (in the present system, a discourse salience state is just a Tarskian, total variable assignment).

The resulting Plural Compositional DRT (PCDRT) system advances the research program in Muskens (1996) of constructing theories and formal systems that integrate different frameworks (e.g. Montague semantics and dynamic semantics): PCDRT unifies in classical type logic the static, compositional analysis of generalized quantification in Montague semantics, Link's static analysis of plurality and van den Berg's Dynamic Plural Logic.

Moreover, PCDRT can be extended in the usual way with additional sorts for eventualities, times and possible worlds, which enables us to account for temporal and modal anaphora and quantification in a way that is parallel to the account of individual-level anaphora and quantification (see, for example, Brasoveanu 2007 for a parallel account of quantificational and modal subordination that extends the present account of donkey anaphora).

The remainder of this section introduces the main components of the PCDRT system and shows how the system deals with a couple of well-known examples and puzzles.

3.1 Conditions, New Dref's, DRS's and the Definition of Truth

A sentence is interpreted as a Discourse Representation Structure (DRS), i.e. as a relation of type $(st)((st)t)$ between an input info state I_{st} and an output info state J_{st}. As shown in (13) below, a DRS is represented as a [**new dref's** | **conditions**] pair, which abbreviates a term of type $(st)((st)t)$ that places two kinds of constraints on the output info state J: (*i*) J differs from the input info state I at most with respect to the **new dref's** and (*ii*) J satisfies all the **conditions**. An example is provided in (14) below.

13. [**new dref's** | **conditions**] := $\lambda I_{st}.\lambda J_{st}.\ I[\textbf{new dref's}]J \wedge \textbf{conditions}J$
14. $[u, u' \mid person\{u\}, book\{u'\}, buy\{u, u'\}] :=$
    $\lambda I_{st}.\lambda J_{st}.\ I[u, u']J \wedge person\{u\}J \wedge book\{u'\}J \wedge buy\{u, u'\}J$

DRS's of the form [**conditions**] that do not introduce new dref's are *tests* and they abbreviate terms of the form $\lambda I_{st}.\lambda J_{st}.\ I{=}J \wedge \textbf{conditions}J$, e.g. $[book\{u'\}] := \lambda I_{st}.\lambda J_{st}.\ I{=}J \wedge book\{u'\}J$.

Conditions, e.g. lexical relations like $buy\{u, u'\}$, are sets of plural info states, i.e. they are terms of type $(st)t$. Lexical relations are *unselectively distributive* with respect to the plural info states they accept, where "unselective" is used in the sense of Lewis (1975). That is, lexical relations universally quantify over variable assignments – or cases, to use the terminology of Lewis (1975): a lexical relation accepts a plural info state I iff it accepts, in a pointwise manner, every single assignment i in the info state I, as shown in (15) below. The first conjunct in (15), i.e. $I{\neq}\emptyset$, rules out the (degenerate) case when the universal quantification in the second conjunct $\forall i_s{\in}I(\ldots)$ (which encodes unselective distributivity) is vacuously satisfied.

An info state I satisfying condition $R\{u_1, \ldots, u_n\}$ can be intuitively depicted by a matrix like the one in (16) below.

15. **Lexical relations in PCDRT**:
    $R\{u_1, \ldots, u_n\} := \lambda I_{st}.\ I{\neq}\emptyset \wedge \forall i_s{\in}I(R(u_1 i, \ldots, u_n i))$,
    for any non-logical constant R of type $e^n t$ [12].

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[12] Where, following Muskens (1996), $e^n t$ is defined as the smallest set of types such that: (*i*) $e^0 t := t$ and (*ii*) $e^{m+1}t := e(e^m t)$.

16. Info state I	…	u_1	…	u_n	…
i	…	x_1 $(=u_1i)$	…	x_n $(=u_ni)$	…
		$R(u_1i, \ldots, u_ni)$, i.e. $R(x_1, \ldots, x_n)$			
i'	…	x_1' $(=u_1i')$	…	x_n' $(=u_ni')$	…
i''	…	x_1'' $(=u_1i'')$	…	x_n'' $(=u_ni'')$	…
…	…	…	…	…	…

Given unselective distributivity, the denotation of lexical relations has a lattice-theoretic ideal structure.

17. $\mathfrak{I}$ is a *complete ideal without a bottom element* (abbreviated as *c-ideal*) with respect to the partial order induced by set inclusion $\subseteq$ on the set $\wp^+(\boldsymbol{D}_s^{\boldsymbol{M}})$ [13] iff: (*i*) $\mathfrak{I} \subseteq \wp^+(\boldsymbol{D}_s^{\boldsymbol{M}})$; (*ii*) $\mathfrak{I}$ is closed under non-empty subsets and under arbitrary unions.
18. For any c-ideal $\mathfrak{I}$, $\mathfrak{I} = \wp^+(\cup\mathfrak{I})$, i.e. c-ideals are complete Boolean algebras without a bottom element.

The definition of lexical relations in (15) above ensures that they always denote c-ideals (in the atomic lattice $\wp(\boldsymbol{D}_s^{\boldsymbol{M}})$). We can in fact characterize them in terms of the supremum of their denotation, as shown in (19) below.

19. **Lexical relations as c-ideals**: For any constant R of type e^nt and sequence of dref's $<u_1, \ldots, u_n>$, let $\mathbb{I}(R, <u_1, \ldots, u_n>) := \lambda i_s.\ R(u_1i, \ldots, u_ni)$, abbreviated $\mathbb{I}^R$ whenever the sequence $<u_1, \ldots, u_n>$ can be recovered from context. Then, $R\{u_1, \ldots, u_n\} = \wp^+(\mathbb{I}^R)$ [14].

The fact that lexical relations denote c-ideals endows the PCDRT notion of dynamic meaning with a range of desirable formal properties, e.g., as shown in (23) below, DRS's (which are terms of type $(st)((st)t)$) can be defined in terms of simpler relations of type $s(st)$.

The other component of the definition of DRS's in (13) above is new dref introduction. We already have a Dynamic Ty2 notion of dref introduction, i.e. random assignment of value to a dref u. This notion, symbolized as $i[u]j$, relates two assignments i_s and j_s and can be informally paraphrased as: assignments i and j differ at most with respect to the value they assign to the dref u (see the **Appendix** for the exact definition).

The problem posed by the definition of new dref introduction in dynamic system based on plural info states is how to generalize the Dynamic Ty2 notion of new dref introduction, which is a relation between variable assignments, to a relation between sets of variable assignments (i.e. plural info states) I_{st} and J_{st}. The PCDRT definition is just the pointwise generalization of the Dynamic Ty2 notion, as shown in (20) below[15].

20. **New dref's in PCDRT**: $[u] := \lambda I_{st}.\lambda J_{st}.\ \forall i_s{\in}I(\exists j_s{\in}J(i[u]j)) \wedge \forall j_s{\in}J(\exists i_s{\in}I(i[u]j))$

Informally, $I[u]J$ means that each input assignment i has a $[u]$-successor output assignment j and, vice-versa, each output assignment j has a $[u]$-predecessor input assignment i. This ensures that we preserve the values and structure associated with the previously introduced dref's u', u'' etc. The definition in (20) treats the structure and value components of a plural info state in parallel, since we non-deterministically introduce both of them, namely: (*i*) some new (random) values for u and, also, (*ii*) some new (random) structure associating the u-values and the values of any other (previously introduced) dref's u', u'' etc.

The fact that the PCDRT definition of new dref introduction treats the dynamics of value and structure in parallel distinguishes it from most dynamic systems based on plural info states,

[13] Where $\wp^+(\boldsymbol{D}_s^{\boldsymbol{M}}) := \wp(\boldsymbol{D}_s^{\boldsymbol{M}})\backslash\{\emptyset\}$ and $\boldsymbol{D}_s^{\boldsymbol{M}}$)is the domain of entities of type s in model $\boldsymbol{M}$.

[14] Convention: $\wp^+(\emptyset_{st}) = \emptyset_{(st)t}$.

[15] This definition is equivalent to the definition of random assignment in van den Berg (1994).

including van den Berg (1996), Krifka (1996b) and Nouwen (2003), which only introduce values non-deterministically, while any newly introduced set of values is *deterministically* associated with a particular structure.

The explicit PCDRT distinction between the two informational components of an info state, i.e. values and structure, and their parallel treatment is motivated both empirically and theoretically. Empirically, the definition in (20) enables us to account for mixed reading donkey sentences like (2) above. Recall that, intuitively, we want to allow the credit cards to vary from book to book. That is, we want the restrictor of the *every*-quantification in (2) to non-deterministically introduce some set of u''-cards and non-deterministically associate them with the u'-books and let the nuclear scope filter the non-deterministically assigned values and structure by requiring each u''-card to be used to pay for the corresponding u'-book.

Theoretically, the PCDRT definition in (20) is the natural generalization of the Dynamic Ty2 definition insofar as it preserves its formal properties: just as $i[u]j$ as an equivalence relation of type $s(st)$ between variable assignments, $I[u]J$ as an equivalence relation of type $(st)((st)t)$ between sets of variable assignments (i.e. between plural info states).

Moreover, the fact that $[u]$ is an equivalence relation enables us to simplify the PCDRT definition of DRS's as shown in (23) below.

21. **PCDRT dynamic conjunction**: $D; D' := \lambda I_{st}.\lambda J_{st}.\ \exists H_{st}(DIH \wedge D'HJ)$.
22. $[u_1, \ldots, u_n] := [u_1]; \ldots; [u_n]$
23. **DRS's in terms of c-ideals over relations of type** $s(st)$.
    For any DRS $D := [u_1, \ldots, u_n \mid C_1, \ldots, C_m]$, where the conditions $C_1, \ldots, C_m$ are c-ideals, let $\mathbb{R}^D := \lambda i_s.\lambda j_s.\ i[u_1, \ldots, u_n]j \wedge j \in ((\cup C_1) \cap \ldots \cap (\cup C_m))$ [16]. Then, $D := \lambda I_{st}.\lambda J_{st}.\ \exists \mathbb{R}_{s(st)} \neq \emptyset(I = \mathbf{Dom}(\mathbb{R}) \wedge J = \mathbf{Ran}(\mathbb{R}) \wedge \mathbb{R} \subseteq \mathbb{R}^D)$, i.e. $D := \lambda I_{st}.\lambda J_{st}.\ \exists \mathbb{R} \in \wp^+(\mathbb{R}^D)(I = \mathbf{Dom}(\mathbb{R}) \wedge J = \mathbf{Ran}(\mathbb{R}))$ [17].

The PCDRT definition of truth – which has the expected form, namely existential quantification over output info states (a.k.a. existential closure) – is provided in (24) below.

24. **Truth.** A DRS D of type $(st)((st)t)$ is *true* with respect to an input info state I_{st} iff $\exists J_{st}(DIJ)$.

I will conclude this subsection with a brief comparison of the definition of lexical relations in (15) above, which is distributive at the discourse level (i.e. relative to a plural info state), with the alternative definition in (25) below, which is collective at the discourse level, e.g. the condition $book\{u\}$ requires the *sum* of all the individuals in uI, i.e. $\oplus uI$, to be in the set denoted by the static property *book* of type et [18].

The discourse-level collective definition in (25) is the PCDRT counterpart of the definition of tests in the Dynamic Plural Logic (DPlL) of van den Berg (1996). The collective definition is a sensible choice in DPlL because, in this system, only discourse-level plurality is acknowledged and non-atomic individuals, i.e. domain-level pluralities, can be obtained only

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[16] Where $i[u_1, \ldots, u_n]j := i([u_1]; \ldots ;[u_n])j$. Obviously, in this case, dynamic conjunction ';' is defined as relation composition over terms of type $s(st)$, i.e. $[u]; [u'] := \lambda i_s.\lambda j_s.\ \exists h_s(i[u]h \wedge h[u']j)$, where $[u]$ and $[u']$ are Dynamic Ty2 terms of type $s(st)$.

[17] Where $\mathbf{Dom}(\mathbb{R}) := \{i_s: \exists j_s(\mathbb{R}ij)\}$ and $\mathbf{Ran}(\mathbb{R}) := \{j_s: \exists i_s(\mathbb{R}ij)\}$.

[18] We can derive the intuitively correct distributive interpretation of the English noun *book* even if we assume the collective interpretation of lexical relations in (25) if we make the standard assumption that certain (uses of) static lexical relations are closed under sums, i.e. they are cumulative, e.g. $\forall x_e \forall y_e(book(x) \wedge book(y) \rightarrow book(x \oplus y))$ and, also, distributive at the domain level, e.g. if an individual x is a book, then its atomic parts are also books, i.e. $\forall x_e(book(x) \rightarrow \forall y_e \leq x(\mathbf{atom}(y) \rightarrow book(y)))$. Thus, because the discourse-level collective $book(\oplus uI)$ is domain-level cumulative and distributive, we correctly derive the fact that any atom that is a part of the sum individual $\oplus uI$ is a book, i.e. $\forall y_e \leq \oplus uI(\mathbf{atom}(y) \rightarrow book(y))$.

by summing over plural info states, i.e. over discourse-level pluralities. Thus, interpreting discourse-level plurality collectively (by default) is the only way to capture in DPlL the idea (going back to Link 1983) that, at the domain-level, atomic and non-atomic individuals are, by default, on a par (we need to add a predicate **atom** to the system to distinguish the atomic individuals).

25. $R\{u_1, \ldots, u_n\} := \lambda I_{st}.\ R(\oplus u_1 I, \ldots, \oplus u_n I)$,
for any non-logical constant R of type $e^n t$,
e.g. $book\{u\} := \lambda I_{st}.\ book(\oplus uI)$ and $buy\{u, u'\} := \lambda I_{st}.\ buy(\oplus uI, \oplus u'I)$.

However, since PCDRT acknowledges both discourse-level and domain-level plurality, such a choice is not forced upon us anymore. We can allow for a default collective interpretation of domain-level plurality while maintaining that discourse-level plurality is, by default, interpreted distributively. Interpreting discourse-level pluralities distributively by default is motivated by the fact that a discourse-level plurality is, ultimately, just a set of variable assignments – and one of the primary uses of variable assignments is to encode quantificational dependencies, which they do one assignment at a time, i.e. distributively.

This does not mean that we exclude the possibility of discourse-level collective readings for plurals. We do need them, as shown by the interpretation of the plural pronoun $they_u$ in discourse (26) below. Informally, sentence (26a) introduces a quantificational dependency between girls and purses that is distributively encoded in the output set of variable assignments: the output plural info state I is such that $u'I$ is the set of all girl-atoms and, for each assignment $i \in I$, ui is the purse-atom that Linus bought for the corresponding girl-atom $u'i$. Sentence (26b), however, *collectively* elaborates on the set of purchased purses: we consider the sum individual $\oplus uI$ consisting of all and only the previously introduced purse-atoms and we predicate of this sum individual that its atoms are identical except for the color.

26. **a.** Linus bought an^{u} alligator purse for every$^{u'}$ girl in his class. **b.** They$_{\oplus u}^{u''}$ were identical except for the color.

The fact that plural pronouns can be interpreted collectively at the discourse-level is compatible with the distributive definition of lexical relations in (15) above – that is, there is no need to generalize to the worst case and let lexical relations be collective at the discourse-level[19]. Instead, I will assume, in the spirit of Kamp & Reyle (1993), that plural pronouns can be optionally interpreted as summing over the dref (or dref's) they are anaphoric to. The existence of such discourse-level sums is independently motivated by discourses like (27) and (28) below, in which the plural pronoun *they* introduces the sum u'' of the two previously introduced dref's u and u'.

27. I saw John^{u} and Mary$^{u'}$ yesterday. They$_{u \oplus u'}^{u''}$ had just gotten married.
28. I saw a^{u} man and a$^{u'}$ woman yesterday. They$_{u \oplus u'}^{u''}$ had just gotten married.

Correspondingly, the plural pronoun in (26b) above is interpreted as $they_{\oplus u}^{u''}$, i.e. as introducing the sum of the previously introduced dref u. The relevant conditions are defined in (29) and (30) below. We will return to the interpretation of singular and plural pronouns in section **3.3** below.

29. $u''=u \oplus u' := \lambda I_{st}.\ I \neq \emptyset \wedge \forall i_s \in I(u''i=ui \oplus u'i)$
30. $u'=\oplus u := \lambda I_{st}.\ I \neq \emptyset \wedge \forall i_s \in I(u'i=\oplus uI)$

[19] I am indebted to Donka Farkas and Kyle Rawlins (p.c.) for discussion of this point.

With the basic dynamic system now in place, we can turn to the compositional interpretation of pronouns, indefinites and generalized determiners.

3.2 Compositionality

Given the underlying type logic, compositionality at sub-clausal level follows automatically and standard techniques from Montague semantics become available.

In more detail, the compositional aspect of interpretation in an extensional Fregean / Montagovian framework is largely determined by the types for the (extensions of the) 'saturated' expressions, i.e. names and sentences. Let us abbreviate them as **e** and **t**.

An extensional static logic with domain-level plurality identifies **e** with e (atomic and non-atomic individuals) and **t** with t (truth-values). The denotation of the noun `book` is of type **et**, i.e. et: `book` $\rightsquigarrow \lambda x_e.\ book_{et}(x)$. The generalized determiner `every` is of type (**et**)((**et**)**t**), i.e. $(et)((et)t)$: `every` $\rightsquigarrow \lambda S_{et}.\lambda S'_{et}.\ \forall x_e(S(x) \rightarrow S'(x))$.

We go dynamic with respect to both value and structure by making the 'meta-types' **e** and **t** more complex, i.e. by assigning finer-grained meanings to names and sentences. More precisely, PCDRT assigns the following dynamic types to the 'meta-types' **e** and **t**: **t** abbreviates $(st)((st)t)$, i.e. a sentence is interpreted as a DRS, and **e** abbreviates se, i.e. a name is interpreted as a dref for individuals.

The denotation of the noun `book` is still of type **et**, as shown in (31) below. The denotations of generalized determiners, indefinite articles and pronouns are provided in the following two subsections. Determiners and articles have denotations of the expected type, i.e. (**et**)((**et**)**t**), while pronouns anaphoric to a dref u are interpreted as the Montagovian quantifier-lift of the dref u (of type **e**), i.e. their type is (**et**)**t**.

31. `book` $\rightsquigarrow \lambda v_{\mathbf{e}}.\ [book\{v\}]$, i.e. `book` $\rightsquigarrow \lambda v_{\mathbf{e}}.\lambda I_{st}.\lambda J_{st}.\ I{=}J \wedge book\{v\}J$

The **Appendix** provides a rough-and-ready syntax for a fragment of English containing the donkey sentences in (2) and (3) above and compositionally defines its semantics in terms of a type-driven translation procedure from English into PCDRT.

3.3 Pronouns and Indefinites

A pronoun anaphoric to a dref u is interpreted as the Montagovian quantifier-lift of the dref u (of type **e**), i.e. its type is (**et**)**t**. Singular number morphology on pronouns contributes domain-level atomicity, as shown in (33) below. For simplicity, I take the $\mathbf{atom}\{u\}$ condition to be asserted and not presupposed – but section **5** below will remedy this shortcoming.

Plural number morphology on pronouns makes a fairly weak contribution: it just indicates the absence of a domain-level atomicity requirement. The stronger requirement of domain-level non-atomicity that is associated with many uses of plural pronouns can be derived in various ways, e.g., following Sauerland (2003), we can assume that a Maximize Presupposition principle of the kind proposed in Heim (1991) requires us to use singular pronouns whenever we can.

32. $\mathbf{atom}\{u\} := \lambda I_{st}.\ \mathbf{atom}(\oplus uI)$
33. $he_u \rightsquigarrow \lambda P_{\mathbf{et}}.\ [\mathbf{atom}\{u\}]; P(u)$
34. $they_u \rightsquigarrow \lambda P_{\mathbf{et}}.\ P(u)$ [20]

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[20] Anaphoric definite articles receive similar translations, namely $the_{\mathbf{sg}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [\mathbf{atom}\{u\}]; P(u); P'(u)$ and $the_{\mathbf{pl}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ P(u); P'(u)$.

The fact that singular pronouns contribute an **atom**$\{u\}$ condition enables us to derive the incompatibility between collective predicates and singular pronouns exemplified in (6) above, while allowing for collective predicates with plural pronouns, as in (8).

Also, the **atom**$\{u\}$ condition on singular pronouns captures the intuition that deictic (i.e. discourse-initial) uses of singular pronouns refer to atomic individuals. In particular, it is crucial that the **atom**$\{u\}$ condition is collectively interpreted relative to a plural info state I (i.e. at the discourse level). This ensures two things: (*i*) any two assignments i and i' in the info state I assign the same individual x to u, i.e. $\forall i_s{\in}I\forall i'_s{\in}I(ui{=}ui')$; (*ii*) the individual x assigned to u throughout the info state I is an atomic individual, i.e. $\forall i_s{\in}I(\mathbf{atom}(ui))$. Moreover, since the **atom** condition can apply only to entities of type e, which are elements of $\wp^+(\mathbf{IN}) := \wp(\mathbf{IN})\backslash\{\varnothing\}$, we are guaranteed that any info state I satisfying **atom**$\{u\}$ is non-empty, hence we do not need a separate conjunct of the form $I{\neq}\varnothing$ in definition (32).

In addition to the default meaning for plural pronouns in (34), we also need sum-based meanings that are discourse-level collective to account for examples (26), (27) and (28) above[21] – they are provided in (35) and (36) below. The sum-based meaning in (36) (together with the notion of dynamic generalized quantification introduced in the following subsection) enables us to account for examples like (37) below (see Kanazawa 2001: 397, (65)), where singular donkey anaphora interacts with sum-denoting plural pronouns.

35. $\mathit{they}_{\oplus u}{}^{u'} \rightsquigarrow \lambda P_{\mathbf{et}}.\ [u' \mid u'{=}\oplus u];\ P(u')$
36. $\mathit{they}_{u\oplus u'}{}^{u''} \rightsquigarrow \lambda P_{\mathbf{et}}.\ [u'' \mid u''{=}u{\oplus}u'];\ P(u'')$ [22]
37. `Every man who introduced a`^{u} `friend to me`$^{u'}$ `thought we`$_{u\oplus u'}{}^{u''}$ `had something in common.`

Let us turn now to indefinite articles. As (38) below shows, their PCDRT translation has the expected type (**et**)((**et**)**t**), i.e. it takes two dynamic properties P (the restrictor) and P' (the nuclear scope) as arguments and returns a DRS (i.e. a term of type **t**) as value. This DRS consists of two sub-DRS's that are dynamically conjoined: the first one, namely $[u]$, introduces a new dref u (the dref with which the indefinite article is indexed); the second sub-DRS, i.e. **dist**([**atom**$\{u\}$]; $P(u)$; $P'(u)$), constrains the value of this newly introduced dref.

38. $a^{\mathbf{wk}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [u];\ \mathbf{dist}([\mathbf{atom}\{u\}];\ P(u);\ P'(u))$

Just as in the case of pronouns, singular number morphology on indefinites contributes domain-level atomicity, i.e. a condition **atom**$\{u\}$. This condition, however, just as the restrictor and nuclear scope DRS's $P(u)$ and $P'(u)$, is within the scope of a discourse-level distributivity operator **dist**, defined in (39) below.

We need the **dist** operator in the translation of indefinites because singular (weak and strong) donkey anaphora is neutral with respect to semantic number – recall that, in (2) above, we are not quantifying only over people that buy exactly one book and have exactly one credit card, but over people that buy one or more books and use one or more of their credit cards to buy them. The fact that the **dist** operator takes scope over the **atom**$\{u\}$ condition contributed by singular number morphology neutralizes the domain-level atomicity requirement, which has to be satisfied only relative to each assignment i in the plural info state I and not relative to the entire info state I, thereby capturing the semantic number neutrality of donkey anaphora.

39. $\mathbf{dist}(D) := \lambda I_{st}.\lambda J_{st}.\ \exists R_{s((st)t)}{\neq}\varnothing(I{=}\mathbf{Dom}(R) \wedge J{=}\cup\mathbf{Ran}(R) \wedge \forall{<}k_s,L_{st}{>}{\in}R(D\{k\}L))$ [23]

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[21] The analysis of (26) also requires a notion of generalized quantification that is externally dynamic – see chapter **6** in Brasoveanu 2007 for the PCDRT formulation of such a notion.

[22] Plural anaphoric definite articles receive similar translations, namely $\mathit{the}_{\mathbf{pl}:\oplus u}{}^{u'} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [u' \mid u'{=}\oplus u];\ P(u');\ P'(u')$ and $\mathit{the}_{\mathbf{pl}:u\oplus u'}{}^{u''} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [u'' \mid u''{=}u{\oplus}u'];\ P(u'');\ P'(u'')$.

Distributively updating an input info state I with a DRS D of type $\mathbf{t} := (st)((st)t)$ means that we update each assignment $i \in I$ with the DRS D and then take the union of the resulting output info states. The operator **dist** is unselectively distributive at the discourse level in the same sense as the lexical relations defined in (15) above: it is discourse-level distributive because it distributes over plural info states and it is unselective in the sense of Lewis (1975) – we update one *case*, i.e. one assignment i in I, at a time[24].

PCDRT enables us to provide a unitary account of weak / strong donkey ambiguities as exhibited by both singular indefinite articles (see (2) above) and cardinal determiners (see (8) and (12) above for the strong and weak readings of `two`). The only difference between weak and strong indefinites (of both kinds) is the absence vs. presence of a maximization operator $\mathbf{max}^u$ taking scope over both the restrictor and the nuclear scope of the indefinites. The translation in (38) above is the one PCDRT associates with weak indefinite articles, while (40) below provides the **max**-based translation for strong indefinite articles.

40. $a^{\mathbf{str}:u} \leadsto \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^u(\mathbf{dist}([\mathbf{atom}\{u\}];\ P(u);\ P'(u)))$

Attributing the weak / strong ambiguity to the indefinites enables us to give a compositional account of the mixed reading sentence in (2) above because we *locally* decide for each indefinite whether it receives a weak or a strong reading.

Moreover, since the only difference between weak and strong indefinites is the absence vs. presence of the $\mathbf{max}^u$ operator, we can think of indefinites as *underspecified* with respect to maximization: the decision to introduce $\mathbf{max}^u$ or not is made online depending on the discourse and utterance context – much like aspectual coercion[25] or the selection of a particular type for the denotation of an expression[26] are context-driven online processes.

The hypothesis that indefinites are ambiguous between / underspecified for a weak vs. strong meaning does not lead to over-generation. As discussed in section **4** below, the weak / strong contrast surfaces only if: (*i*) there is anaphora to the indefinites (if there is no anaphora, weak and strong indefinites are truth-conditionally equivalent) and (*ii*) the indefinites and the anaphoric expressions are embedded in quantificational contexts. Thus, the weak / strong ambiguity is effectively neutralized for anaphora in non-quantificational contexts, e.g. in 'top'-level anaphora discourses like $A^{\mathbf{wk/str}:u}$ `man came in. He`$_u$ `sat down`.

Moreover, as section **5** below shows, taking indefinites to be underspecified for the presence / absence of a **max** operator enables us to account for phenomena that are unrelated to weak / strong donkey readings, namely the uniqueness effects exhibited by singular donkey and non-donkey anaphora.

The $\mathbf{max}^u$ operator, defined in (41) below, ensures that, after we process a strong indefinite, the output plural info state stores (with respect to the dref u) the *maximal* set of individuals satisfying both the restrictor dynamic property P and the nuclear scope dynamic property P'. In contrast, a weak indefinite will non-deterministically store *some* set of individuals satisfying its restrictor and nuclear scope.

41. $\mathbf{max}^u(D) := \lambda I_{st}.\lambda J_{st}.\ ([u];\ D)IJ \wedge \forall K_{st}(([u];\ D)IK \rightarrow uK \subseteq uJ)$

---

[23] Where $\mathbf{Dom}(R) := \{k_s\!: \exists L_{st}(RkL)\}$ and $\mathbf{Ran}(R) := \{L_{st}\!: \exists k_s(RkL)\}$.

[24] The fact that both **dist** operators and lexical relations are unselectively distributive at the discourse level does not mean that **dist** operators are redundant: unlike lexical relations, the operators can take scope over **atom** conditions, **max** operators etc., yielding dynamic and truth-conditional effects that are crucial for natural language representation.

[25] E.g. the iterative interpretation of `Linus sent a letter to the company for years` or `The light is flashing`.

[26] E.g. proper names are type-lifted when they are conjoined with generalized quantifiers.

The first conjunct in (41) introduces u as a new dref and makes sure that each individual in uJ satisfies D, i.e. uJ stores only individuals that satisfy D. The second conjunct enforces the maximality requirement: any other set uK obtained by a similar procedure (i.e. any other set of individuals that satisfies D) is included in uJ, i.e. uJ stores all the individuals that satisfy D. The DRS $\mathbf{max}^u(D)$ can be thought of as dynamic λ-abstraction over individuals: the abstracted variable is the dref u, the scope is the DRS D and the result of the abstraction is a set of individuals uJ containing all and only the individuals that satisfy D.

Moreover, the $\mathbf{max}^u$ operator together with the **dist** operator introduced above enable us to dynamize λ-abstraction over both values and structure: an update of the form $\mathbf{max}^u(\mathbf{dist}(D))$ (like the one contributed by strong indefinites – see (40) above) introduces the maximal set of individuals that satisfies D *distributively*, i.e. the structure that D associates with the dref u (e.g. D might introduce new dref's u', u'' etc. that will stand in particular structural relations to u) is introduced relative to one assignment at a time. This is particularly useful for examples like (3) above, which contain multiple instances of strong donkey anaphora[27].

The weak and strong meanings for cardinal indefinites differ from the ones for indefinite articles only with respect to the domain-level requirement. As (43) and (44) below show, each cardinal indefinite comes with its corresponding domain-level condition requiring the newly introduced individuals to have a particular number of atoms. For example, in the case of `two`, the condition $\mathbf{2\_atoms}\{u\}$ requires each individual to contain exactly two atomic parts.

42. $\mathbf{2\_atoms}\{u\} := \lambda I_{st}.\ \mathbf{2\_atoms}(\oplus uI)$,
where $\mathbf{2\_atoms}(x_e) := |\{y_e\colon y \leq x \wedge \mathbf{atom}(y)\}| = 2$.
43. $\mathtt{two}^{\mathbf{wk}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [u];\ \mathbf{dist}([\mathbf{2\_atoms}\{u\}];\ P(u);\ P'(u))$
44. $\mathtt{two}^{\mathbf{str}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^u(\mathbf{dist}([\mathbf{2\_atoms}\{u\}];\ P(u);\ P'(u)))$

3.4 Generalized Quantification

Selective generalized determiners are relations between two dynamic properties $P_{\mathbf{et}}$ (the restrictor) and $P'_{\mathbf{et}}$ (the nuclear scope), i.e. they have denotations of type $(\mathbf{et})((\mathbf{et})\mathbf{t})$. There are at least three empirical desiderata for any dynamic definition of selective generalized quantification – the definition has to be formulated in such a way that: (*i*) we capture the fact that anaphors in the nuclear scope can have antecedents in the restrictor, (*ii*) we avoid the proportion problem, i.e. the generalized determiner relates sets of individuals and not sets of variable assignments, and (*iii*) we can account for mixed reading (weak & strong) donkey sentences. Thus, the main problem posed by the dynamic definition of generalized quantification is to find a suitable way to extract the restrictor and nuclear scope sets of individuals based on the restrictor and the nuclear scope dynamic properties.

The proposed ways to define a notion of dynamic generalized quantification satisfying these three desiderata fall into two broad classes. The first class of solutions employs a dynamic framework based on singular info states (e.g. classical DRT / FCS / DPL) and analyzes generalized quantification as internally dynamic and externally static. The main idea is that the restrictor set of individuals is extracted based on the restrictor dynamic property, while the

[27] Thus, the **dist** operator enables us to express in PCDRT everything that the classical DRT / FCS / DPL systems can express, because the dynamic update in these systems is defined in a pointwise manner relative to individual variable assignments (i.e. relative to singular info states). In particular, the multiple strong donkey sentence in (3) above does not pose any problems for DRT / FCS / DPL precisely because their notion of dynamic update manipulates one assignment at a time, i.e. it is unselectively distributive.

Note, however, that adding **dist** to PCDRT does not mean that we inherit the problems of classical DRT / FCS / DPL: as the following sections show, PCDRT does not have a proportion problem and can account for weak / strong donkey ambiguities.

nuclear scope set of individuals is extracted based on both the restrictor and the nuclear scope dynamic property, so that the anaphoric connections between them are captured.

The second class of solutions employs a dynamic framework based on plural information states and analyzes generalized quantification as both internally and externally dynamic (see van den Berg 1994, 1996 – and also Krifka 1996b and Nouwen 2003 among others). The main idea is that the restrictor set of individuals is extracted based on the restrictor dynamic property and the nuclear scope set of individuals is the maximal *structured subset* of the restrictor set of individuals that satisfies the nuclear scope dynamic property.

Given that the notion of a dref being a structured subset of another dref required for the van den Berg-style definition involves non-trivial complexities that are orthogonal to the issues at hand, I will define selective generalized quantification following the format of the DRT / FCS / DPL-style definition. However, since PCDRT is a system based on plural info states and formulated in classical type logic, the definition of selective generalized determiners I provide in (45) and (46) below is novel. This definition is intermediate between the two ways of defining dynamic quantification described above and, as such, it is useful in formally exhibiting the commonalities and differences between them; see Brasoveanu (2007) for more discussion and a detailed comparison of the two definitions.

45. **Selective Generalized Determiners in PCDRT – the translation:**

$\mathtt{det}^{u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [\mathbf{det}_u(\mathbf{dist}(P(u)), \mathbf{dist}(P'(u)))]$

46. **Selective Generalized Determiners in PCDRT – the dynamic condition:**

$\mathbf{det}_u(D, D') := \lambda I_{st}.\ I{\neq}\emptyset \wedge \mathbf{DET}(u[DI],\ u[(D; D')I])$,

where $u[DI] := \{\oplus uJ\text{: } ([u \mid \mathbf{atom}\{u\}]; D)IJ\}$

and **DET** is the corresponding static determiner.

The condition $\mathbf{det}_u$ defined in (46) above tests that the static determiner **DET** relates two sets of atomic individuals, namely the restrictor set $u[DI]$ and the nuclear scope set $u[(D; D')I]$. The restrictor set $u[DI]$ is the set of atomic individuals that can be assigned to the individual dref *u* and that satisfy the restrictor DRS; this DRS is $\mathbf{dist}(P(u))$ (see the translation in (45)). The nuclear scope set $u[(D; D')I]$ is the set of atomic individuals that can be assigned to the individual dref *u* and that satisfy the dynamically conjoined restrictor and nuclear scope DRS's; the resulting DRS is $\mathbf{dist}(P(u))$; $\mathbf{dist}(P(u'))$. Dynamically conjoining the restrictor and nuclear scope DRS's ensures that the donkey pronouns in the nuclear scope can be successfully linked to their antecedents in the restrictor.

Thus, since the generalized determiners defined in (45)-(46) above relate sets of individuals, they contribute a selective quantification ("selective" in the sense of Lewis 1975) and thereby avoid the proportion problem of classical DRT / FCS / DPL. Moreover, the determiners are neutral with respect to weak vs. strong donkey readings (they are compatible with either of them) and the selection of a particular donkey reading is exclusively determined by the indefinite articles.

The definitions in (45)-(46) above endow dynamic determiners with two important characteristics. First, the determiners are domain-level atomic and discourse-level distributive relative to the 'variable' *u* they quantify over; this is ensured by the condition $\mathbf{atom}\{u\}$ in the definition of $u[DI]$ in (46)[28]. Second, they are discourse-level distributive relative to all the

[28] The dynamic generalized determiners are domain-level atomic and discourse-level distributive relative to the dref *u* they quantify over because, according to the definition of $u[DI]$ in (46), they relate two sets of atomic individuals and these sets of atomic individuals are required to satisfy the restrictor and nuclear scope dynamic properties one individual at a time (i.e. discourse-level distributivity). Both atomicity and distributivity are enforced by the condition $\mathbf{atom}\{u\}$ because this condition is collectively interpreted relative to a plural info state *I*, hence: (*i*) for any output info state *J*, any two assignments *j* and *j'* in it assign the same individual *x* to *u*, i.e.

dref's introduced and / or retrieved in their restrictor and nuclear scope – in particular, they are discourse-level distributive relative to donkey anaphora; this is ensured by the **dist** operators in (45) over the restrictor $P(u)$ and the nuclear scope $P'(u)$.

The two **dist** operators are in fact superfluous as far as the quantificational dref u is concerned, as shown by the 'unpacked' translation in (47) below: the condition $\mathbf{atom}\{u\}$ that precedes the **dist** operators already ensures that dref u is interpreted distributively at the discourse-level. That is, we could drop the **dist** operators without affecting the core truth-conditions of our generalized determiners.

47. $det^{u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\lambda I_{st}.\lambda J_{st}.\ I{=}J \wedge I{\neq}\emptyset \wedge$
$$\mathbf{DET}(\{\oplus uK\text{: } ([u \mid \mathbf{atom}\{u\}];\ \mathbf{dist}(P(u)))JK\},$$
$$\{\oplus uK\text{: } ([u \mid \mathbf{atom}\{u\}];\ \mathbf{dist}(P(u));\ \mathbf{dist}(P'(u)))JK\})$$

However, this does not mean that the **dist** operators are in general truth-conditionally or dynamically vacuous. They are one of the two crucial ingredients that enable us to derive the semantic number neutrality exhibited by donkey anaphora (the other ingredient being the **dist** operators introduced by singular indefinite articles), because they ensure the vacuous satisfaction of the **atom** conditions contributed by the singular donkey pronouns in the nuclear scope $P'(u)$. We will return to this issue section **5** below, when we discuss donkey uniqueness effects.

This concludes the discussion of the PCDRT system. The next three subsections provide analyses for three kinds of phenomena discussed in the previous static and dynamic literature: bound variable anaphora, quantifier scope ambiguities and proportions. The goal of these subsections is twofold: on the one hand, we see that PCDRT preserves previously obtained results; on the other hand, we are able to further clarify and motivate the system.

3.5 Bound Variable Anaphora

Going compositional at subclausal level requires us to make certain syntactic assumptions. For simplicity, I will work with a basic transformational syntax in the tradition of Chomsky (1981). The **Appendix** provides the complete definitions of the relevant fragment of English and the type-drive translation procedure[29].

"The most important requirement that we impose is that the syntactic component of the grammar assigns indices to all names, pronouns and determiners" (Muskens 1996: 159). The antecedents are indexed with superscripts and dependent elements with subscripts, following the convention in Barwise (1987). I will let indices be both specific and unspecific dref's, e.g. *Dobby*, *Megan*, u, u' etc.

I will also allow variables that have the appropriate dref type, e.g. v_{se}, v'_{se} etc., as indices, but *only* on traces of movement – because they are needed only on them. As Muskens (1996): 169 puts it: "In Montague's PTQ (Montague 197[4]) the Quantifying-in rules served two purposes: (a) to obtain scope ambiguities between noun phrases and other scope bearing elements, such as noun phrases, negations and intensional contexts, and (b) to bind pronouns appearing in the expression that the noun phrase took scope over. In the present set-up the mechanism of discourse referents takes over the second task".

The fact that we use distinct indices for the two purposes (unlike Muskens 1996 or Heim & Kratzer 1998, where natural numbers are used across the board) enables us to keep track of when our indexation makes an essentially dynamic contribution to the semantics and when it

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$\forall j_s{\in}J\forall j'_s{\in}J(uj{=}uj')$ (discourse-level distributivity of dref u); (*ii*) for any output info state J, the individual x assigned to u throughout J is an atomic individual, i.e. $\forall j_s{\in}J(\mathbf{atom}(uj))$ (domain-level atomicity of dref u).

[29] The definitions are based on Muskens (1996), Heim & Kratzer (1998) and Muskens (2005).

is an artifact of the particular scoping mechanism and the particular syntax/semantics interface we employ.

It will, therefore, be straightforward for the reader to reformulate the PCDRT analyses we develop here in her favorite syntactic formalism. The present choice of a particular (version of a particular) syntactic formalism is largely orthogonal to the matters we are concerned with here and is motivated only by presentational considerations: whichever syntactic formalism the reader favors, it is a reasonable expectation that she will have at least a nodding acquaintance with the Y-model of GB syntax.

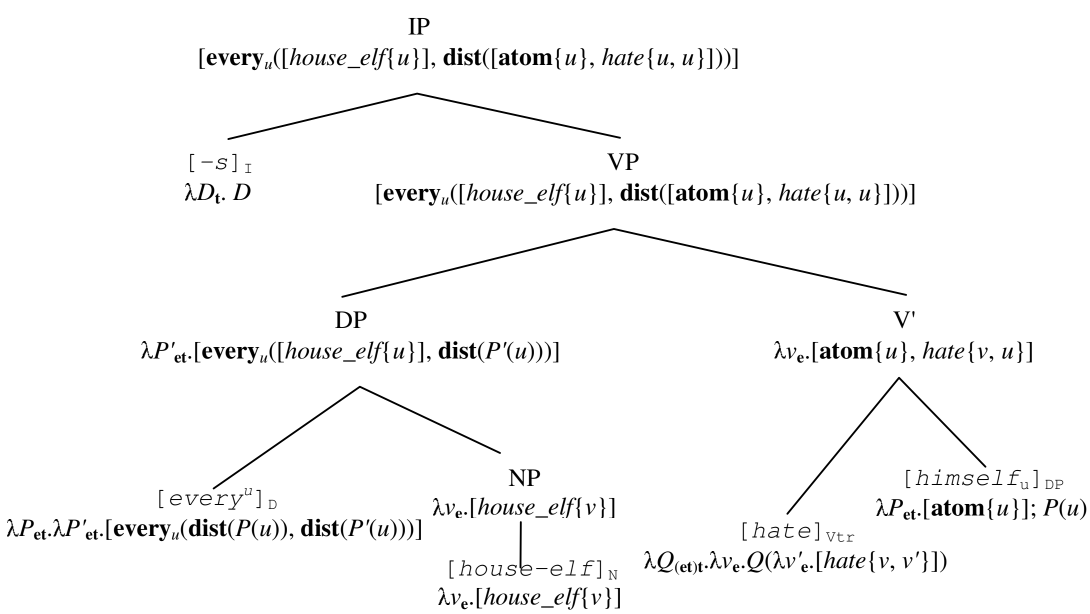
We turn now to the analysis of bound variable anaphora, exemplified in (48) below. In PCDRT (much like in any other compositional dynamic system), we can capture bound anaphora without using the syntactic rule of Quantifier Raising and the corresponding translation rule of Quantifying-In (see the **Appendix** for their exact definitions). We simply need the pronoun to be coindexed with the antecedent.

48. `Every`^{u} `house-elf hates himself`$_{u}$`.`

Coindexation is enough because binding in PCDRT (just like in DRT / FCS / DPL) is actually taken care of by the explicit quantification over assignments built into the meaning of dynamic generalized determiners. In contrast, quantification over assignments is only implicit in classical (static) logic – the paradigm example is λ-abstraction, which manipulates assignments only indirectly, as a function of the variable that is abstracted over. Therefore, if we want to obtain bound variable anaphora in a static system, coindexation, i.e. using the same variable, is not enough. We also need to create a suitable syntactic configuration that places the variable contributed by the pronoun in the scope of the relevant λ-abstractor, thereby ensuring semantic covariation.

Sentence (48) is compositionally translated as shown in (49) below. The final PCDRT representation (simplified based on various PCDRT equivalences, e.g. redundant **dist** operators are dropped) derives the intuitively correct truth-conditions, provided in (50).

49. `Every`^{u} `house-elf hates himself`$_{u}$`.`



50. $\lambda I_{st}.\ I \neq \emptyset \wedge \forall x_e(\mathbf{atom}(x) \wedge house\_elf(x) \rightarrow hate(x, x))$

Informally, the update provided under the IP node in (49) above instructs us to check that each way of filling column *u* (in the input matrix *I*) with a single elf *x* is a way of filling column *u* with elf *x* such that *x* hates himself. This update will be successful iff, in the model under consideration, every house-elf hates himself.

3.6 Quantifier Scope Ambiguities

The basic PCDRT system is compatible with any scoping mechanism in the literature. Since quantifier scope issues are not directly relevant to the matters we are interested in here[30], I will adopt the well-known, Montagovian Quantifying-In / Quantifier Raising mechanism. Consider the sentence in (51) below, which is ambiguous between two quantifier scopings: the surface-based scope $every^{u}$>>$a^{\mathbf{wk}:u'}$, represented as shown in (52), and the reverse scope $a^{\mathbf{wk}:u'}$>>$every^{u}$, represented as shown in (53). I assume, for simplicity, that the indefinite article is weak, but we will see that we obtain identical truth-conditions if the indefinite article is strong.

51. Every^{u} house-elf adores $\text{a}^{\mathbf{wk}:u'}$ witch.
52. $every^{u}$>>$a^{\mathbf{wk}:u'}$:

    $[\mathbf{every}_u([house\_elf\{u\}],\ [u'];\ \mathbf{dist}([\mathbf{atom}\{u'\}, witch\{u'\}, adore\{u, u'\}]))]$
53. $a^{\mathbf{wk}:u'}$>>$every^{u}$:

    $[u'];\ \mathbf{dist}([\mathbf{atom}\{u'\}, witch\{u'\}, \mathbf{every}_u([house\_elf\{u\}],\ [adore\{u, u'\}])])$

Informally, the update in (52) instructs us to check that, for every way of filling column *u* (in the input matrix *I*) with a single elf, there is a way of extending the resulting matrix by filling column *u'* with some witch that said elf adores. This update is successful iff every house-elf is such that s/he adores some witch or other.

In contrast, the update in (53) instructs us to do the following operations on the input matrix *I*: fill column *u'* with one or more (atomic) witches; then, for every single witch *y* in column *u'*, check that each way of extending the matrix with a column *u* that stores a single elf *x* is a way of extending the matrix with a column *u* that stores elf *x* and such that *x* adores witch *y*. This update is successful iff there is at least one witch (e.g. Hermione) such that every house-elf adores her.

The above PCDRT representations are compositionally obtained on the basis of the Logical Forms (LF's) in (54) and (57) below (once again, they are simplified based on various PCDRT equivalences, e.g. redundant **dist** operators are omitted). As (57) shows, the reverse scope is obtained by applying the QR rule to the indefinite DP $\text{a}^{\mathbf{wk}:u'}$ *witch*.

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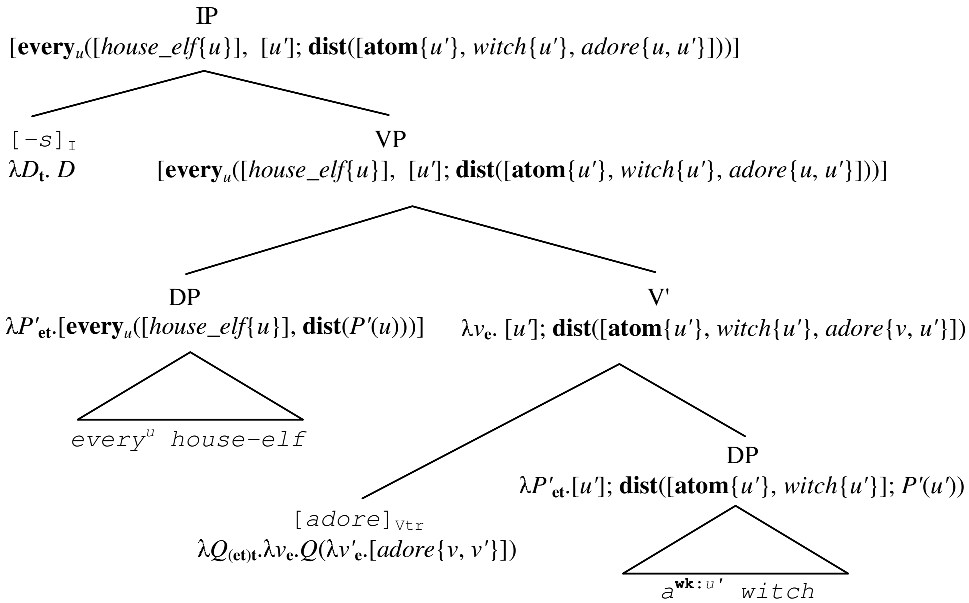
30 This does not mean that PCDRT does not have anything new to contribute to quantifier scope-related matters – see Brasoveanu & Farkas (2007) for a novel account of exceptional wide scope indefinites that makes crucial use of the fact that plural information states store and pass on quantificational dependencies introduced and elaborated upon in discourse.

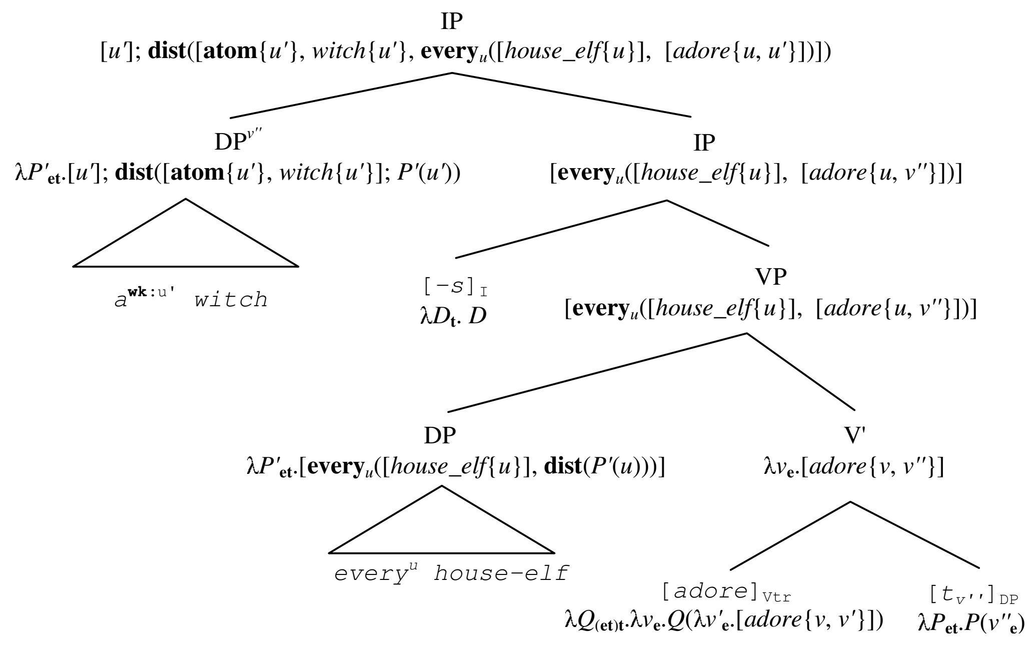
54. $every^{u}$>>$a^{\mathbf{wk}:u'}$: $Every^{u}$ house-elf adores $a^{\mathbf{wk}:u'}$ witch.



55. $\lambda I_{st}.\ I{\neq}\emptyset \wedge \forall x_e(\mathbf{atom}(x) \wedge house\_elf(x)$
$\rightarrow \exists Y_{et}{\neq}\emptyset(\forall y_e{\in}\, Y(\mathbf{atom}(y) \wedge witch(y) \wedge adore(x, y))))$

56. $\lambda I_{st}.\ I{\neq}\emptyset \wedge \forall x_e(\mathbf{atom}(x) \wedge house\_elf(x) \rightarrow \exists y_e(\mathbf{atom}(y) \wedge witch(y) \wedge adore(x, y)))$

57. $a^{\mathbf{wk}:u'}$>>$every^{u}$: $Every^{u}$ house-elf adores $a^{\mathbf{wk}:u'}$ witch.



58. $\lambda I_{st}.\ I{\neq}\emptyset \wedge \exists Y_{et}{\neq}\emptyset(\forall y_e{\in}\, Y(\mathbf{atom}(y) \wedge witch(y)$
$\wedge \forall x_e(\mathbf{atom}(x) \wedge house\_elf(x) \rightarrow adore(x, y))))$

59. $\lambda I_{st}.\ I{\neq}\emptyset \wedge \exists y_e(\mathbf{atom}(y) \wedge witch(y) \wedge \forall x_e(\mathbf{atom}(x) \wedge house\_elf(x) \rightarrow adore(x, y)))$

The representations derive the intuitively correct truth-conditions for the two readings, provided in (55) and (58) above. The quantification over sets of individuals in (55) and (58), i.e. $\exists Y_{et}{\neq}\emptyset(...)$, is used only to make more explicit the connection between truth-conditions and plural info states. In these particular cases (but not in general!), quantification over sets is not essential: (55) is equivalent to (56) above and (58) is equivalent to (59).

The two PCDRT representations that we obtain if the indefinite article is strong are provided in (61) and (64) below. Informally, the update in (61) instructs us to check that, for every way of filling column *u* (in the input matrix *I*) with a single elf, there is a way of extending the resulting matrix by filling column *u'* with some non-empty set containing all and only the witches that said elf adores. This update is successful iff every house-elf is such that s/he adores some witch or other.

The update in (64) instructs us to fill column *u'* (of the input matrix *I*) with some non-empty set containing all the (atomic) witches *y* that satisfy the following condition: for every single witch *y* in column *u'*, check that each way of extending the matrix with a column *u* that stores a single elf *x* is a way of extending the matrix with a column *u* that stores elf *x* and such that *x* adores witch *y*. This update is successful iff there is at least one witch (e.g. Hermione) such that every house-elf adores her.

For each representation, we derive truth-conditions that are ultimately equivalent to the ones above. Yet again, in both (62) and (65), we can do away with quantification over sets of individuals since we can substitute *salva veritate* $\exists y_e(\boldsymbol{F}y)$ for $\exists Y_{et}\neq\emptyset(\forall y_e(\boldsymbol{F}y \leftrightarrow y\in Y))$, where $\boldsymbol{F}$ stands for the predicate that is appropriate in each of the two cases.

60. `Every`^{u} `house-elf adores a`$^{\mathbf{str}:u'}$ `witch.`
61. *every*^{u}*>>a*$^{\mathbf{str}:u'}$:
 $[\mathbf{every}_u([\mathit{house\_elf}\{u\}],\ \mathbf{dist}(\mathbf{max}^{u'}(\mathbf{dist}([\mathbf{atom}\{u'\}, \mathit{witch}\{u'\}, \mathit{adore}\{u, u'\}]))))]$
62. $\lambda I_{st}.\ I\neq\emptyset \wedge \forall x_e(\mathbf{atom}(x) \wedge \mathit{house\_elf}(x)$
 $\rightarrow \exists Y_{et}\neq\emptyset(\forall y_e(\mathbf{atom}(y) \wedge \mathit{witch}(y) \wedge \mathit{adore}(x, y) \leftrightarrow y\in Y)))$
63. $\lambda I_{st}.\ I\neq\emptyset \wedge \forall x_e(\mathbf{atom}(x) \wedge \mathit{house\_elf}(x) \rightarrow \exists y_e(\mathbf{atom}(y) \wedge \mathit{witch}(y) \wedge \mathit{adore}(x, y)))$
64. *a*$^{\mathbf{str}:u'}$*>>every*^{u}:
 $\mathbf{max}^{u'}(\mathbf{dist}([\mathbf{atom}\{u'\}, \mathit{witch}\{u'\}, \mathbf{every}_u([\mathit{house\_elf}\{u\}],\ [\mathit{adore}\{u, u'\}])]))$
65. $\lambda I_{st}.\ I\neq\emptyset \wedge \exists Y_{et}\neq\emptyset(\forall y_e(\mathbf{atom}(y) \wedge \mathit{witch}(y)$
 $\wedge \forall x_e(\mathbf{atom}(x) \wedge \mathit{house\_elf}(x) \rightarrow \mathit{adore}(x, y)) \leftrightarrow y\in Y))$
66. $\lambda I_{st}.\ I\neq\emptyset \wedge \exists y_e(\mathbf{atom}(y) \wedge \mathit{witch}(y) \wedge \forall x_e(\mathbf{atom}(x) \wedge \mathit{house\_elf}(x) \rightarrow \mathit{adore}(x, y)))$

The reader can check that we also obtain the correct truth-conditions for examples in which indefinites take scope relative to downward entailing quantifiers like *no*^{u} *house-elf* or *few*^{u} *house-elves*. In particular, the **dist** operator contributed by weak / strong indefinite articles is crucial for the derivation of the correct truth-conditions: if we omit **dist**, we obtain overly weak truth-conditions when a singular indefinite has wide scope relative to a downward entailing quantifier.

3.7 The Proportion Problem and Weak / Strong Ambiguities

The proportion problem is solved in PCDRT because we work with a selective form of dynamic generalized quantification. The donkey sentences in (67)[31] and (70) below, exemplifying the proportion problem with strong and weak donkey anaphora respectively, are represented in PCDRT as shown in (68) and (71).

These compositionally obtained representations yield the intuitively correct truth-conditions, provided in (69) and (72) below. In words, (69) requires the cardinality of the set of (atomic) witch-loving house-elves that buy an alligator purse for each witch they fall in love with to be greater than the cardinality of the set of (atomic) witch-loving house-elves that fail to buy an alligator purse for at least one witch they fall in love with. Similarly, (72) requires the cardinality of the set of (atomic) dime-owning drivers that put at least one of their dimes in

[31] I take the indefinite *an*$^{\mathbf{wk}:u''}$ *alligator purse* in (67) to be weak only for simplicity – the truth-conditions that we derive if the indefinite is strong are identical.

the meter to be greater than the cardinality of the set of (atomic) dime-owning drivers that do not put any of their dimes in the meter.

67. `Most`^{u} `house-elves who fall in love with a`$^{\mathbf{str}:u'}$ `witch buy her`$_{u'}$ `an`$^{\mathbf{wk}:u''}$ `alligator purse.`

68. $[\mathbf{most}_u(\mathbf{dist}([\mathit{house\_elf}\{u\}];\ \mathbf{max}^{u'}(\mathbf{dist}([\mathbf{atom}\{u'\},\ \mathit{witch}\{u'\},\mathit{fall\_in\_love}\{u,\ u'\}]))),$
$\mathbf{dist}([\mathbf{atom}\{u'\}];\ [u''];\ \mathbf{dist}([\mathbf{atom}\{u''\},\ \mathit{a.purse}\{u''\},\ \mathit{buy}\{u,\ u',\ u''\}])))]$

69. $\lambda I_{st}.\ I{\neq}\varnothing \wedge |\{x_e\text{: } \mathbf{atom}(x) \wedge \mathit{house\_elf}(x) \wedge \exists y_e(\mathbf{atom}(y) \wedge \mathit{witch}(y) \wedge \mathit{fall\_in\_love}(x,\ y))$
$\wedge \forall y'_e(\mathbf{atom}(y') \wedge \mathit{witch}(y') \wedge \mathit{fall\_in\_love}(x,\ y')$
$\rightarrow \exists z_e(\mathbf{atom}(z) \wedge \mathit{a.purse}(z) \wedge \mathit{buy}(x,\ y',\ z)))\}| >$
$|\{x_e\text{: } \mathbf{atom}(x) \wedge \mathit{house\_elf}(x) \wedge \exists y_e(\mathbf{atom}(y) \wedge \mathit{witch}(y) \wedge \mathit{fall\_in\_love}(x,\ y)$
$\wedge \neg\exists z_e(\mathbf{atom}(z) \wedge \mathit{a.purse}(z) \wedge \mathit{buy}(x,\ y,\ z)))\}|$

70. `Most`^{u} `drivers who have a`$^{\mathbf{wk}:u'}$ `dime will put it`$_{u'}$ `in the meter.`

71. $[\mathbf{most}_u([\mathit{driver}\{u\}];\ [u'];\ \mathbf{dist}([\mathbf{atom}\{u'\},\ \mathit{dime}\{u'\},\ \mathit{have}\{u,\ u'\}]),$
$\mathbf{dist}([\mathbf{atom}\{u'\},\ \mathit{put\_in\_meter}\{u,\ u'\}]))]$

72. $\lambda I_{st}.\ I{\neq}\varnothing \wedge |\{x_e\text{: } \mathbf{atom}(x) \wedge \mathit{driver}(x) \wedge \exists y_e(\mathbf{atom}(y) \wedge \mathit{dime}(y) \wedge \mathit{have}(x,\ y)$
$\wedge\ \mathit{put\_in\_meter}(x,\ y))\}| >$
$|\{x_e\text{: } \mathbf{atom}(x) \wedge \mathit{driver}(x) \wedge \exists y_e(\mathbf{atom}(y) \wedge \mathit{dime}(y) \wedge \mathit{have}(x,\ y))$
$\wedge \forall y'_e(\mathbf{atom}(y') \wedge \mathit{dime}(y') \wedge \mathit{have}(x,\ y') \rightarrow \neg\mathit{put\_in\_meter}(x,\ y'))\}|$

4 Solutions to Donkey Problems

This section provides the PCDRT account of the core phenomena introduced in section **1**, namely mixed weak & strong donkey sentences and the incompatibility between singular donkey anaphora and collective predicates. The section concludes with a brief discussion of the neutralization of weak vs. strong contrasts in non-quantificational contexts.

4.1 Mixed Reading Donkey Anaphora and Collective Predicates

The compositionally obtained representation (simplified based on various PCDRT equivalences) for the mixed reading donkey sentence in (2) is given in (73) below; based on this representation, we derive the intuitively correct truth-conditions, provided in (74).

73. $[\mathbf{every}_u(\mathbf{dist}([\mathit{person}\{u\}];\ \mathbf{max}^{u'}(\mathbf{dist}([\mathbf{atom}\{u'\},\ \mathit{book}\{u'\},\ \mathit{buy}\{u,\ u'\}]));$
$[u''];\ \mathbf{dist}([\mathbf{atom}\{u''\},\ \mathit{c.card}\{u''\},\ \mathit{have}\{u,\ u''\}])),$
$\mathbf{dist}([\mathbf{atom}\{u'\},\ \mathbf{atom}\{u''\},\ \mathit{use\_to\_pay}\{u,\ u',\ u''\}]))]$

74. $\lambda I_{st}.\ I{\neq}\varnothing \wedge \forall x_e \forall y_e(\mathbf{atom}(x) \wedge \mathit{person}(x) \wedge \mathbf{atom}(y) \wedge \mathit{book}(y) \wedge \mathit{buy}(x,\ y) \wedge$
$\exists z_e(\mathbf{atom}(z) \wedge \mathit{c.card}(z) \wedge \mathit{have}(x,\ z))$
$\rightarrow \exists z'_e(\mathbf{atom}(z') \wedge \mathit{c.card}(z') \wedge \mathit{have}(x,\ z') \wedge \mathit{use\_to\_pay}(x,\ y,\ z')))$

Informally, the update in (73) can be described as follows. After the input info state is updated with the restrictor of the quantification in (2), we obtain a plural info state that stores, for each atomic *u*-person that is a book buyer and a card owner: (*i*) the maximal set of purchased book atoms, stored relative to the dref *u'* (since the indefinite `a`$^{\mathbf{str}:u'}$ `book` is strong), (*ii*) some non-deterministically introduced set of credit-card atoms, stored relative to the dref *u''* (since the indefinite `a`$^{\mathbf{wk}:u''}$ `credit card` is weak) and, finally, (*iii*) some non-deterministically introduced structure correlating the *u'*-atoms and the *u''*-atoms.

The nuclear scope of the quantification in (2) is anaphoric to both values (in this case, atomic individuals) and structure: we test that the non-deterministically introduced values for *u''* and the non-deterministically introduced structure associating *u''* and *u'* (the structure is tested by means of the **dist** operator) satisfy the nuclear scope update, i.e. we test that, for each assignment in the info state, the *u''*-card stored in that assignment is used to pay for the *u'*-

book stored in the same assignment. That is, the nuclear scope update elaborates on the structure, i.e. the dependency between u'' and u', that was non-deterministically introduced in the restrictor update.

The pseudo-scopal relation between the strong indefinite $a^{\mathbf{str}:u'}$ *book* and the weak indefinite $a^{\mathbf{wk}:u''}$ *credit card* ("pseudo" because, by the Coordinate Structure Constraint, the strong indefinite cannot syntactically take scope over the weak indefinite[32]) emerges as a consequence of the fact that PCDRT uses plural information states, which store and pass on information about both objects and dependencies between them.

The $\mathbf{atom}\{u'\}$ and $\mathbf{atom}\{u''\}$ contributed by the donkey pronouns $it_{u'}$ and $it_{u''}$ are vacuously satisfied as a consequence of two independent contributions. First, we have the $\mathbf{atom}\{u'\}$ and $\mathbf{atom}\{u''\}$ contributed by the singular donkey indefinites $a^{\mathbf{str}:u'}$ *book* and $a^{\mathbf{wk}:u''}$ *credit card*; the singular indefinites, however, introduce these conditions within the scope of two **dist** operators, allowing for output plural info states that store non-singleton sets of atoms relative to the dref's u' and u'' (and correctly so, given that donkey anaphora is not necessarily correlated with uniqueness).

The second component that is essential for the satisfaction of the pronominal **atom** conditions is the **dist** operator contributed by $every^{u}$ that scopes over the entire nuclear scope update. Given that this operator takes scope over the **atom** conditions, they only have to be satisfied relative to each variable assignment in the plural info state and not relative to the entire plural info state. That is, the fact that singular donkey pronouns are embedded in quantificational contexts (in particular, under **dist** operators contributed by generalized determiners) is essential for their neutrality with respect to semantic number.

As the next section shows, singular anaphoric pronouns that are not embedded in quantificational contexts are semantically singular, since the **atom** conditions they contribute have to be satisfied relative to entire plural info states.

The PCDRT representation for sentence (3), provided in (75) below, is largely parallel to the one for sentence (2) except for the fact that both indefinites ($a^{\mathbf{str}:u'}$ *Christmas gift* and $a^{\mathbf{str}:u''}$ *girl*) are strong.

75. $$[\mathbf{every}_u(\mathbf{dist}([boy\{u\}];\ \mathbf{max}^{u'}(\mathbf{dist}([\mathbf{atom}\{u'\}, gift\{u'\}];\ \mathbf{max}^{u''}(\mathbf{dist}([\mathbf{atom}\{u''\}, girl\{u''\}, buy\_for\{u, u', u''\}]))))),\ \mathbf{dist}([\mathbf{atom}\{u''\}];\ \mathbf{max}^{u'''}([d.mate\{u'''\}, of\{u''', u''\}]);\ [\mathbf{atom}\{u'''\}];\ [\mathbf{atom}\{u'\}];\ [a.t.w\{u, u''', u'\}]))]$$

76. $$\lambda I_{st}.\ I\neq\emptyset \wedge \forall x_e \forall R_{e(et)}\neq\emptyset(\mathbf{atom}(x) \wedge boy(x) \wedge \mathbf{Dom}(R) = \{y_e\!: \mathbf{atom}(y) \wedge gift(y) \wedge \exists z_e(\mathbf{atom}(z) \wedge girl(z) \wedge buy\_for(x, y, z))\} \wedge \forall y_e{\in}\mathbf{Dom}(R)(\forall z_e(Ryz \leftrightarrow \mathbf{atom}(z) \wedge girl(z) \wedge buy\_for(x, y, z))) \rightarrow \forall y_e \forall z_e(Ryz \rightarrow \exists z'_e(\mathbf{atom}(z') \wedge \forall z''_e(d.mate(z'') \wedge of(z'',z) \leftrightarrow z''=z') \wedge a.t.w(x,z',y))))$$

Informally, the update in (75) can be described as follows. After the input info state is updated with the restrictor of the quantification, we obtain a plural info state that, for a particular u-boy atom, stores (*i*) relative to u': the maximal set of gift atoms that the u-boy bought for some girl, (*ii*) relative to u'': the maximal set of girl atoms for whom the u-boy bought a gift and (*iii*) the structure associating the u'-atoms and the u''-atoms: for each assignment i in the output info state, the u'-gift stored in i was bought for the u''-girl stored in i.

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[32] That the Coordinate Structure Constraint does apply to this kind of examples is shown by sentence (i) below, where the quantifier $every^{u}$ *HP book* cannot scope out of its own conjunct to bind a pronoun in the other conjunct.

(i) #Every boy who reads $every^{u}$ *Harry Potter* book and recommends it_{u} to his friends is a *Harry Potter* addict.

Yet again, the nuclear scope of the quantification is anaphoric to both values and structure: we require each assignment in the plural info state to be such that the deskmate of the u''-girl in that assignment was asked to wrap the u'-gift in the same assignment. Thus, just as in the previous example, the nuclear scope elaborates on the structured dependency between the two sets of atoms (gifts and girls) introduced in the restrictor. As (76) shows, the dynamics of structure is truth-conditionally captured as quantification over the relation variable $R_{e(et)}$.

The interaction between the **max** and **dist** operators in the restrictor update, i.e. their interspersing $\mathbf{max}^{u'}(\mathbf{dist}(\ldots\mathbf{max}^{u''}\ldots$, ensures that we have structure maximization in addition to value maximization, i.e. not only we store all the u'-gifts and all the u''-girls, but, relative to *each* u'-gift (this is required by the **dist** operator), we store all the corresponding u''-girls. That value maximization and structure maximization are distinct is shown by example (77) below, where, for a given u-man, we do not want to store only the maximal u' and u'' values, i.e. all the u'-paintings that at least one of the friends like and all the u''-friends that like at least one painting, but also the maximal structure correlating u'-paintings and u''-friends. That is, if two different friends happen to like the same painting, there will be two distinct assignments i and i' correlating that painting with each of the friends, so that we can subsequently check that a distinct reproduction of the painting was bought for each friend.

77. Every^{u} man who saw a$^{\mathbf{str}:u'}$ painting that a$^{\mathbf{str}:u''}$ friend of his liked bought a$^{u'''}$ reproduction of it$_{u'}$ for him$_{u''}$.

The possessive $her_{u''}{}^{\mathbf{sg}:u'''}$ *deskmate* in (3) is analyzed as a Russellian definite description that contributes both existence (we introduce the dref u''' by means of $\mathbf{max}^{u'''}$) and uniqueness (relativized to u''-girls), as shown in (78) through (80) below. Note that the **max** operator contributed by Russellian definites has scope only over the restrictor update – in contrast to the **max** operator contributed by strong indefinites, which has scope over both the restrictor and the nuclear scope updates.

78. $the^{\mathbf{sg}:u'} \rightsquigarrow \lambda P_{\mathbf{et}}.\, \lambda P'_{\mathbf{et}}.\, \mathbf{max}^{u'}(P(u'));\, [\mathbf{atom}\{u'\}];\, P'(u')$

79. $her_{u}{}^{\mathbf{sg}:u'} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\, [\mathbf{atom}\{u\}];\, \mathbf{max}^{u'}(P(u');\, [of\{u', u\}]);\, [\mathbf{atom}\{u'\}];\, P'(u')$

80. $her_{u''}{}^{\mathbf{sg}:u'''}$ *deskmate* $\rightsquigarrow$

$$\lambda P_{\mathbf{et}}.\, [\mathbf{atom}\{u''\}];\, \mathbf{max}^{u'''}([d.mate\{u'''\},\, of\{u''', u''\}]);\, [\mathbf{atom}\{u'''\}];\, P(u''')$$ [33]

In both (78) and (79) above, uniqueness is a consequence of combining a **max** operator and an **atom** condition (with the condition outside the scope of the **max** operator) that target the same dref.

The analysis of the plural donkey example in (8) above is completely parallel to the analysis of (3). Similarly, the singular and plural weak donkey sentences in (11) and (12) above receive parallel analyses. The account of the singular and plural sage plant examples in (9) and (10) is discussed in detailed in section **6** below.

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[33] I provide a separate meaning for the possessive *her* only for simplicity. We can in fact analyze the possessive definite description $her_{u''}{}^{\mathbf{sg}:u'''}$ *deskmate* compositionally as being derived from $the^{\mathbf{sg}:u'''}$ $[[deskmate]_{N}\ [of\ she_{u''}]_{PP}]_{NP}$, where the preposition *of* is translated like a transitive verb, as shown in (i) below. The only difference between the type-drive translation in (ii) below and the one in (80) above is the location of the $\mathbf{atom}\{u''\}$ condition contributed by the singular pronoun *she* relative to the $\mathbf{max}^{u'''}$ operator contributed by the Russellian definite article $the^{\mathbf{sg}:u'''}$: the **atom** condition is in the scope of the **max** operator in (ii), but outside the scope of the operator in (80) (as its presuppositional status would actually have it). The two ways of providing a meaning for possessive Russellian descriptions and the resulting PCDRT updates are, in the case at hand, equivalent.

(i) $of \rightsquigarrow \lambda Q_{\mathbf{(et)t}}.\, \lambda v_{\mathbf{e}}.\, Q(\lambda v'_{\mathbf{e}}.\, [of_{e(et)}\{v, v'\}])$

(ii) $the^{\mathbf{sg}:u'''}\ [[deskmate]_{N}\ [of\ she_{u''}]_{PP}]_{NP} \rightsquigarrow$

$$\lambda P_{\mathbf{et}}.\, \mathbf{max}^{u'''}([d.mate\{u'''\},\, \mathbf{atom}\{u''\},\, of\{u''', u''\}]);\, [\mathbf{atom}\{u'''\}];\, P(u''')$$

Finally, the incompatibility between singular donkey anaphora and collective predicates exemplified in (6) above follows in PCDRT from the fact that the singular number morphology on the donkey pronoun $\mathtt{it}_{u'}$ contributes an **atom**{*u'*} condition which contradicts the collective, i.e. non-atomic nature, of the verb `gather`[34],[35].

This concludes the PCDRT account of the core phenomena introduced in section **1**. The present version of PCDRT does not account for example (1) because the definition of selective generalized determiners is externally static; for a version of PCDRT with externally dynamic generalized quantification that can account for (1), see Brasoveanu (2007).

4.2 Neutralization of Weak / Strong Contrasts in Non-Quantificational Contexts

This section argues that the weak / strong ambiguity is neutralized if singular indefinites and pronouns anaphoric to them are not embedded in quantificational contexts. More precisely, if the indefinite in a discourse like (81) below receives a strong reading, the truth conditions that the PCDRT representation derives for this discourse amount to the truth conditions traditionally associated with such existential discourses plus their strengthening due to the scalar implicatures triggered by the use of the singular indefinite article $\mathtt{a}^{u}$ as opposed to cardinal indefinites like $\mathtt{two}^{u}$, $\mathtt{three}^{u}$ etc. (I am assuming a Horn scale of the form $\mathtt{a}^{u}/\mathtt{one}^{u}<\mathtt{two}^{u}<\mathtt{three}^{u}\ldots$).

81. $\mathtt{A}^{u}$ `man came in.` $\mathtt{He}_{u}$ `sat down.`

That is, if the indefinite article is strong, discourse (81) is interpreted as: exactly one man came in and this man sat down. As shown by the representation in (82) below, this interpretation is a consequence of the interaction between the $\mathbf{max}^{u}$ operator contributed by the strong indefinite and the **atom**{*u*} condition contributed by the singular pronoun. Note that this cross-sentential effect is similar to the intra-sentential interaction between the **max** operator and the **atom** condition that enables us to capture the uniqueness component of Russellian definite descriptions (see (78) above).

82. $\mathbf{max}^{u}(\mathbf{dist}([\mathbf{atom}\{u\}, man\{u\}, come\_in\{u\}]))$; $[\mathbf{atom}\{u\}, sit\_down\{u\}]$
83. $\lambda I_{st}.\ I{\neq}\emptyset \wedge \exists x_e(\forall x'_e(\mathbf{atom}(x') \wedge man(x') \wedge come\_in(x') \leftrightarrow x'{=}x) \wedge sit\_down(x))$

If the indefinite in discourse (81) has a weak reading, the derived truth conditions are the standard existential ones not enriched with scalar implicatures, provided in (85) below.

84. $[u]$; $\mathbf{dist}([\mathbf{atom}\{u\}, man\{u\}, come\_in\{u\}])$; $[\mathbf{atom}\{u\}, sit\_down\{u\}]$,
or, equivalently: $[u \mid \mathbf{atom}\{u\}, man\{u\}, come\_in\{u\}, sit\_down\{u\}]$
85. $\lambda I_{st}.\ I{\neq}\emptyset \wedge \exists x_e(\mathbf{atom}(x) \wedge man(x) \wedge come\_in(x) \wedge sit\_down(x))$

Thus, the contrast between weak and strong indefinite articles is neutralized when these indefinites are not embedded under quantifiers in the following sense. First – and this applies equally to indefinites occurring in quantificational contexts –, if there is no anaphora to the

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[34] The PCDRT translation for the verb `gather` is provided in (i) below; the collectivity requirement is explicitly formalized by means of the condition ~[**atom**{*v'*}], modeled, for simplicity, as an assertion and not as a presupposition. See the **Appendix** for the definition of dynamic negation '~'.

(i) `gather` $\rightsquigarrow \lambda Q_{\mathbf{(et)t}}.\lambda v_{\mathbf{e}}.\ Q(\lambda v'_{\mathbf{e}}.\ [\sim[\mathbf{atom}\{v'\}], gather\{v, v'\}])$

[35] One more ingredient is needed to derive the infelicity of examples like (6) above – in addition to the collective predicate `gather` and the **atom** condition contributed by the donkey pronoun, namely: the kind of entities that the donkey indefinite and, in particular, the common noun, denotes. In example (6), the common noun `donkey` is individual denoting – but, if we replace it with a group denoting noun like `pack`, the resulting sentence, provided in (i) below, is felicitous. I am grateful to Alan Munn (p.c.) for emphasizing this point,

(i) $\mathtt{Every}^{u}$ `farmer who owns a` $\mathtt{pack}^{u'}$ `of donkeys gathers` $\mathtt{it}_{u'}$ `around the fire at night.`

indefinite, the weak and strong meanings for the indefinite articles are truth-conditionally identical: they contribute an existential quantification over atomic individuals that satisfy their restrictor and nuclear scope properties. That is, PCDRT correctly derives the fact that sentence (86) below has only one reading.

86. `There is a doctor`$^{\mathbf{wk/str}:u}$ `who is Welsh in London.`

Second, if there is anaphora to the indefinite, the strong indefinite is just the same as the weak indefinite plus the scalar implicature of uniqueness triggered by the Horn scale a^u<two^u<$three^u$…; this applies only to singular anaphors (in particular, to the **atom** conditions contributed by them) that do not occur in quantificational contexts, i.e. that are not in the scope of a **dist** operator[36],[37].

I will conclude with the observation that this analysis of weak / strong contrast neutralization in non-quantificational contexts is closely related to the investigation of the family of closely related meanings for reciprocal expressions in Dalrymple et al (1998) on the one hand, and, on the other hand, to the theory of scalar implicature computation proposed in Chierchia (2006). An investigation of the similarities and differences between these theories is left for future research.

5 Uniqueness Effects

This section provides an independent argument for the hypothesis that weak / strong donkey readings should be attributable to the fact that singular indefinites are ambiguous or, better yet, underspecified with respect to the presence / absence of a **max** operator. In particular, I argue that this variation in the meaning of the indefinite articles enables us to capture the variable nature of the uniqueness effects associated with singular donkey and non-donkey anaphora – where by "non-donkey anaphora", I mean singular anaphora that is not embedded in a quantificational context.

Thus, the PCDRT analysis of indefinites, initially motivated by the variable (weak vs. strong) readings associated with donkey anaphora, enables us to account for an unrelated, independently observed phenomenon: the variability of the uniqueness effects associated with singular (donkey) anaphora.

5.1 Uniqueness Effects and Anaphora in Non-Quantificational Contexts

Whether singular anaphora is associated with uniqueness has been debated at least since Evans (1977, 1980), Parsons (1978), Cooper (1979) and Heim (1982). Evans observes that the example in (87) below (see Evans 1980: 222, (26)[38]) is intuitively interpreted as: there is a *unique* doctor in London and this doctor is Welsh.

87. `There is a`$^{\mathbf{str}:u}$ `doctor in London and he`$_u$ `is Welsh.`

As the representation in (88) and the corresponding truth conditions in (89) below show, PCDRT captures this interpretation if the indefinite $a^{\mathbf{str}:u}$ `doctor` has a strong reading.

88. $\mathbf{max}^u(\mathbf{dist}([\mathbf{atom}\{u\}, doctor\{u\}, in\_London\{u\}])); [\mathbf{atom}\{u\}, Welsh\{u\}]$

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[36] These observations also apply to discourses involving multiple singular anaphors, e.g. A^u *man saw* $a^{u'}$ *woman. He*$_u$ *greeted her*$_{u'}$.

[37] I will not address here the problem of ensuring that we always have singular anaphora to singular indefinites, i.e. that we rule out plural pronouns, definites etc. anaphoric to singular indefinites. Various hypotheses can be formulated, e.g. singular anaphora could be required by syntactic number agreement or singular anaphora could be a consequence of a principle like Maximize Presupposition (Heim 1991), whereby if we can use a singular (as opposed to a plural) anaphor, we have to. The study of this matter is left for future research.

[38] Page references are to Evans (1985).

89. $\lambda I_{st}.\ I{\neq}\emptyset \wedge \exists x_e(\forall x'_e(\mathbf{atom}(x') \wedge doctor(x') \wedge in\_London(x') \leftrightarrow x'{=}x) \wedge Welsh(x))$

The fact that the uniqueness effect is a consequence of combining the meanings of the strong indefinite (in particular, the $\mathbf{max}^u$ operator) and the singular pronoun (in particular, the $\mathbf{atom}\{u\}$ condition) captures the observation in Kadmon (1990) that anaphora is a pre-condition for uniqueness: "[…] indefinite NP's don't always have unique referents. […] When anaphora is attempted, however, the uniqueness effect always shows up" (pp. 279-280).

Kadmon's observation is motivated by the contrast between examples (87) and (86) above: there is no uniqueness effect in (86) because no anaphora is attempted. But, contra Kadmon, uniqueness effects do not necessarily show up when anaphora is attempted. This is shown by the narration-type example in (90) below, from Heim (1982): 31, (29).

90. `There was a`$^{\mathbf{wk}:u}$ `doctor in London. He`$_u$ `was Welsh ...`

91. $[u]$; $\mathbf{dist}([\mathbf{atom}\{u\}, doctor\{u\}, in\_London\{u\}])$; $[\mathbf{atom}\{u\}, Welsh\{u\}]$,
or, equivalently: $[u \mid \mathbf{atom}\{u\}, doctor\{u\}, in\_London\{u\}, Welsh\{u\}]$

92. $\lambda I_{st}.\ I{\neq}\emptyset \wedge \exists x_e(\mathbf{atom}(x) \wedge doctor(x) \wedge in\_London(x) \wedge Welsh(x))$

Thus, native speakers have wavering intuitions with respect to the uniqueness effect associated with singular anaphora. As shown by the representation in (91) above, PCDRT captures this variability in judgments in terms of the presence or absence of the **max** operator in the meaning of indefinite articles: the non-**max** meaning does not yield any uniqueness effects, while the **max**-based meaning does[39].

PCDRT does not have anything to say about which particular reading we select in any given case – and rightfully so, since the choice is sensitive to various factors that are pragmatic in nature and / or are related to the rhetorical structure of the discourse, e.g. the fact that (90), unlike (87), is a narrative, seems to favor non-uniqueness[40].

In sum, besides the variable nature of the uniqueness effects, PCDRT also captures Kadmon's observation that singular anaphora is a necessary (but, contra Kadmon, not sufficient) condition for the occurrence of uniqueness effects. Importantly, the ingredients of the analysis – in particular, the two meanings associated with the indefinite article – are independently motivated by the analysis of weak / strong donkey anaphora.

Moreover, the account is compositional and the **atom** condition contributed by singular number morphology on anaphors is a local constraint on dref values of the same kind as ordinary lexical relations – in contrast to the non-local and non-compositional uniqueness condition proposed in Kadmon (1990) to account for such uniqueness effects[41].

Finally, unlike Kadmon (1990) (see the contrast between the preliminary and final versions of the uniqueness condition stated in Kadmon 1990[42]), PCDRT captures without any additional stipulations the contrast between the *absolute* uniqueness effects instantiated by (87) (where

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[39] PCDRT also makes correct predictions with respect to the examples in (i) and (ii) below, due to Heim (1982): (28), (27) and (27a), which are parallel to the examples in (86), (87) and (90) above.

(i) `A wine glass broke last night. It had been very expensive.`

(ii) `A wine glass which had been very expensive broke last night.`

[40] See Heim (1982), Kadmon (1987, 1990) and Roberts (2003) (among others) for more discussion.

[41] This is the preliminary (simpler) version of the uniqueness condition in Kadmon (1990): 284, (30): "A definite NP associated with a variable X in DRS K is used felicitously only if for every model M, for all embedding functions f, g verifying K relative to M, $f(X)=g(X)$".

[42] The preliminary version of the uniqueness condition is provided in fn. **41** above. The final version of the uniqueness condition is as follows: "Let α be a definite NP associated with a variable Y, let K_{loc} be the local DRS of α, and let K be the highest DRS s.t. K is accessible from K_{loc} and $Y\in U_K$. α is used felicitously only if for every model M, for all embedding functions f, g verifying K relative to M, if $\forall X\in B_K\ f(X)=g(X)$ then $f(Y)=g(Y)$" (Kadmon 1990: 293, (31)), where $B_K := \{X: \exists K'$ accessible from K s.t. $K'\neq K$ and $X\in U_{K'}\}$.

the doctor is absolutely unique) and the *relativized* uniqueness effects exhibited by donkey anaphora that will be discussed in the following subsection: relativized uniqueness follows automatically in PCDRT from the interaction between the independently motivated dynamic meaning for generalized determiners and the **atom** condition contributed by singular donkey pronouns.

5.2 Uniqueness Effects and Donkey Anaphora

The uniqueness effects associated with intra-sentential singular donkey anaphora are, by and large, just as unstable as the ones associated with cross-sentential singular anaphora.

On the one hand, the examples in (93) and (94) below (see Parsons 1978: 19, (4), where the example is attributed to B. Partee, and Cooper 1979: 81, (60)) exhibit uniqueness effects, more precisely: uniqueness effects relativized to each particular value of the dref *u* quantified over by the generalized determiner $every^u$.

93. $Every^u$ man who has $a^{u'}$ son wills $him_{u'}$ all his money.
94. $Every^u$ man who has $a^{u'}$ daughter thinks $she_{u'}$ is the most beautiful girl in the world.

On the other hand, the examples in (95), (96)[43] , (97), (98)[44] and (99) below (some repeated from above), instantiating both weak and strong donkey readings, do not exhibit uniqueness effects[45].

95. $Every^u$ farmer who owns $a^{\mathbf{str}:u'}$ donkey beats $it_{u'}$.
96. $Most^u$ people that owned $a^{\mathbf{str}:u'}$ slave also owned $his_{u'}$ offspring.
97. $Every^u$ driver who had $a^{\mathbf{wk}:u'}$ dime put $it_{u'}$ in the meter.
98. No^u parent with $a^{\mathbf{wk}:u'}$ son still in high school has ever lent $him_{u'}$ the car on a weeknight.
99. $Every^u$ person who buys $a^{\mathbf{str}:u'}$ TV and has $a^{\mathbf{wk}:u''}$ credit card uses $it_{u''}$ to pay for $it_{u'}$.
100. $Everybody^u$ who bought $a^{\mathbf{wk/str}:u'}$ sage plant here bought $eight^{u''}$ others along with $it_{u'}$.[46]

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[43] See Heim (1990): 162, (49).

[44] See Rooth (1987): 256, (48).

[45] Kadmon (1990) is undecided with respect to examples like (95)/(96) and (i) below (see Kadmon 1990: 307, (48)). Kadmon (1990): 307 takes these examples to exhibit uniqueness effects, while mentioning on the following page that some informants disagree and "treat [(i)] as if it said 'at least one dog'; for them, [(i)] doesn't display a uniqueness effect" (Kadmon 1990: 308-309).

(i) $Most^u$ women who own $a^{u'}$ dog talk to $it_{u'}$.

Kanazawa (2001): 391, fn. 5 also claims that relative-clause donkey sentences always exhibit uniqueness effects and distinguishes them from conditional donkey sentences, which do not contribute any form of uniqueness. Note, however, that the uniqueness intuitions associated with relative-clause donkeys are much weaker (if at all present) when we consider examples with *multiple* donkey indefinites like (99), i.e. relative-clause donkey sentences that are closer in form to conditional donkey sentences.

[46] Kadmon (1990): 317 maintains that the donkey anaphora in (100) does in fact contribute a uniqueness presupposition, but the "speakers accept this example because it can't make any difference to truth conditions which sage plant the pronoun *it* stands for, out of all the sage plants that a buyer *x* bought (for each buyer *x*)". But, as Heim (1990): 161 points out, Kadmon's 'supervaluation' analysis (the connection with supervaluation treatments of vagueness is due to Mats Rooth – see Heim 1990: 160, fn. 11) makes incorrect predictions with respect to example (98) above: intuitively, sentence (98) is falsified by any parent who has a son in high school and who has lent him the car on a weeknight even if said parent has another son who never got the car – which is

Intuitively, there is a clear contrast between the two sets of examples. Sentence (93), for example, intuitively quantifies only over men that have a single son (i.e. relativized uniqueness), while sentence (96), for example, quantifies over slave owners in general and not only slave owners that owned a single slave (i.e. no uniqueness).

In general, previous accounts of donkey anaphora are designed to account either for the first set of examples (e.g. uniqueness accounts like Parsons 1978, Cooper 1979, Kadmon 1990 among others), or for the second set of examples (e.g. non-uniqueness accounts like Kamp 1981, Heim 1982, 1990, Neale 1990, Kamp & Reyle 1993 among others). The goal of this subsection is to argue that the availability of **max / dist** operators and **atom** conditions enables PCDRT to provide a flexible account of donkey anaphora that accommodates both kinds of donkey examples in a natural way.

Thus, what we need to account for is a three-way contrast between: (*i*) strong, non-unique donkey readings, e.g. (95) and (96) above, (*ii*) weak, non-unique donkey readings, e.g. (97) and (98) above, and, finally, (*iii*) unique readings (which, intuitively, conflate weak and strong readings), e.g. (93) and (94) above.

The binary distinction between weak and strong indefinite articles can only partly capture this three-way contrast. The other crucial – and independently motivated – component that I will invoke is that the **atom** presupposition contributed by singular number morphology on donkey pronouns can be satisfied in different ways, more precisely: at different places in the PCDRT representation associated with any particular donkey sentence.

Recall that, for simplicity, I took the **atom** condition contributed by pronouns like $\mathtt{him}_{u'}$, $\mathtt{it}_{u'}$ etc. to be part of the assertion. The variable nature of donkey uniqueness effects, however, requires us to do away with this simplifying assumption.

So, let us treat the **atom** condition contributed by singular number morphology on pronouns as a presupposition. We will then expect it to behave just like any other presupposition introduced in the nuclear scope of a generalized determiner – in particular, we would that different loci in the representation will be available for satisfaction or accommodation of this presupposition[47].

This is shown by the example in (101) below (a close variant of the examples in Karttunen & Peters 1979 and Heim 1983): in principle, the presupposition that the *u*-nations have a king, which is triggered by the possessive $\mathtt{its}_u$, can be accommodated relative to the global, intermediate or narrow contexts of interpretation.

101. $\mathtt{Every}^u$ `nation cherishes` $\mathtt{its}_u$ `king.`
102. **GLOBAL** $\mathtt{Every}^u$ **[**$_{\textbf{restrictor}}$ **INTERMEDIATE** `nation`**]**
[$_{\textbf{nuclear scope}}$ **NARROW** `cherishes` $\mathtt{its}_u$ `king`**]**.

However, as van der Sandt (1992) observes, the global resolution is not possible because it would render the dref *u* retrieved by the possessive $\mathtt{its}_u$ unbound, violating the Trapping Principle: "if a presupposition containing a variable *x* is triggered in an environment where *x* is bound by a quantifier *Q*, the presupposition will be accommodated in such a way that *x* remains bound by *Q*" (Beaver & Zeevat 2006: 14).

Therefore, the only available resolution loci for the presupposition triggered by the possessive $\mathtt{its}_u$ are the intermediate and narrow contexts – and, intuitively both readings are acceptable (a fact repeatedly noted in the literature): intermediate accommodation yields a reading to the

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to say that it does make a difference in this case which son the pronoun $\mathtt{him}_u$ in (98) stands for (for more discussion, see also Geurts 2002: 145 et seqq).

[47] See Karttunen & Peters (1979), Heim (1983), van der Sandt (1992) and Beaver & Zeevat (2006) among many others for more discussion.

effect that every nation that has a king cherishes him, while local accommodation produces a reading to the effect that every nation has a king and cherishes him.

I propose to treat the **atom** presupposition contributed by singular donkey pronouns in a parallel way. Global resolution is again not possible because the dref anaphorically retrieved by the donkey pronoun becomes unbound. The remaining two possibilities are the intermediate and narrow resolutions, which account for the variable nature of the donkey uniqueness effects: intermediate resolution (in conjunction with a strong reading for the indefinite) yields uniqueness, while the narrow resolution yields semantic number neutrality.

In more detail: consider the PCDRT representation for the strong, non-unique donkey sentence in (95) above, provided in (103) below (underlining is meant to indicate the presuppositional status of the $\underline{\mathbf{atom}\{u'\}}$ condition). The narrow resolution of the presupposition derives the intuitively correct, number neutral reading: every donkey-owning farmer beats every donkey s/he owns.

103. strong, non-unique donkey readings (narrow presupposition resolution):
$[\mathbf{every}_u(\mathbf{dist}([farmer\{u\}]; \mathbf{max}^{u'}(\mathbf{dist}([\mathbf{atom}\{u'\}, donkey\{u'\}, own\{u, u'\}]))),$
$\mathbf{dist}([\underline{\mathbf{atom}\{u'\}}]; [beat\{u, u'\}]))]$

Consider now the PCDRT representation for the unique donkey sentence in (93) above, given in (104) below. The intermediate resolution of the presupposition derives the intuitively correct reading, i.e. the reading that exhibits relativized uniqueness effects ("relativized" in the sense that the u'-son is unique relative to each value of the dref u): every man who has exactly one son wills all this money to his son.

104. unique donkey readings (intermediate presupposition resolution):
$[\mathbf{every}_u(\mathbf{dist}([man\{u\}]; \mathbf{max}^{u'}(\mathbf{dist}([\mathbf{atom}\{u'\}, son\{u'\}, have\{u, u'\}]))); [\underline{\mathbf{atom}\{u'\}}],$
$\mathbf{dist}([will\_all\_money\{u, u'\}]))]$

The uniqueness effects emerge as a consequence of the interaction between the $\mathbf{max}^{u'}$ operator contributed by the donkey indefinite $a^{u'}$ son and the $\underline{\mathbf{atom}\{u'\}}$ presupposition contributed by the singular donkey pronoun $him_{u'}$; crucially, the $\underline{\mathbf{atom}\{u'\}}$ condition is not embedded under the nuclear scope **dist** operator. The relativized nature of the uniqueness effects follows automatically from the fact that both the operator $\mathbf{max}^{u'}$ and the $\underline{\mathbf{atom}\{u'\}}$ condition are embedded in the restrictor of $every^u$[48].

Once again, PCDRT correctly predicts that uniqueness effects appear only when the dref introduced by the indefinite is anaphorically retrieved. If there is no donkey anaphora, e.g. in

[48] A historical note: Parsons (1978) considers the uniqueness effects associated with the donkey sentence in (93) above and suggests two different ways to capture them. The PCDRT account can be seen as an implementation of the first suggestion: "One might suggest that the feeling of inappropriateness [of sentence (93) when taken to be talking about men that have more than one son] comes explicitly from the use of the pronoun. How would that work? Well, one purported meaning of 'a' is 'one', in the sense of 'exactly one'. Usually this is thought to be a presupposition, implication, or implicature of the utterance rather than part of the content of what is said. But perhaps the use of a singular pronoun can make the import part of the official content. The suggestion then is that 'a' can mean either 'at least one' or 'exactly one'. Normally it means the former, but certain grammatical constructions force the latter reading. The former reading is the 'indefinite' one, and the latter is the 'definite' one." (Parsons 1978: 19). To my knowledge, the present account is the first to take this suggestion seriously.

Parsons' second suggestion is the one that is taken up by D-/E-type approaches that consider pronouns to be numberless Russellian definite descriptions (e.g. Neale 1990): "Sometimes 'the' doesn't mean 'exactly one', but rather 'at least one' or 'every'. It means 'at least one' in *everyone must pay the clerk five dollars* and it means 'every' in *you should always watch out for the other driver*. Or something like this. So perhaps the treatment of pronouns as paraphrases is correct, but we have to tailor the meaning of 'the' for the situation at hand. For example, in our sample sentence we need to read *the donkey he owns* as *every donkey he owns*. This response would involve specifying some method for determining which reading of *the* is appropriate in a given paraphrase; I haven't carried this out" (Parsons 1978: 20).

`Every farmer who owns a donkey is happy`, there will be no uniqueness effects – and no weak / strong donkey ambiguities for that matter: we obtain the same truth-conditions, irrespective of which reading the indefinite article has.

PCDRT does not decide which presupposition resolution (narrow vs. intermediate) is preferred in a particular case – and correctly so, since this is a pragmatic decision based on various factors, including non-linguistic ones like world knowledge about wills[49].

However, the present analysis predicts that, *ceteris paribus*, the narrow resolution of the presupposition will be preferred to the intermediate one when we have donkey anaphora embedded under distributive generalized determiners like `every`, `no` etc. since the presupposition is automatically satisfied if it has narrow 'scope' while, if the presupposition has intermediate 'scope', it needs to be accommodated[50].

Finally, let us turn to the weak donkey sentence in (97) above. The two representations corresponding to the narrow and intermediate presupposition resolution are provided in (105) and (106) below. These representations derive identical truth conditions – hence, PCDRT accounts for the fact that, intuitively, the donkey sentence in (97) has only one interpretation.

105. weak donkey readings (narrow presupposition resolution):
[**every**$_u$(**dist**([*driver*{*u*}]; [*u'*]; **dist**([**atom**{*u'*}, *dime*{*u'*}, *have*{*u*, *u'*}])),
dist([<u>**atom**{*u'*}</u>]; [*put_in_meter*{*u*, *u'*}]))]

106. weak donkey readings (intermediate presupposition resolution):
[**every**$_u$(**dist**([*driver*{*u*}]; [*u'*]; **dist**([**atom**{*u'*}, *dime*{*u'*}, *have*{*u*, *u'*}])); [<u>**atom**{*u'*}</u>],
dist([*put_in_meter*{*u*, *u'*}]))]

6 Comparison with Alternative Approaches

PCDRT differs from previous dynamic and static approaches to singular / plural donkey anaphora in a couple of respects. The first difference is conceptual: PCDRT explicitly encodes the idea that reference to structure is as important as reference to value and that the two should be treated in parallel, i.e. both of them should be non-deterministically introduced (see the definition of dref introduction in section **3.1** above). This is in contrast to van den Berg (1996), Krifka (1996b) and Nouwen (2003), where only value is non-deterministically introduced, while structure is deterministic.

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[49] Note that the PCDRT way of representing donkey sentences allows – at least in principle – for one more way in which the <u>**atom**{*u'*}</u> can be narrowly resolved, namely in the nuclear scope of the quantification but outside the scope of the **dist** operator contributed by the generalized determiner, as shown in (i) below. The reading encoded by this representation can be paraphrased as: every man who has a son actually has exactly one son and wills all his money to his son.

(i) [**every**$_u$(**dist**([*man*{*u*}]; **max**$^{u'}$(**dist**([**atom**{*u'*}, *son*{*u'*}, *have*{*u*, *u'*}]))),
[<u>**atom**{*u'*}</u>]; **dist**([*will_all_money*{*u*, *u'*}]))]

Intuitively, this reading closely resembles the 'narrow resolution' reading of sentence (101) above. I leave it for future research to determine if the reading in (i) is actually attested and, if so, how to provide an account that allows for two distinct narrow 'scope' resolutions of the <u>**atom**</u> presupposition contributed by singular number morphology on donkey anaphora.

[50] This, of course, assumes that presupposition satisfaction is in general preferable to presupposition accommodation (see van der Sandt 1992 for more discussion).

However, we seem to need an additional principle requiring the <u>**atom**</u> presupposition contributed by singular pronouns to be resolved as highly as possible (this principle should override the general preference for satisfaction over accommodation) if we want PCDRT to capture the fact that the only intuitively available readings for the discourse A^{u} `man entered. Everyone`$^{u'}$ `looked at him`$_u$ are the weak one (there is a man such that he entered and everyone looked at him) and the unique one (a single man entered and everyone looked at him) –but not the strong, non-unique one (everyone looked at every man that entered).

The PCDRT analysis of reference to structure as *discourse* reference to structure (i.e. in parallel to discourse reference to values) also contrasts with the analysis of reference to structure by means of (dref's for) choice and / or Skolem functions. Although such functions could be used to capture (donkey) anaphora to structure, they would have variable arity depending on how many simultaneous anaphoric connections there are, e.g., there are two simultaneous anaphoric connections in sentences (2) and (3) above. Therefore, since the arity of the Skolem functions is determined by the discourse context, it should be encoded in the database that stores discourse information, namely, in the information state and not in the representation associated with a lexical item, be it the donkey pronoun and / or its antecedent.

The second difference is empirical: the motivation for plural information states is provided by *singular* and *intra-sentential* donkey anaphora, in contrast to much of the previous literature (van den Berg 1996, Krifka 1996b, Nouwen 2003 and Asher & Wang 2003 among others) which relies on plural and cross-sentential anaphora of the kind instantiated by (1) above.

Intra-sentential donkey anaphora to structure provides a much stronger argument for the idea that plural info states are *semantically* necessary. To see this, consider anaphora to value first. A pragmatic account is plausible for cases of cross-sentential anaphora, e.g. in `A man came in. He sat down`, the pronoun `he` can be taken to refer to whatever man is pragmatically brought to salience by the use of the indefinite in the first sentence. However, a pragmatic account is less plausible for cases of intra-sentential donkey anaphora: no particular donkey is brought to salience in `Every farmer who owns a donkey beats it`.

Similarly, a pragmatic account of anaphora to structure is plausible for cases of cross-sentential anaphora like (1) or (5) above. Consider sentence (1) again: the first conjunct correlates each girl with the gift(s) that Linus bought for her and the second conjunct elaborates on this correlation – for each girl, Linus asked her deskmate to wrap the corresponding gift(s). That is, the wrapping structure is the same as the buying structure, but the identity of structure might be a pragmatic addition to semantic values that are unspecified for structure (e.g. the second conjunct could be interpreted cumulatively). However, a pragmatic approach is less plausible for cases of intra-sentential donkey anaphora to structure instantiated by (3) and (8) above.

Thirdly, PCDRT differs from the previous dynamic approaches to plural anaphora insofar as it models plural reference and plural discourse reference as two distinct and independent notions. The previous dynamic approaches basically fall into two classes based on the way in which they conflate these two notions.

The approaches in the first class (van den Berg 1994, 1996, Nouwen 2003, Asher and Wang 2003 among others) make plural reference dependent on plural discourse reference, i.e. they allow the variable assignments to store only atomic individuals and non-atomic individuals can be accessed in discourse only by summing over plural info states. These dynamic approaches (much like the E-type approach in Neale 1990) find it difficult to capture the intuitively correct truth-conditions of plural sage plant examples like the one in (10) above (see section **6.5** below for a detailed discussion) and, to they extent they can derive the correct truth-conditions, they fail to capture the intuitive parallels between singular and plural (donkey) anaphora, e.g. between (10) and the singular sage plant example in (9) or between (3) and (8) above.

The approaches in the second class (e.g. Krifka 1996b, building on Barwise 1987 and Rooth 1987) make plural discourse reference dependent on plural reference, i.e. the central notion of *parametrized sum individual* associates each atom that is part of a non-atomic individual with a variable assignment that 'parametrizes' / is dependent on that atom, e.g. the non-atomic / sum individual under discussion might contain all and only the farmer atoms that are donkey owners and each farmer atom is associated with a variable assignment that stores (relative to a new dref) a donkey atom that the farmer owns.

Besides the fact that these approaches, just like the previous ones, have difficulties with plural sage plant examples and with the parallels between singular and plural (donkey) anaphora, they predict that we cannot access the dependent individuals (e.g. the donkeys owned by some farmer or other) *directly*, but only via anaphora to and / or quantification over the sum individuals they depend on. As sentence (26) above shows, this prediction is incorrect: the second sentence in (26) anaphorically retrieves the alligator purses directly and not as a function of the girls they are dependent on.

Moreover, such approaches require an independent notion of cover (see, for example, Krifka 1996b) to account for the codistributivity effects associated with the interpretation of discourses like `Three`^{u} `soldiers aimed at five`$^{u'}$ `targets. They`$_{u}$ `/ The`$_{u}$ `soldiers hit them`$_{u'}$ `/ the`$_{u'}$ `targets`[51]. The fact that PCDRT countenances both notions of plurality, i.e. both non-atomic individuals and plural info states, enables it to encode covers by letting each assignment i in a plural info state I be such that the sum of soldier atoms ui aimed at and hit the sum of target atoms $u'i$.

The remainder of this section provides a detailed comparison between PCDRT and a variety of previous approaches to donkey anaphora, weak / strong ambiguities and uniqueness effects.

The accounts of weak / strong donkey readings fall (roughly) into three categories. First, we have accounts that locate the weak / strong ambiguity at the level of the generalized determiner (e.g. the determiner `every` in the classic example `Every farmer who owns a donkey beats it`); most dynamic accounts fall into this category, including Rooth (1987), Van Eijck & de Vries (1992), Dekker (1993), Kanazawa (1994a, b), but also the D-/E-type approach in Heim (1990). These approaches are discussed in section **6.1**.

Second, we have accounts that locate the ambiguity at the level of the donkey pronoun, e.g. the D-/E-type approaches in van der Does (1993) and Lappin & Francez (1994); these approaches are discussed in section **6.2**. The hybrid dynamic/E-type approach pursued in Chierchia (1995) is also discussed in this section.

Finally, we have an account that locates the ambiguity at the level of the indefinite article, namely van den Berg (1994, 1996). This approach is discussed in section **6.3**.

Subsections **6.4** through **6.8** discuss several kinds of donkey sentences that are either novel or rarely addressed in the previous literature. These sentences are discussed separately from the above three-way classification because the problems they raise often cut across the three classes of approaches to weak / strong donkey anaphora.

The final subsection (**6.9**) suggests that PCDRT can be seen as a unification of dynamic and D-/E-type situation-based accounts – a unification that extends their empirical coverage and helps separate the linguistic issues genuinely at stake from the more idiosyncratic, technical aspects of these approaches.

6.1 Weak / Strong Determiners and Donkey Sentences with Nuclear Scope Negation

We will first discuss approaches that locate the weak / strong ambiguity at the level of generalized determiners. This category comprises dynamic approaches, e.g. Rooth (1987), Van Eijck & de Vries (1992), Dekker (1993), Kanazawa (1994a, b) and the D-/E-type approach in Heim (1990).

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[51] The first sentence in this discourse is based on an example in Kamp & Reyle (1993). The second sentence is from Winter (2000).

Locating the weak / strong donkey ambiguity in the meaning of selective generalized determiners is a natural choice for dynamic approaches. This is due to the fact that, in classical dynamic semantics (Kamp 1981, Heim 1982 and Kamp & Reyle 1993), indefinites do not have any quantificational force whatsoever, so all the truth-conditional effects associated with donkey anaphora have to be built into whatever element in the environment gives the quantificational force of the indefinite.

In contrast, the proposal pursued in this paper is that indefinites should be endowed with a minimal quantificational force of their own: (*i*) just as in DPL (Groenendijk & Stokhof 1991), they contribute an existential quantification; (*ii*) what is new is that the indefinites can also specifiy whether the existential quantification they introduce is maximal or not, i.e. whether they introduce in discourse *some* witness set or the *maximal* witness set that satisfies the nuclear scope update.

I will put forth two arguments against taking meaning of selective generalized determiners to be the locus of weak / strong donkey ambiguities.

The first one has to do with the syntax/semantics aspect of the interpretation of donkey sentences, in particular, with the requirement of (strict) compositionality. If we attribute the weak / strong ambiguity to the determiner and we want to derive the intuitively correct truth-conditions for the mixed weak & stong donkey sentence in (2) above, we basically need to pack an entire logical form into the meaning of the generalized determiner $\mathtt{every}$, which needs to non-locally / non-compositionally determine: (*i*) the readings associated with different indefinites and (*ii*) their relative pseudo-scope[52] – recall that, in, the weak indefinite co-varies with the strong indefinite since the credit card can vary from book to book).

Besides non-compositionality, this strategy greatly increases the number of lexical entries for each determiner, e.g., depending on the number of simultaneous donkey anaphors and their relative pseudo-scope, we will have: $\mathtt{every}_{u}$, $\mathtt{every}_{u}^{\mathbf{str}:u'}$, $\mathtt{every}_{u}^{\mathbf{wk}:u'}$, $\mathtt{every}_{u}^{\mathbf{str}:u'>>\mathbf{wk}:u''}$, $\mathtt{every}_{u}^{\mathbf{wk}:u''>>\mathbf{str}:u'}$ etc.

In contrast, PCDRT locates the weak / strong ambiguity at the level of indefinite articles, which requires only two distinct representation for the indefinite article – or, if you will, a single representation underspecified for the presence / absence of a **max** operator. Moreover, the pseudo-scopal relation between the two indefinites in (2) follows automatically from the fact that PCDRT uses plural info states, which store and pass on information about both objects and dependencies between them that are introduced and elaborated upon in discourse.

The second reason not to locate the weak / strong ambiguity in the generalized determiners has to with the semantics/pragmatics side of the interpretation of donkey sentences, namely: (*i*) the variety of factors that influence which reading is selected in any given instance of donkey anaphora and (*ii*) the defeasible character of the generalizations correlating these factors and the resulting readings. Some of these factors are:

- the logical properties of the determiners – see Kanazawa (1994a, b);
- world-knowledge – see the 'dime' example in Pelletier & Schubert (1989) and, also, the examples and discussion in Geurts (2002);
- the information (focus-topic-background) structure of the sentence – see Kadmon (1987), Heim (1990);
- the kind of predicates that are used, i.e. total vs. partial predicates – see Krifka (1996a) and references therein;

[52] I call the semantic relation between the two indefinites in (2) pseudo-scope because, by the Coordinate Structure Constraint, the strong indefinite cannot syntactically take scope over the weak indefinite.

- whether the donkey indefinite is referred back to by a donkey pronoun – see Bäuerle & Egli (1985)[53].

Given the variety of factors that influence which reading is selected in any given instance of donkey anaphora and also the defeasible character of the generalizations correlating these factors and the resulting readings, I think that the most conservative hypothesis is to locate the weak / strong ambiguity at the level of the donkey anaphora itself and let more general and defeasible pragmatic mechanisms decide which meaning is selected in any particular case.

One of the most prominent accounts of donkey anaphora that locates the weak / strong ambiguity (or, more neutrally: the weak / strong variation) in the meaning of the generalized determiners is proposed in Kanazawa (1994a, b) (but see also Heim 1990). The remainder of this section is dedicated to the discussion of this account.

To begin, we should note that Kanazawa's account is ultimately pragmatic, much like the PCDRT indefinite-based account. In fact, except for the fact that Kanazawa chooses to make generalized determiners – and not the indefinites – underspecified for weak / strong readings, all the observations below also apply to the PCDRT account.

> "The primary assumption I make is the following: […] The grammar rules in general underspecify the interpretation of a donkey sentence.
> Thus, I assume that, for any donkey sentence, the grammar only partially characterizes its meaning, with which a range of specific interpretations are compatible. So the truth value of donkey sentences in particular situations may be left undecided by the grammar. This may not be such an outrageous idea; it may explain the lack of robust intuitions about donkey sentences.
> For the sake of concreteness, I assume that the underspecified interpretation of a donkey sentence Det N' VP assigned to by the grammar can be represented using an indeterminate dynamic generalized determiner *Q* which is related to the static generalized determiner Q denoted by Det and which satisfies certain natural properties. […]
> Even if its interpretation is underspecified, a sentence may be assigned a definite truth-value in special circumstances. […] It is not unreasonable to suppose that people are capable of assessing the truth value of a donkey sentence without resolving the 'vagueness' of the meaning given by the grammar when there is no need to do so. […] underspecification causes no problems for people in assigning a truth value to a donkey sentence in situations where the uniqueness condition for the donkey pronoun is met."
> (Kanazawa 1994a: 151-152)

In particular, the situations in which the model-level "uniqueness condition" is met are precisely the situations in which the PCDRT weak and strong meanings for the indefinite article are conflated: the weak indefinite introduces some witness set which, by the model-level uniqueness condition, is the only, hence also the maximal, witness set.

Thus, just like PCDRT, Kanazawa's account defers the task of weak / strong disambiguation to pragmatics. The two accounts differ in what particular lexical items should be disambiguated – and Kanazawa's main argument for locating the weak / strong variation at the level of generalized determiners is that there is a reliable correlation between the monotonicity properties of the determiners and the (most salient) reading associated with donkey anaphora.

Basically, if the direction of the monotonicity of the determiners is the same in both arguments (e.g. `no`, which is ↓MON↓, and `two` and `a`, which are ↑MON↑), the donkey anaphora is always weak, while if the direction of the motonicity is different in the two arguments (e.g. `every`, which is ↓MON↑, and `not every`, which is ↑MON↓), the donkey anaphora is preferrably strong.

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[53] Apud Heim (1990).

I agree with Kanazawa's observation that the strong donkey reading is not available with indefinites like `two` and `a` as main determiners – and, as shown in section **6.8** below, PCDRT captures this generalization.

However, the correlation between the direction of monotonicity and choice of donkey reading makes two undesirable predictions. First, the determiner `every` should be associated only – or preferrably – with strong readings. But the well-known dime example shows that this monotonicity-based bias for strong readings is easily trumped by world knowledge[54].

Second, the monotonicity-based bias can be systematically overridden for most other determiners in a particular kind of construction that involves nuclear scope negation[55]. This observation – together with the above list of five unrelated factors that influence the choice between weak and strong readings – provides support for the arguably more conservative hypothesis that the source of the weak / strong ambiguity should be located in the donkey anaphora itself and not in some other element in their linguistic environment.

I use "nuclear scope negation" as a cover term for negative items, e.g. sentential negation or negative verbs like *fail*, *forget* and *refuse*, that occur within the nuclear scope of a quantification and that semantically take scope over the other elements in the nuclear scope. To my knowledge, the only examples of nuclear scope negation discussed in the previous literature are the ones provided in (107) (see van der Does 1993: 18, (27c)), (108) (see Kanazawa 1994a: 117, fn. 16), (109) and (110) (see Lappin & Francez 1994: 401, (22a)) below[56].

107. A boy who had an^u apple in his rucksack didn't give it$_u$ to his sister.
108. No man who had a^u credit card failed to use it$_u$.
109. Every person who had a^u dime in his pocket did not put it$_u$ into the meter.
110. Every person who had a^u dime in his pocket refused to put it$_u$ into the meter.

The generalization that seems to emerge is that nuclear scope negation requires the strong donkey reading[57]. Sentence (107) is interpreted as asserting that there is some boy such that, for *every* apple in his rucksack, he didn't give that apple to his sister. Sentence (108) is interpreted as asserting that no man is such that, for *every* credit card of his, he failed to use that card, i.e. no man failed to use every credit card of his – or, equivalently, every man used some credit card or other.

The examples in (107) and (108) form minimal pairs with sentences (111) and (112) below, where there is no nuclear scope negation and where the most salient donkey reading is the weak one. The examples in (109) and (110) contrast in the same way with the classical weak reading example in Pelletier & Schubert (1989).

111. A boy who had an^u apple in his rucksack gave it$_u$ to his sister.
112. No man who had a^u credit card used it$_u$ (to pay the bill).

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[54] For more discussion, see Kanazawa (1994a): 122-124 and Geurts (2002).

[55] I am grateful to Hans Kamp (p.c.) for pointing out to me that there seems to be a systematic correlation between sentential negation and donkey readings. Most of the empirical observations in this subsection emerged during or as a result of our conversations.

[56] Geurts (2002) also mentions the examples due to van der Does (1993) and Kanazawa (1994a), but he believes that "such examples are hard to find" (Geurts 2002: 131).

[57] See Lappin & Francez (1994) for observations that point towards the same generalization (p. 408 in particular) and for a critique of Kanazawa (1994a) based on sentences (109) and (110) (pp. 410-411)

We can observe a similar contrast for non-monotone intersective determiners of the form `exactly n`, which Kanazawa (1994a) also takes to favor the weak reading. The most salient reading of (113) below is the strong donkey reading: exactly two men are such that, for *every* credit card they had, they failed to use that card. The most salient reading of (114) is the weak one: exactly two men used *some* credit card they had.

113. Exactly two men who had a^{u} credit card failed to use it$_{u}$ / didn't use it$_{u}$ / forgot to use it$_{u}$.
114. Exactly two men who had a^{u} credit card used it$_{u}$.

The same observation applies to the *only*-based donkey examples in (115) and (116) below and to the pairs of *at least n–*, *at most n–* and *most*-sentences in (117)-(118), (119)-(120) and (121)-(122).

115. Only two men who had a^{u} credit card failed to use it$_{u}$ / didn't use it$_{u}$ / forgot to use it$_{u}$.
116. Only two men who had a^{u} credit card used it$_{u}$.
117. At least two men who had a^{u} quarter put it$_{u}$ in the meter.
118. At least two men who had a^{u} quarter refused to put it$_{u}$ in the meter / forgot to put it$_{u}$ in the meter.
119. At most two men who had a^{u} quarter put it$_{u}$ in the meter.
120. At most two men who had a^{u} quarter refused to put it$_{u}$ in the meter / forgot to put it$_{u}$ in the meter.
121. Most men who had a^{u} nice suit wore it$_{u}$ at the town meeting. [58]
122. Most men who had a^{u} nice suit refused to wear it$_{u}$ at the town meeting / forgot to wear it$_{u}$ at the town meeting / didn't wear it$_{u}$ at the town meeting.

In contrast, note that negation with scope over the entire donkey quantification does not have a similar strengthening effect, as the examples in (123), (124) and (125) below show. Consider (124) for example: its strong reading is that not every man who had a credit card is such that, for *every* credit card he had, he used that card to pay the bill – an assertion that borders on triviality. Intuitively, sentence (124) asserts that not every man who had a credit card used *some* credit card of his to pay the bill – or, equivalently, that there is a man who had a credit card and who didn't use any of his cards to pay, i.e. the weak donkey reading.

123. Not every man who had a^{u} dime put it$_{u}$ in the meter.
124. Not every man who had a^{u} credit card used it$_{u}$ to pay the bill.
125. Not every person who buys a^{u} book on amazon.com and has a$^{u'}$ credit card uses it$_{u'}$ to pay for it$_{u}$.

However, just like the other generalizations about the distribution of weak vs. strong donkey readings proposed in the literature, the correlation between nuclear scope negation and strong donkey readings is not without exception. A wide-scope negation cancels the strengthening effect of nuclear scope negation, as the examples in (126) and (127) below show. Note also that the weak donkey sentences in (126) and (127) below and the ones in (123), (124) and (125) above show that ↑MON↓ determiners like *not every* and *not all* reliably tolerate weak readings, contra Kanazawa (1994a): 118 et seqq.

126. Not every man who had a^{u} credit card failed to use it$_{u}$.

[58] This example is based on Kanazawa (2001): 386, (17).

127. `Not every man who had a nice suit refused to wear it`$_u$ `at the town meeting / forgot to wear it`$_u$ `at the town meeting.`

All these examples indicate that, if there are any correlations between negation and weak / strong donkey readings or between monotonicity properties and weak / strong donkey readings, these correlations are only defaults and cannot be locally and deterministically established by taking into account only one particular item, be it the generalized determiner (as Kanazawa argues) or the nuclear scope negation. We need to simultaneously take into account various lexical items and, in addition to this, factors of a different nature, e.g. world knowledge about how credit card payment normally happen or about how people normally wear their suits (not all of them at the same time, even if they are very nice).

I will conclude this section with the example in (128) below, which provides one more exception to the correlation between nuclear scope negation and strong donkey readings. The most salient reading of (128) is that every man who placed a suitcase on the belt took back every suitcase after it was X-rayed, i.e. no man who placed a suitcase on the belt failed, for *some* such suitcase, to take it back, i.e. the weak donkey reading.

128. (At the airport "self check-in", where customers place their suitcase / suitcases on the belt to have them X-rayed:)
`No man who placed a`^{u} `suitcase on the belt forgot to take it`$_u$ `back after it`$_u$ `was X-rayed / failed to take it`$_u$ `back after it`$_u$ `was X-rayed.`

I leave for future research an analysis of the default correllations between donkey readings and nuclear scope negation – but I hope to have established that the volatile nature of weak / strong ambiguities makes the indefinite-based PCDRT account more plausible than the alternative strategy of locating these ambiguities in the generalized determiners.

6.2 Weak / Strong Pronouns and Donkey Sentences with DP Conjunctions

D-/E-type accounts of donkey anaphora fall into two categories with respect to the problem posed by weak / strong ambiguities. If they address the problem at all (e.g. Neale 1990 and Elbourne 2005 do not), they either locate the weak / strong ambiguity in the meaning of the generalized determiner, e.g. Heim (1990), or in the meaning of the donkey pronoun, e.g. van der Does (1993) and Lappin & Francez (1994).

In this section, I will focus on accounts that take the donkey pronoun to be the source of the weak / strong ambiguity – in particular, the account in Lappin & Francez (1994), but the general argument also applies to van der Does (1993).

Lappin & Francez (1994): 403 propose to analyze donkey pronouns as functions from individuals to *i*-sums (i.e. individual sums, a.k.a. plural / sum / non-atomic individuals), e.g., in the classical donkey example *Every farmer who owns a donkey beats it*, the pronoun *it* denotes a function f that, for every donkey-owning farmer x, returns some *i*-sum $f(x)$ of donkeys that x owns, i.e. the sum of some subset of the donkeys that x owns.

Strong donkey readings are obtained by placing a maximality constraint on the function f, which requires f to select, for each x in its domain, the supremum of its possible values, i.e., in the case at hand, the maximal *i*-sum of donkeys that x owns. Weak donkey readings are obtained by suspending the maximality constraint, i.e. f is a choice function from x to one of the *i*-sums of donkeys that x owns.

I will use donkey sentences with DP conjunctions in subject position like (129) to distinguish between the D-/E-type strategy of locating the weak / strong ambiguity in the donkey pronoun and the PCDRT strategy of locating it in the donkey indefinite. Sentence (129) is a mixed reading donkey sentence – its interpretation is that every company that hired a Moldavian promoted *every* Moldavian it hired within two weeks, while there is no company that hired some Transylvanian and promoted *some* Transylvanian it hired within two weeks.

129. (Today's newspaper claims that, based on the most recent statistics:)
`Every`^{u} `company that hired a`$^{u'}$ `Moldavian man, but no`$^{u''}$ `company that hired a`$^{u'}$ `Transylvanian man promoted him`$_{u'}$ `within two weeks of hiring.`

The crucial aspect of sentence (129) is that, intuitively, the same pronon `it` is anaphoric to both indefinites. This example is problematic for approaches that locate the weak / strong ambiguity in the donkey pronouns (e.g. Lappin & Francez 1994 and van der Does 1993) because there is only one pronoun in (129), but two distinct donkey readings. That it, assuming that there are no covert syntactic manipulations duplicating the donkey pronoun, either this pronoun is subject to the maximality constraint, hence it delivers only strong donkey readings, or the maximality constraint is suspended and the pronoun delivers only weak readings.

Sentence (130) below makes the same point as (129) – the only difference is that, in (130), we conjoin two DP's headed by the same generalized determiner[59]. Example (130) can be felicitously uttered in the following context: there is this Sunday fair where, among other things, people come to sell their young puppies – and they do want to get rid of all of them before they are too old. Also, the fair entrance fee is one dollar. Now, the fair rules are strict: all the puppies need to be checked for fleas at the gate and, at the same time, the one dollar bills also need to be checked for authenticity because of the many faux-monnayeurs in the area. So:

130. `Everyone`^{u} `who has a`$^{u'}$ `puppy and everyone`$^{u''}$ `who has a`$^{u'}$ `dollar brings it`$_{u'}$ `to the gate to be checked.`

The most salient interpretation of sentence (130) is that every potential seller brings *all* her puppies to the gate to be checked, while every potential buyer needs to bring only *one* of her dollars, i.e. anaphora to *a puppy*$^{u'}$ is strong, while anaphora to *a*$^{u''}$ *dollar* is weak.

The sentences in (129) and (130) pose an even more severe problem for the hybrid approach to weak / strong ambiguities proposed in Chierchia (1995), where the weak reading is derived within a dynamic framework and the strong reading is attributed to a D-/E-type reading of the donkey pronoun. Given that Chierchia (1995) agrees with the observation that examples like (129) and (130) above involve a single pronoun (he actually uses examples of the same form to argue for a semantic as opposed to a syntactic approach to donkey anaphora), his approach is faced with the problem of deriving, by means of a single pronoun, two different donkey readings which are furthermore claimed to involve two different kinds of semantic representations for the pronoun.

One more move is still possible for the D-/E-type approach of Lappin & Francez (1994). Following a suggestion in Chierchia (1995): 116-117, the donkey pronouns in (129) and (130) can be taken to denote the union of two different functions, a maximal one that is contributed by the first DP in their respective sentences and a non-maximal, choice-based one that is contributed by the second DP.

However, this strategy does not always work because the union of two functions is not necessarily a function. In particular, suppose that, in (129), the very same company *x* hired both a Moldavian man and a Transylvanian man; the first function will return the Moldavian man as value for the argument *x*, while the second function will return the Transylvanian man,

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[59] Example (130) is based on an example due to Sam Cumming (p.c.).

so the result of their union is not a function and, therefore, not a suitable kind of meaning for a donkey pronoun[60].

PCDRT can account for mixed reading DP conjunction donkey sentences without any additional stipulations – we only need to specify, for each indefinite, whether it receives a weak or a strong reading. The dynamic meaning for `and` (or `but`), which conjoins two dynamic quantifiers of type (**et**)**t** in (129) and (130), is easily obtained based on the static definition of generalized conjunction in Partee & Rooth (1983) (see the **Appendix** for details). The compositionally obtained representation for sentence (129) is provided in (131) below; as the reader is invited to check, this representation derives the intuitively correct truth-conditions.

131. $[\mathbf{every}_u(\mathbf{dist}([company\{u\}];\ \mathbf{max}^{u'}(\mathbf{dist}([\mathbf{atom}\{u'\},\ moldavian\{u'\},\ hire\{u,\ u'\}]))),$
$\mathbf{dist}([\mathbf{atom}\{u'\}, promote\{u, u'\}]))];$
$[\mathbf{no}_{u''}([company\{u''\}]; [u']; \mathbf{dist}([\mathbf{atom}\{u'\}, transylvanian\{u'\}, hire\{u'', u'\}]),$
$\mathbf{dist}([\mathbf{atom}\{u'\}, promote\{u'', u'\}]))]$

Moreover, the PCDRT account also predicts that the same indefinite cannot be interpreted as strong with respect to one pronoun and weak with respect to another – not unless the two readings concide, which happens only if there is only one possible witness for the indefinite and, therefore, only one possible witness set (the singleton set containing that witness) which, trivially, is also the maximal witness set.

This prediction seems to be borne out[61]: the donkey sentences in (132) and (133) below are felicitous only if every man under consideration bought exactly one suit or had exactly one credit card. To put it differently, sentence (132) is infelicitous in a situation in which every man bought two suits, a grey one and a black one, and they wore the grey suits at the morning party and the black suites at the evening party. Similarly, sentence (133) is infelicitous if every man has a MasterCard and a Visa and they use their MasterCards to pay for the food and their Visas to pay for the drinks. In each sentence, the two pronouns intuitively refer to the same entity.

132. Every man who bought a^u suit wore it_u at the morning ceremony, but refused to wear it_u at the evening party.
133. Every man who had a^u credit card used it_u to pay for the food, but didn't use it_u to pay for the drinks.

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[60] We can take the function union approach one step further and assume that, when we take the union of two functions *f* and *f'*, we require the resulting function to return, for any *x* that is in the domain of both *f* and *f'*, the sum of the individuals *f*(*x*) and *f'*(*x*). This "union & sum" strategy could yield the correct truth-conditions for example (130) where, for a person *x*, *x* brings to the gate to be checked every individual in the *i*-sum formed out of *x*'s puppies and one of *x*'s dollar bills – but it will not yield the intuitively correct truth-conditions for (129).

Moreover, the "union & sum" strategy (and D-/E-type approaches in general) predict that the sum should be available for subsequent *singular* cross-sentential anaphora – if the function that provides the meaning of the pronoun is salient enough the first time around, it should still be salient enough immediately afterwards. However, subsequent singular anaphora to puppy-dollar sums is unacceptable: *Everyone who has a^u puppy and everyone who has a^u dollar brings it_u to the gate to be checked. #They do so because the rules of the fair require that it_u (should) be checked*.

Subsequent plural anaphora, however, is perfectly acceptable -- to see this, replace it_u with $they_u$ in the second sentence of the discourse above – but there is no obvious way in which D-/E-type approaches can account for this asymmetry between singular and plural donkey anaphora with split antecedents. In contrast, PCDRT can account for this asymmetry without any additional stipulations; for more details, see the discussion of donkey anaphora with split antecedents in section **6.6** below.

[61] I am indebted to Roger Schwarzschild and Stanley Peters (p.c.) for emphasizing the need for such an argument and to Sam Cumming and Will Starr for the acceptability judgments.

In contrast, the D-/E-type analysis in Lappin & Francez (1994) (the points also applies to the hybrid approach in Chierchia 1995) incorrectly predicts that sentences (132) and (133) should be felicitous even in the non-unique scenarios described above. Nothing in these approaches forces a unique reading for (132) and (133) and nothing requires the two pronouns in each sentence to refer to the same entity.

6.3 Weak / Strong Indefinites and Mixed Reading Donkey Sentences

Let us turn now to the approach in van den Berg (1994, 1996), which, just as PCDRT, locates the weak / strong donkey ambiguity in the meaning of the indefinites.

There are various formal differences between van den Berg's Dynamic Plural Logic (DPlL) and PCDRT, e.g. DPlL is formulated in three-valued logic, it does not provide a compositional interpretation procedure for natural language discourses, DPlL acknowledges only discourse-level pluralities, lexical relations are interpreted collectively at the discourse level and not distributively, the introduction of new dref's treats values and structure asymmetrically (only values are introduced non-deterministically)[62] etc. I will ignore all these differences here[63] and focus on the only two that are directly relevant to the matter at hand, namely the DPlL definition of maximization and the analysis of singular indefinite articles.

The DPlL notion of dynamic maximization (see van den Berg 1994: 15, (45) and van den Berg 1996: 139, (3.1)) is different from the PCDRT notion in one important respect: it is is a weaker version of the $\mathbf{max}^u$ operator insofar as it does not require the existence of a supremum – it simply requires an output state to non-deterministically store a (locally) maximal set[64]. A PCDRT rendering of DPlL maximization is given in (134) below, where '$\subset$' stands for strict inclusion. This two operators stand in the relation given in (135) below.

134. $\mathbf{max\text{-}wk}^u(D) := \lambda I_{st}.\lambda J_{st}.\ ([u];\ D)IJ \wedge \neg\exists K_{st}(([u];\ D)IK \wedge uJ \subset uK)$
135. $\mathbf{max}^u(D) \subseteq \mathbf{max\text{-}wk}^u(D)$

DPlL crucially requies the weaker form of maximization $\mathbf{max\text{-}wk}^u$ (as opposed to the PCDRT one) to be able to account for weak / strong ambiguities. The reason for this is that DPlL takes indefinites to be generalized quantifiers and generalized quantifiers are defined in terms of maximization[65] – hence, a maximization operator is used to give the meaning of both weak and strong donkey indefinites.

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[62] A PCDRT rendering of the definition of new dref introduction in van den Berg (1996) is provided in (i) below. Unlike the PCDRT definition in (20) above, this definition treats structure deterministically. In fact, as shown in (iii), van den Berg's random assignment can be defined in terms of the PCDRT random assignment and the **enough_assignments** condition in (ii); **enough_assignments** is a closure condition closely related to Axiom **4** (Enough assignments) of Dynamic Ty2 (see the **Appendix** for the exact definition of this axiom). It follows from (iii) that the two definitions are related as shown in (iv) below.

(i) $\{u\} := \lambda I_{st}.\lambda J_{st}.\ \exists X_{et} \neq \emptyset (J = \{j_s\!: \exists i_s \in I(i[u]j \wedge uj \in X)\})$

(ii) $\mathbf{enough\_assignments}\{u\} := \lambda I_{st}.\ \forall x_e \in uI \forall i_s \in I(\exists i'_s \in I(i[u]i' \wedge ui' = x))$

(iii) $\{u\} := \lambda I_{st}.\lambda J_{st}.\ I[u]J \wedge \mathbf{enough\_assignments}\{u\}J$,
i.e. $\{u\} := [u \mid \mathbf{enough\_assignments}\{u\}]$ in DRT-style abbreviation.

(iv) $\{u\} \subseteq [u]$.

[63] See chapter **5** in Brasoveanu 2007 for a more detailed comparison.

[64] For example, assume that if we update a given input info state I with a DRS of the form $[u]$; D, we get three possible output states J_1, J_2 and J_3 such that $uJ_1=\{a\}$, $uJ_2=\{a, b\}$ and $uJ_3=\{a, c\}$. The PCDRT supremum-based form of maximization will simply discard the input info state I altogether because there is no supremum in the set $\{uJ_1, uJ_2, uJ_3\}$. The weak, maxima-based form of maximization will retain the input info state I and the corresponding output states J_2 and J_3, but not J_1.

[65] See chapter **6** in Brasoveanu (2007) for a similar PCDRT definition of dynamic generalized quantification – which, crucially, does not include indefinites.

In the case of weak indefinites, however, DPlL needs to neutralize the maximization effect (since people usually do not put *all* their dimes in the meter), so an additional *singular* condition (basically the same as the PCDRT **atom**$\{u\}$ condition) is added, which requires the weak indefinite dref to store a singleton set relative to a plural info state. Obviously, this can work only in tandem with weak maximization, since strong maximization together with the **atom** condition requires model-level uniqueness and yields the Russellian analysis of definite descriptions, not the desired weak donkey indefinites.

The DPlL meanings for weak and strong indefinites are provided in (136) below, rendered in a compositional PCDRT format for ease of comparison. The two meanings correspond to the DPlL collective and distributive existential quantification respectively (see van den Berg 1994: 18-19 and van den Berg 1996: 158-159, 163-164).

136. **A PCDRT version of van den Berg's analysis of weak / strong indefinites:**
weak indef's: $a^{\mathbf{wk}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max\text{-}wk}^{u}([\mathbf{atom}\{u\}];\ P(u);\ P'(u))$
strong indef's: $a^{\mathbf{str}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max\text{-}wk}^{u}(\mathbf{dist}([\mathbf{atom}\{u\}]);\ \mathbf{dist}(P(u));\ \mathbf{dist}(P'(u)))$

The DPlL analysis can account for simple instances of weak / strong donkey ambiguities, but it does not generalize to mixed weak & strong donkey sentences like (2) above. The reason is that DPlL weak indefinites always introduce singleton sets, while sentence (2) is compatible with situations in which the value of the weak indefinite $a^{\mathbf{wk}:u''}$ `credit card` is different for different values of the strong indefinite $a^{\mathbf{str}:u'}$ `book`, i.e. with situations in which the credit cards vary from book to book. The DPlL analysis incorrectly pairs all the *u'*-books with the same *u''*-credit card – as shown by the update in (137) below (simplified based on various PCDRT equivalences), which represents the restrictor of the quantification in sentence (2).

137. $[person\{u\}];\ \mathbf{max\text{-}wk}^{u'}(\mathbf{dist}([\mathbf{atom}\{u\}]);\ [book\{u'\},\ buy\{u, u'\}]);$
$\mathbf{max\text{-}wk}^{u''}([\mathbf{atom}\{u''\},\ c.card\{u''\},\ have\{u, u''\}])$

Moreover, extracting the strong indefinite $a^{\mathbf{str}:u'}$ `book` out of its VP-conjunct and syntactically scoping it over the weak indefinite $a^{\mathbf{wk}:u''}$ `credit card` is not possible because the resulting syntactic structure violates the Coordinate Structure Constraint.

I will conclude this section with the observation that DPlL could, in principle, provide an alternative analysis of mixed weak & strong donkey sentences if it were extended with a form of anaphoric / relativized atomicity of the kind defined in (139) below. If the **atom** condition contributed by the weak indefinite is relativized to the strong indefinite, the value of the weak indefinite will vary with the value of the strong indefinite and we will be able to adequately represent the restrictor of the quantification in sentence (2), as shown in (138) below.

138. $[person\{u\}];\ \mathbf{max\text{-}wk}^{u'}(\mathbf{dist}([\mathbf{atom}\{u'\}]);\ [book\{u'\},\ buy\{u, u'\}]);$
$\mathbf{max\text{-}wk}^{u''}([\mathbf{atom}_{u'}\{u''\},\ c.card\{u''\},\ have\{u, u''\}])$

139. $\mathbf{atom}_{u'}\{u''\} := \lambda I_{st}.\ I{\neq}\emptyset \wedge \forall x_e{\in}u'I(\mathbf{atom}(\oplus u''I_{u'=x})),$
where $I_{u'=x} := \{i_s{\in}I\colon u'i{=}x\}.$

This DPlL analysis, however, lacks independent motivation. On the theoretical and formal side, the meaning of weak indefinites is needlessly involved: they contribute a maximization operator whose (weak) maximization effect is effectively neutralized by the **atom** condition. The only motivation for this is the uniform treatment of weak and strong indefinites as generalized determiners, which can be independently shown to have unwelcome empirical consequences, as discussed in section **6.8** below.

On the empirical side, it is not clear how to justify that the indefinite $a^{\mathbf{wk}:u''}$ `credit card` in sentence (2) contributes an anaphoric condition $\mathbf{atom}_{u'}\{u''\}$, despite the fact that it is not anaphorically dependent in any obvious way on the strong indefine $a^{\mathbf{str}:u'}$ `book`.

6.4 Interactions between Donkey Anaphora and Quantifier Scope Ambiguities

This subsection shows that, unlike various D-/E-type approaches to donkey anaphora, PCDRT can account for donkey sentences that have an additional quantifier / cardinal indefinite in the nuclear scope of the main generalized quantification, like (140) (based on Kanazawa 2001: 397, (63a)), (141) and (142) below.

140. $\texttt{Every}^{u}$ `man who brought` $\texttt{a}^{\mathbf{str}:u'}$ `friend to the party introduced` $\texttt{him}_{u'}$ `to` $\texttt{four}^{\mathbf{wk}:u''}$ `people.`
141. $\texttt{Every}^{u}$ `man who brought` $\texttt{a}^{\mathbf{str}:u'}$ `friend to the party introduced` $\texttt{him}_{u'}$ `to` $\texttt{every}^{u''}$ `movie star there.`
142. $\texttt{Every}^{u}$ `man who brought` $\texttt{a}^{\mathbf{str}:u'}$ `friend to the party introduced` $\texttt{him}_{u'}$ `to exactly` $\texttt{two}^{u''}$ `movie stars.`

Such donkey sentences pose problems for D-/E-type accounts of donkey anaphora that analyze donkey pronouns as plural / sum individuals (e.g. Lappin & Francez 1994 and Krifka 1996a) or as covert (universal) quantifiers (e.g. Neale 1990 and van der Does 1993) because these accounts predict readings that are not available.

In particular, the only intuitively available reading for (140) is that every man who brought a friend to the party introduced each and every friend he brought to four people (possibly different from friend to friend) – that is, if we ignore the reading where $\textit{four}^{\mathbf{wk}:u''}$ *people* has widest scope, i.e. it scopes over the quantifier $\textit{every}^{u}$ *man*[66].

As Kanazawa (2001): 397 observes, (140) does not allow for a cumulative interpretation (in the sense of Scha 1981) of the form: every man who brought one or more friends to the party introduced them (possibly as a group or in subgroups) to four people (as a group or in subgroups). However, sum-based D-/E-type approaches (Lappin & Francez 1994 and Krifka 1996a among others) predict that such a cumulative interpretation is available.

A similar problem is posed by example (142), the only intuitively available reading of which is that every man who brought a friend to the party introduced each friend to exactly two movie stars, possibly different for different friends – again, I ignore the reading in which *exactly* $\textit{two}^{u''}$ *movie stars* has widest scope[67]. Crucially, each friend was introduced to no more than two movie stars.

However, D-/E-type accounts that take donkey pronouns to be covert (universal) quantifiers (Neale 1990 and van der Does 1993 among others) predict that a third reading should be available, in which *exactly* $\textit{two}^{u''}$ *movie stars* takes scope over the donkey pronoun, that is: for every man that brought a friend to the party, there are only two movie stars to which every single friend was introduced – but any particular friend might have been introduced to more than two movie stars.

PCDRT correctly predicts that the donkey sentences in (140), (141) and (142) have only one reading (ignoring the reading in which the embedded quantifier takes widest scope). The representations for (140) and (142) are provided in (143) and (144) below (simplified in various ways) – and, as the reader can check, these representations derive the intuitively correct truth conditions. The two crucial components of the PCDRT analysis are: (*i*) the **dist** operator contributed by the determiner $\textit{every}^{u}$ that scopes over the entire nuclear scope

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[66] For simplicity, I assume that the cardinal $\textit{four}^{\mathbf{wk}:u''}$ has a weak reading; the strong reading yields identical truth conditions for the example under discussion.

[67] This reading might become more salient if a partitive construction is used, e.g. *Every man who brought a friend to the party introduced him to exactly two of the movie stars there*. I am grateful to Roger Schwarzschild for this observation (p.c.).

update, i.e. over both the singular donkey pronoun and the embedded quantifier / cardinal indefinite, and (*ii*) the $\mathbf{atom}\{u'\}$ condition contributed by the singular donkey pronoun $\texttt{him}_{u'}$.

143. $[\mathbf{every}_u(\mathbf{dist}([man\{u\}];\ \mathbf{max}^{u'}(\mathbf{dist}([\mathbf{atom}\{u'\}, friend\{u'\}, bring\_to\_party\{u, u'\}]))),$
$\mathbf{dist}([\mathbf{atom}\{u'\}];\ [u''];\ \mathbf{dist}([\mathbf{4\_atoms}\{u''\}, person\{u''\}, introd\{u, u', u''\}])))]$

144. $[\mathbf{every}_u(\mathbf{dist}([man\{u\}];\ \mathbf{max}^{u'}(\mathbf{dist}([\mathbf{atom}\{u'\}, friend\{u'\}, bring\_to\_party\{u, u'\}]))),$
$\mathbf{dist}([\mathbf{atom}\{u'\}];\ [\mathbf{exactly\_two}_{u''}([m.star\{u''\}], [introd\{u, u', u''\}])])))]$

6.5 Plural Sage Plant Examples

The well-known sage plant example in (9) above and its plural counterpart in (10) – repeated with minor modifications in (145) and (146) below – provide further instances of donkey anaphora interacting with embedded quantifiers. These examples have only one intuitively available reading, with the embedded quantifiers $\textit{eight}^{u''}\ \textit{other}_{u'}\ \textit{sage plants}$ and $\textit{seven}^{u''}\ \textit{other}_{u'}\ \textit{sage plants}$ having narrow scope with respect to $\textit{everybody}^{u}$ (they cannot take widest scope because the dref *u'* retrieved by $\textit{other}_{u'}$ is 'bound' by the donkey indefinites $a^{u'}/two^{u'}\ \textit{sage plant/s}$).

145. Everybody^{u} who bought a$^{u'}$ sage plant here bought eight$^{u''}$ other$_{u'}$ sage plants along with it$_{u'}$.

146. Everybody^{u} who bought two$^{u'}$ sage plants here bought seven$^{u''}$ other$_{u'}$ sage plants along with them$_{u'}$.

Plural sage plant examples like (146) above pose problems for approaches that acknowledge only one kind of plurality, be it domain-level (e.g. Neale 1990, Lappin & Francez 1994 and Krifka 1996a) or discourse-level (e.g. van den Berg 1996 and Asher & Wang 2003). These approaches can account for singular sage plant examples like (145) by making use of a distributivity operator (domain-level or discourse-level, as the case may be) over the plural individual stored by dref *u'* and consisting of all the purchased sage plants. However, for the plural example in (146), we need to 'distribute' over the purchased sage plants in such a way that we look at all the *pairs* of sage plant atoms (and not at individual sage plants) – and it is not clear how to define such an operator or what item in (146) contributes it[68].

Moreover, if the above mentioned accounts were to be extended somehow with an account of plural sage plant examples, the necessary additions would obscure the parallel between examples (145) and (146): intuitively, their interpretation proceeds in the same way, modulo the fact that donkey anaphora is singular in one case and plural in the other.

The fact that PCDRT acknowledges both domain-level and discourse-level pluralities enables us to account for both kinds of examples in way that captures the intuitive parallel between singular and plural sage plant anaphora. The representations (simplified in various ways) of discourses (145) and (146) are provided in (148) and (149) below[69].

147. $\textit{other}_{u} \rightsquigarrow \lambda v_{\mathbf{e}}.\ [other_u\{v\}],$
where $other_u\{u'\} := \lambda I_{st}.\ \{x_e\colon x \leq \oplus uI \wedge \mathbf{atom}(x)\} \cap \{x'\colon x' \leq \oplus u'I \wedge \mathbf{atom}(x')\} = \emptyset.$

148. $[\mathbf{every}_u([person\{u\}];\ [u'];\ \mathbf{dist}([\mathbf{atom}\{u'\}, s.plant\{u'\}, buy\{u, u'\}]),$
$[u''];\ \mathbf{dist}([\mathbf{8\_atoms}\{u''\}, other_{u'}\{u''\}, s.plant\{u''\}, buy\{u, u''\}, \mathbf{atom}\{u'\}]))]$

149. $[\mathbf{every}_u([person\{u\}];\ [u'];\ \mathbf{dist}([\mathbf{2\_atoms}\{u'\}, s.plant\{u'\}, buy\{u, u'\}]),$
$[u''];\ \mathbf{dist}([\mathbf{7\_atoms}\{u''\}, other_{u'}\{u''\}, s.plant\{u''\}, buy\{u, u''\}]))]$

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68 For a more detailed discussion of the 'plural sage plant' issue, see Kanazawa (2001): 393 et seqq. (with respect to domain-level plurality approaches) and van den Berg (1996): 164-165 (with respect to discourse-level plurality approaches).

69 For simplicity, I take all the indefinites ($a^{u'}$, $\textit{eight}^{u''}$, $\textit{two}^{u'}$ and $\textit{seven}^{u''}$) to be weak, but any combination of weak / strong readings yields identical truth conditions.

6.6 Donkey Anaphora with Split Antecedents

Approaches that acknowledge only one kind of plurality (e.g. Neale 1990, Lappin & Francez 1994 and Krifka 1996a on the one hand and van den Berg 1996 and Asher & Wang 2003 on the other hand) face a similar 'distributivity' problem with donkey examples like the one in (150) below (repeated from (37) above). The plural donkey pronoun $we_{u\oplus u'}{}^{u''}$ in (150) has two split antecedents, both singular, and the sentence intuitively quantifies over all the plural individuals that consist of two atoms, one being the *u'*-speaker and the other a *u*-friend (for every such *u*-friend). Once again, it is not clear how the above mentioned approaches can define an operator that would 'distribute' over such pairs of atoms and what particular lexical item contributes such an operator.

150. $\texttt{Every}^{u'''}$ `man who introduced` $\texttt{a}^{u}$ `friend to` $\texttt{me}^{u'}$ `thought` $\texttt{we}_{u\oplus u'}{}^{u''}$ `had something in common.`

In PCDRT, however, we can distribute at the discourse level (by means the **dist** operator contributed by the determiner $every^{u'''}$) over domain-level *u''*-pluralities (contributed by the pronoun $we_{u\oplus u'}{}^{u''}$). The compositionally obtained representation for sentence (150) is provided in (152) below and, as the reader is invited to check, this representation derives the intuitively correct truth conditions.

151. $me^{u'} \rightsquigarrow \lambda P_{\mathbf{et}}.\ [u' \mid u'=Speaker];\ P(u')$ [70]

152. $[\mathbf{every}_{u'''}(\mathbf{dist}([man\{u'''\}];\ \mathbf{max}^{u}(\mathbf{dist}([\mathbf{atom}\{u\}, friend\{u\}];$
$[u' \mid u'=Speaker,\ introd\{u''', u, u'\}]))),$
$\mathbf{dist}([u'' \mid u''=u\oplus u',\ think\_smth\_in\_common\{u''', u''\}]))]$

6.7 Donkey Anaphora and Exceptional Wide Scope

Consider the donkey sentence in (153) below, where the donkey pronoun $it_{u'}$ is syntactically trapped in the relative clause that is part of the restrictor of $most^{u''}$.

153. $\texttt{Every}^{u}$ `linguist who works on` $\texttt{a}^{\mathbf{str}:u'}$ `difficult problem is interested to read` $\texttt{most}^{u''}$ `papers that were written about` $\texttt{it}_{u'}$.

The structure of (153) is similar to the structure of the well-known examples of exceptional wide scope indefinites, e.g. $Every^{u}$ *linguist studied* $every^{u''}$ *conceivable solution that* $some^{u'}$ *problem might have* [71]. In particular, we are interested in the intermediate scope reading of this sentence, namely: every linguist is such that, for some problem (possibly different for different linguists), the linguist studied every conceivable solution that the problem might have.

The availability of this reading, i.e. the fact that the indefinite $some^{u'}$ *problem* can scope over the quantifier $every^{u''}$ *solution* despite being syntactically trapped in its restrictor, poses problems for independently motivated, syntactically restricted scoping mechanisms, e.g. Quantifier Raising / Quantifying-In. Moreover, the analysis of this reading is often taken

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[70] Where *Speaker* is a designated discourse referent that is introduced by default at the beginning of any discourse and that stores the speaker / author of that discourse, e.g., if Dobby is the speaker, then the discourse-initial update will have the form [*Speaker* | *Speaker*=*Dobby*], where *Dobby* is a specific discourse referent, i.e., basically, a rigid designator: $Dobby := \lambda i_s.\ dobby_e$. See Bittner (2007) for more discussion of such start-up updates in a related dynamic framework.

[71] Exceptional wide scope was first noticed in Farkas (1981) and Fodor & Sag (1982); see Abusch (1994), Reinhart (1997), Winter (1997), Kratzer (1998), Chierchia (2001) and Schwarzschild (2002) among others for more discussion. The example in the text is from Chierchia (2001).

to require additional machinery, e.g. special quantifier storage or movement mechanisms, choice-function variables and / or bound implicit arguments.

The donkey sentence in (153) poses parallel problems for D-/E-type approaches that analyze donkey pronouns as sum individuals (e.g. Lappin & Francez 1994 and Krifka 1996a) or as covert (universal) quantifiers (e.g. Neale 1990 and van der Does 1993). To see this, note that the only intuitively available reading for (153) is the following: for any given linguist, for each difficult problem s/he works on (i.e. the donkey anaphora is strong and interpreted 'distributively'), the linguist is interested to read most papers written about it.

In particular, sentence (153) is not true in a situation in which Joe Linguist works on two difficult problems, namely donkey anaphora and weak crossover effects, and there are seven papers exclusively about donkey anaphora, seven paper exclusively about weak crossover and three papers about both topics, adding up to seventeen papers, ten of which are about donkey anaphora and ten of which are about weak crossover. Joe Linguist wants to read only the three papers written about both topics and he is not interested in any of the other papers. Sentence (153) is false in this case because three papers are not most of the ten papers written about donkey anaphora or weak crossover.

However, D-/E-type accounts of donkey anaphora that analyze donkey pronouns as plural / sum individuals or as covert (universal) quantifiers incorrectly predict that sentence (153) is true in the above situation. The first kind of approaches take the referent of the donkey pronoun $\mathtt{it}_{u'}$ to be the sum individual consisting of both difficult problems and, in our scenario, Joe Linguist is in fact interested to read most – in fact, all – papers written about both donkey anaphora and weak crossover. The second kind of approaches make the same prediction because Joe Linguist is interested to read most – in fact, all – papers written about every problem that he works on (i.e. about both problems mentioned above: donkey anaphora and weak crossover)[72].

Thus, both kinds of D-/E-type approaches fail to derive the intuitively correct interpretation for sentence (153). Achieving this would require: (*i*) a distributivity operator over the sum individual denoted by $\mathtt{it}_{u'}$ to take exceptional wide scope, i.e. to scope over $\mathtt{most}^{u''}$ $\mathtt{papers}$ (for the first kind of approaches) or (*ii*) the quantifier contributed by the donkey pronoun itself to take exceptional wide scope over $\mathtt{most}^{u''}$ $\mathtt{papers}$ (for the second kind of approaches). And deriving the exceptional wide scope of the distributivity operator or the donkey pronoun would require additional resources, e.g. special quantifier storage or movement mechanisms and / or choice-function variables.

In contrast, PCDRT can derive the intuitively available reading for this kind of examples without making use of any extra machinery. The representation for sentence (153) (simplified in various ways) is provided in (154) below. The crucial components of the account are: (*i*) the **dist** operator contributed by the determiner $\mathtt{every}^{u}$ and scoping over the entire nuclear scope update, i.e. over the embedded determiner $\mathtt{most}^{u''}$ and the donkey pronoun $\mathtt{it}_{u'}$ (which is interpreted *in situ*); (*ii*) the **atom**{*u'*} condition contributed by the singular pronoun $\mathtt{it}_{u'}$. The reader is invited to check that this representation derives the intuitively correct truth conditions.

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[72] The two kinds of D-/E-type approaches also make incorrect predictions with respect to the 'complementary' scenario in which Joe Linguist wants to read the seven papers written only about donkey anaphora (which count as *most* papers about donkey anaphora) and the seven papers written only about weak crossover (which also count as *most* papers about weak crossover), but he does not want to read any of the three papers written about both donkey anaphora and weak crossover. Intuitively, sentence (153) is true in this scenario, but the D-/E-type approaches under discussion incorrectly predict that the sentence should be false.

154. $[\mathbf{every}_u(\mathbf{dist}([ling\{u\}];\ \mathbf{max}^{u'}(\mathbf{dist}([\mathbf{atom}\{u'\},\ diff\_problem\{u'\},\ work\_on\{u,u'\}]))),$
$\mathbf{dist}([\mathbf{most}_{u''}(\mathbf{dist}([paper\{u''\},\ \mathbf{atom}\{u'\},\ written\_about\{u'',u'\}]),$
$[interested\_to\_read\{u,u''\}])]))]$

PCDRT also enables us to give a novel solution to the original problem of exceptional wide scope indefinites – again, without resorting to movement, special storage mechanisms, choice-function variables or bound implicit arguments. The account makes crucial use of the fact that plural information states store and pass on both the (sets of) objects that are introduced in discourse and the quantificational dependencies between them; for more details, see Brasoveanu & Farkas (2007).

6.8 Uniqueness Effects and Donkey Anaphora Embedded under Indefinites

Donkey sentences like (155) below[73] with cardinal indefinites as their main determiner lack strong, non-unique readings. That is, (155) cannot be interpreted as: three people that have a house in my neighborhood want to sell every house they own in my neighborhood. The only intuitively available interpretations are the weak reading (three people that have a house in my neighborhood want to sell a house they have in my neighborhood) and the unique reading (three people that have exactly one house in my neighborhood want to sell it).

155. `Three`^{u} `people that have a`$^{u'}$ `house in my neighborhood want to sell it`$_{u'}$`.`

PCDRT correctly predicts that the strong, non-unique donkey reading is unavailable because cardinal indefinites do not have a **dist** operator scoping over the nuclear scope update (unlike discourse-level distributive generalized determiners like `every` etc.) – and, since there is no nuclear scope **dist** operator, the narrow and intermediate resolutions of the $\underline{\mathbf{atom}\{u'\}}$ presupposition contributed by `it`$_{u'}$ are conflated (see section **5.2** above for the account of the uniqueness effects associated with donkey anaphora).

We therefore have only two possible PCDRT representations for sentence (155), provided in (156) and (157) below[74]; the two representations differ only with respect to the interpretation of the donkey indefinite `a`$^{u'}$ `house`: if the indefinite is weak, we obtain the weak reading for sentence (155), while if the indefinite is strong, we obtain the unique reading (a result of the interaction between the $\mathbf{max}^{u'}$ operator and the $\underline{\mathbf{atom}\{u'\}}$ condition).

156. weak donkey reading:
$[u];\ \mathbf{dist}([\mathbf{3\_atoms}\{u\},\ person\{u\}];\ [u'];\ \mathbf{dist}([\mathbf{atom}\{u'\},\ house\{u'\},\ have\{u,u'\}]);$
$[\underline{\mathbf{atom}\{u'\}}];\ [want\_to\_sell\{u,u'\}])$ [75]

157. unique donkey reading:
$[u];\ \mathbf{dist}([\mathbf{3\_atoms}\{u\},\ person\{u\}];\ \mathbf{max}^{u'}(\mathbf{dist}([\mathbf{atom}\{u'\},\ house\{u'\},\ have\{u,u'\}]));$
$[\underline{\mathbf{atom}\{u'\}}];\ [want\_to\_sell\{u,u'\}])$ [76]

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[73] Example (155) has been brought to my attention by Donka Farkas (p.c.). She pointed out to me that donkey sentences with cardinal indefinites as the main determiner seem to lack strong readings – they have only weak or unique donkey readings.

[74] For simplicity, I take the non-donkey indefinite `three`^{u} `people` to receive a weak reading; the strong reading yields identical truth-conditions.

[75] This representation is equivalent to the simpler one in (i) below:

(i) $[u];\ \mathbf{dist}([\mathbf{3\_atoms}\{u\},\ person\{u\}];\ [u' \mid \underline{\mathbf{atom}\{u'\}},\ house\{u'\},\ have\{u,u'\},\ want\_to\_sell\{u,u'\}])$

[76] Note that the representations in (156) and (157) deliver the domain-level collective interpretation of sentence (155), i.e. the three people under discussion collectively own a house that they want to sell. To obtain the domain-level distributive interpretation (note that domain-level distributivity is distinct from discourse-level distributivity), we need to assume – together with much of the literature; see Winter 2000 for a recent discussion

Similarly, PCDRT correctly predicts that only the weak and unique readings are available for examples like (158) below, where the main determiner is a singular indefinite article[77].

158. `A`^{u} `person that has a`$^{u'}$ `house in my neighborhood wants to sell it`$_{u'}$`.`

The contrast between the donkey readings available with generalized determiners and the readings available with indefinites (cardinal indefinites or singular indefinite articles) was not noticed in the previous literature – therefore, many of the approaches to donkey anaphora and uniqueness effects fail to capture it.

For example, van den Berg (1994, 1996) assimilates indefinites to generalized determiners and incorrectly predicts that strong donkey anaphora should be possible under indefinites just as it is possible under generalized determiners – see, in particular, the van den Berg-style meaning for strong indefinites in section **6.3** above. Similarly, the account of donkey uniqueness effects in Kadmon (1990) fails to capture the generalized determiner vs. indefinite contrast because the uniqueness effects are not relativized to the kind of determiner under

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– a covert distributivity operator *each* modeled as a VP modifier and scoping over the VP's *have a*$^{u'}$ *house in my neighborhood* and *want to sell it*$_{u'}$, as shown in (i) below.

Note that *each* is distributive both at the domain level (see **dist**([**atom**$\{u\}$])) and at the discourse level (see $\mathbf{dist}_u(P(u))$). Moreover, we need to introduce *selective* discourse-level distributivity $\mathbf{dist}_u(D)$ (in addition to the unselective **dist** operator) to capture the fact that *each* distributes over the values of a particular dref.

The unique donkey reading in (vii) below is another instance of relativized uniqueness: each of the three persons under consideration has a unique house, but the total number of houses is three.

(i) `Three`^{u} `people that each`$^{u''}$ `(have a`$^{u'}$ `house in my neighborhood) each`$_{u''}$`(want to sell it`$_{u'}$`).`

(ii) $each^{u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda v_{\mathbf{e}}.\ \mathbf{dist}([u \mid \oplus u{=}\oplus v]);\ \mathbf{dist}([\mathbf{atom}\{u\}]);\ \mathbf{dist}_u(P(u))$

(iii) $each_{u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda v_{\mathbf{e}}.\ \mathbf{dist}([\oplus u{=}\oplus v]);\ \mathbf{dist}([\mathbf{atom}\{u\}]);\ \mathbf{dist}_u(P(u))$

(iv) $\oplus u{=}\oplus u' := \lambda I_{st}.\ \oplus uI{=}\oplus u'I$

(v) $\mathbf{dist}_u(D) := \lambda I_{st}.\lambda J_{st}.\ I{\neq}\emptyset \wedge uI{=}uJ \wedge \forall x_e{\in}uI(DI_{u=x}J_{u=x})$,
where $I_{u=x} := \{i_s{\in}I{:}\ ui{=}x\}$.

(vi) weak donkey reading (obtained after various simplifications):
$[u];\ \mathbf{dist}([\mathbf{3\_atoms}\{u\},\ person\{u\}];\ \mathbf{dist}([u'' \mid \oplus u''{=}\oplus u]);\ \mathbf{dist}([\mathbf{atom}\{u''\}]);$
$\mathbf{dist}_{u''}([u' \mid \underline{\mathbf{atom}\{u'\}},\ house\{u'\},\ have\{u, u'\},\ want\_to\_sell\{u, u'\}]))$

(vii) unique donkey reading (obtained after various simplifications):
$[u];\ \mathbf{dist}([\mathbf{3\_atoms}\{u\},\ person\{u\}];\ \mathbf{dist}([u'' \mid \oplus u''{=}\oplus u]);\ \mathbf{dist}([\mathbf{atom}\{u''\}]);$
$\mathbf{dist}_{u''}(\mathbf{max}^{u'}(\mathbf{dist}([\mathbf{atom}\{u'\},\ house\{u'\},\ have\{u, u'\}]));\ [\underline{\mathbf{atom}\{u'\}}];\ [want\_to\_sell\{u, u'\}]))$

The fact that *each*^{u} introduces a new dref enables us to account for the following observation in Kamp & Reyle (1993), which presents a problem for theories that conflate plural discourse reference and plural reference (e.g. van den Berg 1996). Consider the example in (viii) below (see Kamp & Reyle 1993: 324, (4.35)) and its counterpart with a cardinal indefinite in (ix). Assume that, for both examples, we interpret the VP distributively relative to the subject DP, as shown by the insertion of the covert *each*$^{u''}$ operator in (x).

(viii) `The lawyers hired a secretary they liked.`

(ix) `Three lawyers hired a secretary they liked.`

(x) `The`$_{u}$`/Three`^{u} `lawyers each`$^{u''}$`(hired a`$^{u'}$ `secretary they`$_{u''/u}$ `liked).`

Even if we disambiguate the interpretation of the VP and make it distributive, the examples are still ambiguous with respect to the interpretation of the pronoun *they* in the relative clause, which can be either (*i*) distributive: each lawyer hired a secretary s/he liked (see Kamp & Reyle 1993: 324, (4.37)), or (*ii*) collective: each lawyer hired a secretary all the lawyers liked (see Kamp & Reyle 1993: 325, (4.39)). In PCDRT, this ambiguity is captured as an ambiguity in the established anaphoric connection: the pronoun *they* is anaphoric to either the subject DP dref *u*, which derives the collective reading, or the dref *u'* contributed by the covert distributor *each*$^{u''}$, which derives the distributive reading.

[77] I am indebted to Maria Bittner, Hans Kamp and Roger Schwarzschild for bringing this kind of examples to my attention and pointing out the restriction on their possible readings.

which donkey anaphora is embedded (although DRT, which is the underlying framework used in Kadmon 1990, does distinguish between the two kinds of determiners).

6.9 Unifying Dynamic Semantics and Situation Semantics

I will conclude the paper with the suggestion that PCDRT effectively unifies dynamic and situation-based D-/E-type approaches of the kind proposed in Heim (1990) (among others) in a way that remains faithful to many of their respective goals and underlying intuitions – while extending their empirical coverage and separating the linguistic issues at stake from the more idiosyncratic, formal aspects of any given approach..

In particular, we have taken the entities of type *s* in PCDRT to model the variable assignments (i.e. the discourse salience states) of dynamic semantics. But we can just as well take the entities of type *s* to be partial situations as they are used in Heim (1990) – with the added advantage that PCDRT (just as any other dynamic approach) does not have the problem of indistinguishable participants (a.k.a. Kamp's bishop problem) and does not need to address the issues raised by the formal link condition.

Moreover, two major differences between dynamic and D-/E-type approaches to anaphora mentioned in Heim (1990): 137 are effectively invalidated by PCDRT. These differences (see the contrasting items (ii)-(iii) and (ii')-(iii') in Heim 1990: 137) concern:

- the treatment of anaphoric pronouns: they are "plain bound variables" in dynamic approaches, while D-/E-type approaches analyze them as "semantically equivalent to (possibly complex) definite descriptions" (Heim 1990: 137);
- the treatment of quantificational determiners: they are "capable of binding multiple variables" in dynamic approaches, while they "bind just one variable each" (Heim 1990: 137) in D-/E-type approaches.

In PCDRT, anaphoric pronouns are basically analyzed as individual-level dref's, i.e. as functions from entities of type *s* to individuals (type *e*). Depending on how we prefer to intuitively think about the entities of type *s*, i.e. as variable assignments or as partial situations, anaphoric pronouns are variables, i.e. they are the equivalent of projection functions on sequences, a.k.a. variable assignments, or they are definite descriptions characterizing a unique individual in a given partial situation.

Similarly, quantificational structures contributed by determiners are analyzed as having the general form in (46) above, i.e. $\mathbf{det}_u(D, D')$. Insofar as these quantificational structures operate over the DRS's D and D', hence over relations between info states, they are capable of binding multiple variables, but insofar as they contribute a particular dref u that is crucial in relating the two updates D and D', they bind one variable each.

Moreover, the way in which PCDRT uses the discourse-level distributivity operator **dist** contributed by generalized determiners to effectively neutralize the **atom** presupposition contributed by singular donkey pronouns (see section **5.2** above) is strongly reminiscent of the way in which minimal situations are used in Heim (1990) to ensure the vacuous satisfaction of the uniqueness presupposition contributed by D-/E-type donkey pronouns (which are analyzed as covert Russellian definite descriptions).

Finally, given the close formal parallels between the Compositional DRT of Muskens (1996) and situation-based D-/E-type approaches, it seems to me that, if the situation-based approaches are to be extended to account for the variety of donkey sentences discussed in the present paper (including mixed weak & strong donkey sentences like (2), the variable nature of uniqueness effects etc.), they will have to introduce operators over sets of situations, updates of such sets etc. that will be fairly similar to the PCDRT notions of plural info state, dynamic update, maximization, distributivity and selective generalized quantification.

Appendix. Plural Compositional DRT: The Formal System

Dynamic Ty2

The definition of types in (1) below isolates a subset of types as the types of dref's: these are functions from assignments (type *s*) to static objects of arbitrary type. We restrict our dref's to functions from variable assignments to *static* objects of arbitrary types because, if we allow for arbitrary dref types, e.g. $s(st)$, we might run into counterparts of Russell's paradox – see Muskens (1995b): 179-180, fn. 10.

1. **Dynamic Ty2 –** the set of dref types **DRefTyp** and the set of types **Typ**.
   a. The set of basic static types **BasSTyp**: $\{t, e\}$ (truth-values and individuals).
   b. The set of static types **STyp**: the smallest set including **BasSTyp** and such that, if $\sigma,\tau \in$ **STyp**, then $(\sigma\tau) \in$ **STyp**.
   c. The set of dref types **DRefTyp**: the smallest set such that, if $\sigma \in$ **STyp**, then $(s\sigma) \in$ **DRefTyp**.
   d. The set of basic types **BasTyp**: **BasSTyp**$\cup\{s\}$ ('variable assignments').
   e. The set of types **Typ**: the smallest set including **BasTyp** and such that, if $\sigma,\tau \in$ **Typ**, then $(\sigma\tau) \in$ **Typ**.
2. **Dynamic Ty2** – terms.
   a. Basic expressions: for any type $\tau \in$ **Typ**, there is a denumerable set of τ-constants $\mathbf{Con}_\tau$ and a denumerably infinite set of τ-variables $\mathbf{Var}_\tau = \{v_{\tau,0}, v_{\tau,1}, \ldots\}$.
   i. $\mathbf{Con}_e = \{john, mary, dobby, \ldots, a, a', \ldots, b, b', \ldots, a_0, a_1, a_2, \ldots\}$
   ii. $\mathbf{Var}_e = \{x, x', \ldots, y, y', \ldots, z, z', \ldots, x_0, x_1, \ldots\}$
   iii. $\mathbf{Con}_{et} = \{donkey, farmer, house\_elf, witch, \ldots, leave, drunk, walk, \ldots\}$
   iv. $\mathbf{Con}_{e(et)} = \{fall\_in\_love, own, beat, have, \ldots\}$
   v. $\mathbf{Var}_\tau = \{f, f', f'', \ldots f_0, f_1, f_2, \ldots\}$, for any $\tau \in$ **STyp**;
   vi. $\mathbf{Con}_{se} = \{u, u', u'', \ldots, u_0, u_1, u_2, \ldots\}$
   vii. $\mathbf{Var}_\tau = \{v, v', v'', \ldots, v_0, v_1, v_2, \ldots\}$, for any $\tau \in$ **Typ.**
   b. For any type $\tau \in$ **Typ**, the set of τ-terms $\mathbf{Term}_\tau$ is the smallest set such that:
   i. $\mathbf{Con}_\tau \cup \mathbf{Var}_\tau \subseteq \mathbf{Term}_\tau$;
   ii. $\alpha(\beta) \in \mathbf{Term}_\tau$ if $\alpha \in \mathbf{Term}_{\sigma\tau}$ and $\beta \in \mathbf{Term}_\sigma$ for any $\sigma \in$ **Typ**;
   iii. $(\lambda v.\ \alpha) \in \mathbf{Term}_\tau$ if $\tau=(\sigma\rho)$, $v \in \mathbf{Var}_\sigma$ and $\alpha \in \mathbf{Term}_\rho$ for any $\sigma,\rho \in$ **Typ**;
   iv. $(\alpha=\beta) \in \mathbf{Term}_\tau$ if $\tau=t$ and $\alpha,\beta \in \mathbf{Term}_\sigma$ for any $\sigma \in$ **Typ**;
   v. $(i[\delta]j) \in \mathbf{Term}_\tau$ if $\tau=t$ and $i,i' \in \mathbf{Var}_s$ and $\delta \in \mathbf{Term}_\sigma$, for any $\sigma \in$ **DRefTyp**.
   c. Abbreviation: $John_{se} := \lambda i_s.\ john_e$, $Mary_{se} := \lambda i_s.\ mary_e$.
3. **Dynamic Ty2 – frames, models, assignments, interpretation and truth.**
   a. A *standard frame* $\boldsymbol{F}$ for Dynamic Ty2 is a set $\boldsymbol{D} = \{\boldsymbol{D}_\tau: \tau \in \mathbf{Typ}\}$ such that $\boldsymbol{D}_e$, $\boldsymbol{D}_t$ and $\boldsymbol{D}_s$ are pairwise disjoint sets ($\boldsymbol{D}_t=\{\mathrm{T}, \mathrm{F}\}$) and $\boldsymbol{D}_{\sigma\tau} = \{\boldsymbol{f}\!: \boldsymbol{f}$ is a total function from $\boldsymbol{D}_\sigma$ to $\boldsymbol{D}_\tau\}$, for any $\sigma,\tau \in$ **Typ**.
   The domain of type *e* is the power set a given non-empty set IN of entities, i.e. $\boldsymbol{D}_e = \wp^+(\mathsf{IN})$, where $\wp^+(\mathsf{IN}) := \wp(\mathsf{IN}) \backslash \{\varnothing\}$. The sum of two individuals $x_e \oplus y_e$ is the union of the sets *x* and *y*. For a set of atomic and/or non-atomic individuals X_{et}, the sum of the individuals in *X* (i.e. their union) is $\oplus X$. The part-of relation over individuals $x \leq y$ (*x* is a part of *y*) is the partial order induced by inclusion $\subseteq$ over the set $\wp^+(\mathsf{IN})$. The atomic individuals are the singleton subsets of IN, identified by the predicate $\mathbf{atom}(x) := \forall y \leq x(y=x)$.
   b. A *model* $\boldsymbol{M}$ for Dynamic Ty2 is a pair $<\boldsymbol{F}^{\boldsymbol{M}}, \|\cdot\|^{\boldsymbol{M}}>$ such that:
   i. $\boldsymbol{F}^{\boldsymbol{M}}$ is a standard frame for Dynamic Ty2;

ii. $\|\cdot\|^{M}$ assigns an object $\|\alpha\|^{M} \in D^{M}{}_{\tau}$ to each $\alpha \in \mathbf{Con}_{\tau}$ for any $\tau \in \mathbf{Typ}$, i.e. $\|\cdot\|^{M}$ respects typing;

iii. M satisfies the following axioms[78]:

- **Axiom1** (Unspecific dref's): $\mathbf{udref}(\delta)$, for any unspecific dref name δ of any type $(s\tau) \in \mathbf{DRefTyp}$, e.g. u_0, u_1, … but not *John*, *Mary*, …[79];
- **Axiom2** (Dref's have unique dref names): $\mathbf{udref}(\delta) \wedge \mathbf{udref}(\delta') \rightarrow \delta \neq \delta'$, for any two distinct dref names δ and δ' of type τ, for any type $\tau \in \mathbf{DrefTyp}$ (i.e. we ensure that we do not accidentally update δ' when we update δ);
- **Axiom3** (Identity of assignments): $\forall i_s j_s (i[]j \rightarrow i=j)$;
- **Axiom4** (Enough assignments): $\forall i_s \forall v_{s\tau} \forall f_{\tau} (\mathbf{udref}(v) \rightarrow \exists j_s (i[v]j \wedge vj=f))$, for any type $\tau \in$ **STyp**.

c. An ***M**-assignment* θ is a function that assigns to each variable $v \in \mathbf{Var}_{\tau}$ an element $\theta(v) \in D^{M}{}_{\tau}$ for any $\tau \in \mathbf{Typ}$. Given an M-assignment θ, if $v \in \mathbf{Var}_{\tau}$ and $d \in D^{M}{}_{\tau}$, then $\theta^{v/d}$ is the M-assignment identical with θ except that it assigns d to v.

d. The interpretation function $\|\cdot\|^{M,\theta}$ is defined as follows:

i. $\|\alpha\|^{M,\theta} = \|\alpha\|^{M}$ if $\alpha \in \mathbf{Con}_{\tau}$ for any $\tau \in \mathbf{Typ}$;

ii. $\|\alpha\|^{M,\theta} = \theta(\alpha)$ if $\alpha \in \mathbf{Var}_{\tau}$ for any $\tau \in \mathbf{Typ}$;

iii. $\|\alpha(\beta)\|^{M,\theta} = \|\alpha\|^{M,\theta}(\|\beta\|^{M,\theta})$;

iv. $\|\lambda v.\ \alpha\|^{M,\theta} = \langle \|\alpha\|^{M,\theta^{v/d}} : d \in D^{M}{}_{\sigma}\rangle$ if $v \in \mathbf{Var}_{\sigma}$;

v. $\|\alpha=\beta\|^{M,\theta} = $ T if $\|\alpha\|^{M,\theta} = \|\beta\|^{M,\theta}$; F otherwise.

vi. $\|i[\delta]j\|^{M,\theta} = $ T if $\delta \in \mathbf{Term}_{\sigma}$, $\sigma \in \mathbf{DrefTyp}$, $\|\forall v_{\sigma}(\mathbf{udref}(v) \wedge v \neq \delta \rightarrow vi=vj)\|^{M,\theta}$ = T and $\|\forall v_{\tau}(\mathbf{udref}(v) \rightarrow vi=vj\|^{M,\theta}$ = T for all $\tau \neq \sigma$, $\tau \in \mathbf{DrefTyp}$; F otherwise.

e. **Truth:**

i. A formula $\phi \in \mathbf{Term}_t$ is *true* in M relative to θ iff $\|\phi\|^{M,\theta}$ = T.

ii. A formula $\phi \in \mathbf{Term}_t$ is *true* in M iff it is true in M relative to any assignment θ.

Plural Compositional DRT – The Basic System

4. **Plural Compositional DRT**.

a. **Atomic conditions** – type $(st)t$ (lexical relations $R\{u_1, \ldots, u_n\}$ are c-ideals; conditions that are interpreted collectively at the discourse-level, e.g. **atom**, **2_atoms**, $u'=\oplus u$, $u=u_1\oplus\ldots\oplus u_n$ etc., are not c-ideals):

i. $R\{u_1, \ldots, u_n\} := \lambda I_{st}.\ I \neq \emptyset \wedge \forall i_s \in I(R(u_1 i, \ldots, u_n i))$, for any non-logical constant R of type $e^n t$, where $e^n t$ is defined as follows: $e^0 t := t$ and $e^{m+1}t := e(e^m t)$;

ii. $u_1=u_2 := \lambda I_{st}.\ I \neq \emptyset \wedge \forall i_s \in I(u_1 i = u_2 i)$;

iii. $\mathbf{atom}\{u\} := \lambda I_{st}.\ \mathbf{atom}(\oplus uI)$,
where $\mathbf{atom}(x_e) := \forall y_e \leq x(y=x)$;

iv. $\mathbf{2\_atoms}\{u\} := \lambda I_{st}.\ \mathbf{2\_atoms}(\oplus uI)$,
where $\mathbf{2\_atoms}(x_e) := |\{y_e : y \leq x \wedge \mathbf{atom}(y)\}|=2$;

v. $u'=\oplus u := \lambda I_{st}.\ I \neq \emptyset \wedge \forall i_s \in I(u'i = \oplus uI)$;

vi. $u=u_1\oplus\ldots\oplus u_n := \lambda I_{st}.\ I \neq \emptyset \wedge \forall i_s \in I(ui = u_1 i \oplus \ldots \oplus u_n i)$.

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78 The axioms / axiom schemata are based on Muskens (1995b, 1996).

79 **udref** is a non-logical constant intuitively identifying the 'variable' dref's, i.e. the non-constant functions of type $s\tau$ (for any $\tau \in \mathbf{STyp}$) intended to model DPL-like variables.. In fact, **udref** stands for an infinite family of non-logical constants of type (τt) for any $\tau \in \mathbf{DRefTyp}$. Alternatively, we can assume a polymorphic type logic with infinite sum types, in which **udref** is a polymorphic function. For a discussion of sum types, see for example Carpenter (1998): 69 et seqq.

b. **Atomic DRS's (DRS's containing one atomic condition)** – type $(st)((st)t)$ (the domain $\mathbf{Dom}(D)$ and range $\mathbf{Ran}(D)$ of an atomic DRS D are c-ideals, where $\mathbf{Dom}(D) := \{I_{st}: \exists J_{st}(DIJ)\}$ and $\mathbf{Ran}(D) := \{J_{st}: \exists I_{st}(DIJ)\}$):
   i. $[R\{u_1, \ldots, u_n\}] := \lambda I_{st}.\lambda J_{st}.\ I{=}J \wedge R\{u_1, \ldots, u_n\}J$
   ii. $[u_1{=}u_2] := \lambda I_{st}.\lambda J_{st}.\ I{=}J \wedge (u_1{=}u_2)J$

c. **Condition-level connectives (negation, anaphoric closure, disjunction, implication), i.e. non-atomic conditions**:
   i. $\sim D := \lambda I_{st}.\ I{\neq}\varnothing \wedge \forall H_{st}(H{\neq}\varnothing \wedge H{\subseteq}I \rightarrow \neg\exists K_{st}(DHK))$,
      i.e. $\sim D := \lambda I_{st}.\ I{\neq}\varnothing \wedge \forall H_{st}{\neq}\varnothing(H{\subseteq}I \rightarrow H{\notin}\mathbf{Dom}(D))$,
      where D is a DRS (type $(st)((st)t)$).
   If $\mathbf{Dom}(D)$ is a c-ideal, hence $\mathbf{Dom}(D) = \wp^+(\cup\mathbf{Dom}(D))$, $\sim D$ is the unique maximal c-ideal disjoint from $\mathbf{Dom}(D)$: $\sim D = \wp^+(\boldsymbol{D}_s^{\boldsymbol{M}}\backslash\cup\mathbf{Dom}(D))$.
   ii. $!D := \lambda I_{st}.\ \exists K_{st}(DIK)$, i.e. $!D := \mathbf{Dom}(D)$.
   If $\mathbf{Dom}(D)$ is a c-ideal, $\sim[\sim D] = !D$.
   iii. $D_1 \vee D_2 := \lambda I_{st}.\ \exists K_{st}(D_1IK \vee D_2IK)$, i.e. $D_1 \vee D_2 := \mathbf{Dom}(D_1)\cup\mathbf{Dom}(D_2)$.
   iv. $D_1 \rightarrow D_2 := \lambda I_{st}.\ \forall H_{st}(D_1IH \rightarrow \exists K_{st}(D_2HK))$, i.e. $D_1 \rightarrow D_2 := \lambda I_{st}.\ D_1I \subseteq \mathbf{Dom}(D_2)$, where $DI := \{J_{st}: DIJ\}$, i.e. $D_1 \rightarrow D_2 := (\wp^+(\boldsymbol{D}_s^{\boldsymbol{M}})\backslash\mathbf{Dom}(D_1)) \cup \{I_{st}{\in}\mathbf{Dom}(D_1): D_1I \subseteq \mathbf{Dom}(D_2)\}$.

d. **Tests (generalizing 'atomic' DRS's)**:
   $[C_1, \ldots, C_m] := \lambda I_{st}.\lambda J_{st}.\ I{=}J \wedge C_1J \wedge \ldots \wedge C_mJ$ [80],
   where $C_1, \ldots, C_m$ are conditions (atomic or not) of type $(st)t$. The domain $\mathbf{Dom}(D)$ and range $\mathbf{Ran}(D)$ of any test D is a c-ideal if all the conditions are c-ideals.

e. **DRS-level connectives (dynamic conjunction)**:
   $D_1; D_2 := \lambda I_{st}.\lambda J_{st}.\ \exists H_{st}(D_1IH \wedge D_2HJ)$,
      where D_1 and D_2 are DRS's (type $(st)((st)t)$).

f. **Quantifiers (random assignment of value to a dref)**:
   $[u] := \lambda I_{st}.\lambda J_{st}.\ \forall i_s{\in}I(\exists j_s{\in}J(i[u]j)) \wedge \forall j_s{\in}J(\exists i_s{\in}I(i[u]j))$
   If a DRS D has the form $[u_1, \ldots, u_n \mid C_1, \ldots, C_m]$, where the conditions $C_1, \ldots, C_m$ are c-ideals, we have that:
   i. $\mathbf{Ran}(D) = C_1\cap \ldots \cap C_m = \wp^+((\cup C_1)\cap \ldots \cap(\cup C_m))$;
   ii. $\mathbf{Dom}(D) = \wp^+(\{i_s: \exists j_s(i[u_1, \ldots, u_n]j \wedge j{\in}(\cup C_1)\cap \ldots \cap(\cup C_m))\})$, where ;
      Since $i[u_1, \ldots, u_n]j$ is reflexive, $\mathbf{Ran}(D){\subseteq}\mathbf{Dom}(D)$;
   iii. Let $\mathbb{R}^D := \lambda i_s.\lambda j_s.\ i[u_1, \ldots, u_n]j \wedge j{\in}((\cup C_1)\cap \ldots \cap(\cup C_m))$. Then, $D := \lambda I_{st}.\lambda J_{st}.\ \exists\mathbb{R}_{s(st)}{\neq}\varnothing(I{=}\mathbf{Dom}(\mathbb{R}) \wedge J{=}\mathbf{Ran}(\mathbb{R}) \wedge \mathbb{R}{\subseteq}\mathbb{R}^D) = \lambda I_{st}.\lambda J_{st}.\ \exists\mathbb{R}{\in}\wp^+(\mathbb{R}^D)(I{=}\mathbf{Dom}(\mathbb{R}) \wedge J{=}\mathbf{Ran}(\mathbb{R})$ [81].

g. **Selective maximization:**
   $\mathbf{max}^u(D) := \lambda I_{st}.\lambda J_{st}.\ \exists H_{st}(I[u]H \wedge DHJ) \wedge \forall K_{st}(\exists H_{st}(I[u]H \wedge DHK) \rightarrow uK{\subseteq}uJ)$, where D is a DRS of type $(st)((st)t)$,
   i.e. $\mathbf{max}^u(D) := \lambda I_{st}.\lambda J_{st}.\ ([u]; D)IJ \wedge \forall K_{st}(([u]; D)IK \rightarrow uK{\subseteq}uJ)$.
   The $\mathbf{max}^u$ operator does not preserve the c-ideal structure of the domain or range of the embedded DRS. Multiply embedded $\mathbf{max}^u$ operators can be reduced as follows:
   $\mathbf{max}^u(D; \mathbf{max}^{u'}(D')) = \mathbf{max}^u(D; [u']; D'); \mathbf{max}^{u'}(D')$,
   if the following two conditions obtain:
   i. u is not reintroduced in D';

80 Alternatively, $[C_1, \ldots, C_m]$ can be defined using dynamic conjunction: $[C_1, \ldots, C_m] := \lambda I_{st}.\lambda J_{st}.\ ([C_1]; \ldots; [C_m])IJ$, where $[C] := \lambda I_{st}.\lambda J_{st}.\ I{=}J \wedge CJ$.

81 Where: $\mathbf{Dom}(\mathbb{R}) := \{i_s: \exists j_s(\mathbb{R}ij)\}$ and $\mathbf{Ran}(\mathbb{R}) := \{j_s: \exists i_s(\mathbb{R}ij)\}$.

ii. D' is of the form $[u_1, \ldots, u_n \mid C_1, \ldots, C_m]$, where $C_1, \ldots, C_m$ are c-ideals[82].

h. **Distributivity (i.e. unselective, discourse-level distributivity)**:
$\mathbf{dist}(D) := \lambda I_{st}.\lambda J_{st}.\ \exists R_{s((st)t)}{\neq}\emptyset(I{=}\mathbf{Dom}(R) \wedge J{=}\cup\mathbf{Ran}(R) \wedge \forall{<}k_s,L_{st}{>}\in R(D\{k\}L))$,
where $\mathbf{Dom}(R) := \{k_s: \exists L_{st}(RkL)\}$ and $\mathbf{Ran}(R) := \{L_{st}: \exists k_s(RkL)\}$[83].

i. **Selective generalized determiners:**
$\mathbf{det}_u(D_1, D_2) := \lambda I_{st}.\ I{\neq}\emptyset \wedge \mathbf{DET}(u[D_1I],\ u[(D_1; D_2)I])$,
where $u[DI] := \{\oplus uJ: ([u \mid \mathbf{atom}\{u\}]; D)IJ\}$
and **DET** is the corresponding static determiner.

j. **Truth:** A DRS D (type $(st)((st)t)$) is true with respect to an input info state I_{st} iff $\exists J_{st}(DIJ)$, i.e. iff $I\in\mathbf{Dom}(D)$ (or, equivalently, $I\in\ !D$).

5. **Additional abbreviations.**

a. **DRS-level quantifiers (multiple random assignment, existential quantification, maximal existential quantification)**:
i. $[u_1, \ldots, u_n] := [u_1]; \ldots; [u_n]$
ii. $\exists u(D) := [u]; D$
iii. $\exists^{\mathbf{m}}u(D) := \mathbf{max}^u(D)$

b. **Condition-level quantifiers (universal quantification)**:
$\forall u(D) := {\sim}([u]; {\sim}D)$, i.e. $\forall u(D) := {\sim}\exists u({\sim}D)$[84].

c. **DRS's (a.k.a. linearized 'boxes')**:
$[u_1, \ldots, u_n \mid C_1, \ldots, C_m] := \lambda I_{st}.\lambda J_{st}.\ ([u_1, \ldots, u_n]; [C_1, \ldots, C_m])IJ$,
where $C_1, \ldots, C_m$ are conditions (atomic or not),
i.e. $[u_1, \ldots, u_n \mid C_1, \ldots, C_m] := \lambda I_{st}.\lambda J_{st}.\ I[u_1, \ldots, u_n]J \wedge C_1J \wedge \ldots \wedge C_mJ$.

Syntax of a Fragment of English

Indexation

"The most important requirement that we impose is that the syntactic component of the grammar assigns indices to all names, pronouns and determiners" (Muskens 1996: 159). The indices are specific and unspecific dref's, e.g. u, u', u_1, *Dobby* etc. Variables that have the appropriate dref type, e.g. v_{se}, v'_{se}, $v_{0,se}$, $v_{1,se}$ etc., are also allowed as indices – but only on traces of movement. The antecedents are indexed with superscripts and dependent elements with subscripts, following the convention in Barwise (1987).

Phrase Structure and Lexical Insertion Rules

The Y-model of syntax has four components: D-structure (DS), S-Structure (SS), Logical Form (LF) and Phonological Form (PF). We will be interested in the first three, in particular in the level of LF, which provides the input to the semantic interpretation procedure.

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[82] See chapter **5** in Brasoveanu (2007) for the proof.

[83] Distributivity operators are sometimes redundant. In general, we cannot omit **dist** operators with scope over a **max** operator, over an **atom** condition or over any other condition that is interpreted collectively at the discourse level. Otherwise, **dist** operators with scope only over conditions that are c-ideals (e.g. lexical relations like $person\{u\}$, $buy\{u, u'\}$ etc. or random assignments (e.g. $[u, u']$) or any combination thereof are redundant because these updates are closed under arbitrary unions and subsets. Also, **dist** is idempotent, i.e. $\mathbf{dist}(\mathbf{dist}(D)) = \mathbf{dist}(D)$, for any DRS D. See the Appendix of chapter **6** in Brasoveanu (2007) for the exact definitions of closure under subsets and arbitrary unions and the basic ideas for the relevant proofs.

[84] The definitions of dynamic universal and existential quantifiers preserve their DRT / FCS / DPL partial duality if we quantify over DRS's whose domains are c-ideals: ${\sim}\exists u(D) = \forall u({\sim}D)$, if $\mathbf{Dom}(D)$ is a c-ideal (hence $\mathbf{Dom}(D) = \mathbf{Dom}([{\sim}[{\sim}D]])$).

The DS component consists of all the trees that can be generated by the phrase structure rules PS1-PS12 and the lexical insertion rules LI1-LI11 in (6) below. We could in fact do away with rule PS1 (the necessary recursion is already built into PS2), but I will keep it as a reminder that sequencing two sentences in discourse occurs at a supra-sentential, textual level.

6. **Phrase structure rules and lexical insertion rules.**

(**PS 1**) Txt → (Txt) CP	(**PS 5**) VP → DP V'	(**PS 9**) V_{di}' → V_{di} DP
(**PS 2**) CP → (CP) IP	(**PS 6**) V' → V_{in}	(**PS 10**) DP → D NP
(**PS 3**) CP → C IP	(**PS 7**) V' → V_{tr} DP	(**PS 11**) NP → N (CP)
(**PS 4**) IP → I VP	(**PS 8**) V' → V_{di}' DP	(**PS 12**) X → X^+ Conj X
(**LI 1**) D → a^u, $every^u$, $most^u$, few^u, no^u, $some^u$, any^u, $a^{u'}$, $every^{u'}$, ...	(**LI 5**) N → farmer, house-elf, donkey, ...	(**LI 9**) I → ∅, doesn't, don't, -ed, -s, didn't, …
(**LI 2**) DP → he_u, she_u, it_u, $he_{u'}$, ..., he_{John}, she_{Mary}, t_v, $t_{v'}$, ...	(**LI 6**) V_{tr} → own, beat, ...	(**LI 10**) C → if
(**LI 3**) DP → $John^u$, $Mary^u$, ...	(**LI 7**) V_{in} → sleep, walk, ...	(**LI 11**) Conj → and, or
(**LI 4**) DP → who, whom, which	(**LI 8**) V_{di} → buy, give, ...	

Subjects are assumed to be VP-internal and this is where they remain by default even at LF (they are raised out of VP only at PF). In this way, we can interpret sentential negation as having scope over quantifiers in subject position. Similarly, V-heads move to the inflectional I-head only at PF.

Relativization and Quantifier Raising

DS and SS are connected via the obligatory movement rule of Relativization (REL). A tree Θ' *follows by REL* from a tree Θ iff Θ' is the result of replacing some sub-tree of Θ of the form $[_{CP}\ [_{IP}\ X\ [_{DP}\ wh]\ Y]\ ]$, where *X* and *Y* are (possibly empty) strings and *wh* is either *who*, *whom* or *which*, by a tree $[_{CP}\ [_{DP}\ wh]^v\ [_{CP}\ [_{IP}\ X\ t_v\ Y]\ ]\ ]$, where *v* is a fresh variable index (not occurring in Θ as a superscript). REL is basically CP adjunction. Formally, SS is the smallest set of trees that includes DS and is closed under REL; thus, DS⊆SS.

7. *Relativization (REL)*: $[_{CP}\ [_{IP}\ X\ [_{DP}\ wh]\ Y\ ]\ ] \rightarrow [_{CP}\ [_{DP}\ wh]^v\ [_{CP}\ [_{IP}\ X\ t_v\ Y\ ]\ ]\ ]$

LF is the syntactic component that is the input to our semantics; this is the level where quantifier scope ambiguities are resolved. We define an optional rule of Quantifier Raising (QR) (May 1977) which adjoins DP's to IP's or DP's to VP's (we need VP-adjunction for ditransitive verbs among other things) and which is basically the Quantifying-In rule of Montague (1974). LF is defined as the smallest set of trees that includes SS and is closed under QR; thus, SS⊆LF.

A tree Θ' *follows by QR* from a tree Θ iff: **(a)** Θ' is the result of replacing some sub-tree Σ of Θ of the form $[_{IP}\ X\ [_{DP}\ Z]\ Y]$ by a tree $[_{IP}\ [_{DP}\ Z]^v\ [_{IP}\ X\ t_v\ Y]\ ]$, where *v* is a fresh variable index (not occurring in Θ as a superscript); or **(b)** Θ' is the result of replacing some sub-tree Σ of Θ of the form $[_{VP}\ X\ [_{DP}\ Z]\ Y]$ by a tree $[_{VP}\ [_{DP}\ Z]^v\ [_{VP}\ X\ t_v\ Y]\ ]$, where *v* is a fresh variable index (not occurring in Θ as a superscript). The conditions on the QR rule are that *Z* is not a pronoun or a *wh*-word and that $[_{DP}\ Z]$ is not a proper sub-tree of a DP sub-tree $[_{DP}\ W]$ of Σ [85].

8. *Quantifier Raising* (*QR*):
   a. $[_{IP}\ X\ [_{DP}\ Z]\ Y\ ] \rightarrow [_{IP}\ [_{DP}\ Z]^v\ [_{IP}\ X\ t_v\ Y\ ]\ ]$

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[85] For example, if the DP sub-tree $[_{DP}\ W]$ of Σ contains a relative clause which in its turn contains $[_{DP}\ Z]$, we do not want to QR $[_{DP}\ Z]$ all the way out of the relative clause.

b. $[_{VP}\ X\ [_{DP}\ Z]\ Y\ ] \ \rightarrow\ [_{VP}\ [_{DP}\ Z]^{v}\ [_{VP}\ X\ t_v\ Y\ ]\ ]$

Type-Driven Translation

Table (9) below provides examples of basic meanings for the lexical items in our fragment of English. The first column contains the lexical item, the second column its Dynamic Ty2 translation and the third column its type, assuming these two abbreviations: $\mathbf{t} := (st)((st)t)$ and $\mathbf{e} := se$. The abbreviated types have exactly the form they have in Montague semantics, e.g. the translation of the intransitive verb *sleep* is of type **et**, the translation of the pronoun *he* is of type (**et**)**t**, the translations of the indefinite article `a` and of the determiner `every` are of type (**et**)((**et**)**t**) etc. The list of basic meanings constitutes rule **TR0** of our type-driven translation procedure.

9. **TR 0: PCDRT Basic Meanings (TN – Terminal Nodes).**

Lexical Item	Translation	Type $\mathbf{e} := se$ $\mathbf{t} := (st)((st)t)$
$[\mathtt{sleep}]_{\mathrm{Vin}}$	$\rightsquigarrow \lambda v_{\mathbf{e}}.\ [sleep_{et}\{v\}]$	**et**
$[\mathtt{own}]_{\mathrm{Vtr}}$	$\rightsquigarrow \lambda Q_{(\mathbf{et})\mathbf{t}}.\ \lambda v_{\mathbf{e}}.\ Q(\lambda v'_{\mathbf{e}}.\ [own_{e(et)}\{v, v'\}])$	((**et**)**t**)(**et**)
$[\mathtt{gather}]_{\mathrm{Vtr}}$	$\rightsquigarrow \lambda Q_{(\mathbf{et})\mathbf{t}}.\lambda v_{\mathbf{e}}.\ Q(\lambda v'_{\mathbf{e}}.\ [\sim[\mathbf{atom}\{v'\}],\ gather\{v, v'\}])$	((**et**)**t**)(**et**)
$[\mathtt{buy}]_{\mathrm{Vdi}}$	$\rightsquigarrow \lambda Q_{(\mathbf{et})\mathbf{t}}.\ \lambda Q'_{(\mathbf{et})\mathbf{t}}.\ \lambda v_{\mathbf{e}}.\ Q(\lambda v'_{\mathbf{e}}.\ Q'(\lambda v''_{\mathbf{e}}.\ [buy_{e(e(et))}\{v, v', v''\}]))$	(**ett**)((**ett**)(**et**))
$[\mathtt{house\text{-}elf}]_{\mathrm{N}}$	$\rightsquigarrow \lambda v_{\mathbf{e}}.\ [house\_elf_{et}\{v\}]$	**et**
$[\mathtt{he}_{u}]_{\mathrm{DP}}$	$\rightsquigarrow \lambda P_{\mathbf{et}}.\ [\mathbf{atom}\{u\}];\ P(u)$	(**et**)**t**
$[\mathtt{the}_{\mathbf{sg}:u}]_{\mathrm{D}}$	$\rightsquigarrow \lambda P_{\mathbf{et}}.\ \lambda P'_{\mathbf{et}}.\ [\mathbf{atom}\{u\}];\ P(u);\ P'(u)$	(**et**)((**et**)**t**)
$[\mathtt{they}_{u}]_{\mathrm{DP}}$	$\rightsquigarrow \lambda P_{\mathbf{et}}.\ P(u)$	(**et**)**t**
$[\mathtt{the}_{\mathbf{pl}:u}]_{\mathrm{D}}$	$\rightsquigarrow \lambda P_{\mathbf{et}}.\ \lambda P'_{\mathbf{et}}.\ P(u);\ P'(u)$	(**et**)((**et**)**t**)
$[\mathtt{they}_{\oplus u}{}^{u'}]_{\mathrm{DP}}$	$\rightsquigarrow \lambda P_{\mathbf{et}}.\ [u' \mid u'{=}\oplus u];\ P(u')$	(**et**)**t**
$[\mathtt{the}_{\mathbf{pl}:\oplus u}{}^{u'}]_{\mathrm{D}}$	$\rightsquigarrow \lambda P_{\mathbf{et}}.\ \lambda P'_{\mathbf{et}}.\ [u' \mid u'{=}\oplus u];\ P(u');\ P'(u')$	(**et**)((**et**)**t**)
$[\mathtt{they}_{u\oplus u'}{}^{u''}]_{\mathrm{DP}}$	$\rightsquigarrow \lambda P_{\mathbf{et}}.\ [u'' \mid u''{=}u\oplus u'];\ P(u'')$	(**et**)**t**
$[\mathtt{the}_{\mathbf{pl}:u\oplus u'}{}^{u''}]_{\mathrm{D}}$	$\rightsquigarrow \lambda P_{\mathbf{et}}.\ \lambda P'_{\mathbf{et}}.\ [u'' \mid u''{=}u\oplus u'];\ P(u'');\ P'(u'')$	(**et**)((**et**)**t**)
$[\mathtt{t}_{v}]_{\mathrm{DP}}$	$\rightsquigarrow \lambda P_{\mathbf{et}}.\ P(v_{\mathbf{e}})$	(**et**)**t**
$[\mathtt{he}_{Dobby}]_{\mathrm{DP}}$	$\rightsquigarrow \lambda P_{\mathbf{et}}.\ P(Dobby_{\mathbf{e}})$	(**et**)**t**
$[\mathtt{Dobby}^{u}]_{\mathrm{DP}}$	$\rightsquigarrow \lambda P_{\mathbf{et}}.\ [u \mid u{=}Dobby];\ P(u)$	(**et**)**t**
$[\mathtt{who}]_{\mathrm{DP}}$	$\rightsquigarrow \lambda P_{\mathbf{et}}.\ P$	(**et**)(**et**)
$[\mathtt{\emptyset/\text{-}ed/\text{-}s}]_{\mathrm{I}}$	$\rightsquigarrow \lambda D_{\mathbf{t}}.\ D$	**tt**
$[\mathtt{doesn't}]_{\mathrm{I}}$/ $[\mathtt{didn't}]_{\mathrm{I}}$	$\rightsquigarrow \lambda D_{\mathbf{t}}.\ [\sim D]$	**tt**
$[\mathtt{a}^{\mathbf{wk}:u}]_{\mathrm{D}}$	$\rightsquigarrow \lambda P_{\mathbf{et}}.\ \lambda P'_{\mathbf{et}}.\ [u];\ \mathbf{dist}([\mathbf{atom}\{u\}];\ P(u);\ P'(u))$, i.e. $\lambda P_{\mathbf{et}}.\ \lambda P'_{\mathbf{et}}.\ \exists u(\mathbf{dist}([\mathbf{atom}\{u\}];\ P(u);\ P'(u)))$, i.e. existence and introduction of some witness set	(**et**)((**et**)**t**)
$[\mathtt{two}^{\mathbf{wk}:u}]_{\mathrm{D}}$	$\rightsquigarrow \lambda P_{\mathbf{et}}.\ \lambda P'_{\mathbf{et}}.\ [u];\ \mathbf{dist}([\mathbf{2\_atoms}\{u\}];\ P(u);\ P'(u))$	(**et**)((**et**)**t**)
$[\mathtt{a}^{\mathbf{str}:u}]_{\mathrm{D}}$	$\rightsquigarrow \lambda P_{\mathbf{et}}.\ \lambda P'_{\mathbf{et}}.\ \mathbf{max}^{u}(\mathbf{dist}([\mathbf{atom}\{u\}];\ P(u);\ P'(u)))$, i.e. $\lambda P_{\mathbf{et}}.\ \lambda P'_{\mathbf{et}}.\ \exists^{\mathbf{m}}u(\mathbf{dist}([\mathbf{atom}\{u\}];\ P(u);\ P'(u)))$, i.e. existence and introduction of the maximal witness set (maximality relative to restrictor P and nuclear scope P')	(**et**)((**et**)**t**)
$[\mathtt{two}^{\mathbf{str}:u}]_{\mathrm{D}}$	$\rightsquigarrow \lambda P_{\mathbf{et}}.\ \lambda P'_{\mathbf{et}}.\ \mathbf{max}^{u}(\mathbf{dist}([\mathbf{2\_atoms}\{u\}];\ P(u);\ P'(u)))$	(**et**)((**et**)**t**)
$[\mathtt{the}^{\mathbf{sg}:u}]_{\mathrm{D}}$	$\rightsquigarrow \lambda P_{\mathbf{et}}.\ \lambda P'_{\mathbf{et}}.\ \mathbf{max}^{u}(P(u));\ [\mathbf{atom}\{u\}];\ P'(u)$, i.e. $\lambda P_{\mathbf{et}}.\ \lambda P'_{\mathbf{et}}.\ \exists^{\mathbf{m}}u(P(u));\ [\mathbf{atom}\{u\}];\ P'(u)$, i.e. existence and uniqueness relative to restrictor P, i.e. the Russellian analysis	(**et**)((**et**)**t**)
$[\mathtt{her}_{u}{}^{\mathbf{sg}:u'}]_{\mathrm{D}}$	$\rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [\mathbf{atom}\{u\}];\ \mathbf{max}^{u'}(P(u');\ [of\{u', u\}]);\ [\mathbf{atom}\{u'\}];\ P'(u')$	(**et**)((**et**)**t**)

9. **TR 0: PCDRT Basic Meanings (TN – Terminal Nodes).**

Lexical Item	**Translation**	**Type** $\mathbf{e} := se$ $\mathbf{t} := (st)((st)t)$
$[the^{\mathbf{pl}:u}]_{D}$	$\rightsquigarrow \lambda P_{\mathbf{et}}.\ \lambda P'_{\mathbf{et}}.\ \mathbf{max}^{u}(P(u));\ P'(u)$, i.e. $\lambda P_{\mathbf{et}}.\ \lambda P'_{\mathbf{et}}.\ \exists^{\mathbf{m}}u(P(u));\ P'(u)$, i.e. existence and maximality relative to restrictor P, i.e. Link's analysis	$(\mathbf{et})((\mathbf{et})\mathbf{t})$
$her_{u}^{\mathbf{pl}:u'}]_{D}$	$\rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [\mathbf{atom}\{u\}];\ \mathbf{max}^{u'}(P(u');\ [of\{u', u\}]);\ P'(u')$	$(\mathbf{et})((\mathbf{et})\mathbf{t})$
$[det^{u}]_{D}$ $(every^{u},\ no^{u},$ $most^{u}$ etc.)	$\rightsquigarrow \lambda P_{\mathbf{et}}.\ \lambda P'_{\mathbf{et}}.\ [\mathbf{det}_{u}(\mathbf{dist}(P(u)),\ \mathbf{dist}(P'(u)))]$	$(\mathbf{et})((\mathbf{et})\mathbf{t})$
$[and]_{Conj}$	$\rightsquigarrow \lambda v_{1}.\ \ldots\ \lambda v_{n}.\ v_{1} \sqcap \ldots \sqcap v_{n}$	$\tau(\ldots(\tau\tau)\ldots)$
$[or]_{Conj}$	$\rightsquigarrow \lambda v_{1}.\ \ldots\ \lambda v_{n}.\ v_{1} \sqcup \ldots \sqcup v_{n}$	$\tau(\ldots(\tau\tau)\ldots)$

Following Partee & Rooth (1983), the set of dynamically conjoinable types is defined as follows.

10. **PCDRT Dynamically Conjoinable Types (DCTyp).** The set of PCDRT dynamically conjoinable types **DCTyp** is the smallest subset of **Typ** s.t. $\mathbf{t}\in\mathbf{DCTyp}$ ($\mathbf{t} := (st)((st)t)$) and, if $\tau\in\mathbf{DCTyp}$, then $(\sigma\tau)\in\mathbf{DCTyp}$ for any $\sigma\in\mathbf{Typ}$.
11. **Generalized Pointwise Dynamic Conjunction ⊓ and Disjunction ⊔.** For any two terms α and β of type τ, for any $\tau\in\mathbf{DCTyp}$:
    a. $\alpha \sqcap \beta := (\alpha; \beta)$ if $\tau=\mathbf{t}$ and $\alpha \sqcap \beta := \lambda v_{\sigma}.\ \alpha(v) \sqcap \beta(v)$ if $\tau=(\sigma\rho)$
    b. **Abbreviation.** $\alpha_{1} \sqcap \alpha_{2} \sqcap \ldots \sqcap \alpha_{n} := (\ldots(\alpha_{1} \sqcap \alpha_{2}) \sqcap \ldots \sqcap \alpha_{n})$
    c. $\alpha \sqcup \beta := [\alpha \vee \beta]$ if $\tau=\mathbf{t}$ and $\alpha \sqcup \beta := \lambda v_{\sigma}.\ \alpha(v) \sqcup \beta(v)$ if $\tau=(\sigma\rho)$
    d. **Abbreviation.** $\alpha_{1} \sqcup \alpha_{2} \sqcup \ldots \sqcup \alpha_{n} := (\ldots(\alpha_{1} \sqcup \alpha_{2}) \sqcup \ldots \sqcup \alpha_{n})$

Based on **TR0**, we can obtain the translation of more complex LF structures by specifying how the translation of a mother node depends on the translations of its daughters. There are five such rules.

12. **TR 1 – Non-branching Nodes (NN).** If A $\rightsquigarrow \alpha$ and A is the only daughter of B, then B $\rightsquigarrow \alpha$.
13. **TR 2 – Functional Application (FA).** If A $\rightsquigarrow \alpha$ and B $\rightsquigarrow \beta$ and A and B are the only daughters of C, then C $\rightsquigarrow \alpha(\beta)$, provided that this is a well-formed term.
14. **TR 3 – Generalized Sequencing (GSeq) (i.e. Sequencing + Predicate Modification[86]).** If A $\rightsquigarrow \alpha$, B $\rightsquigarrow \beta$, A and B are the only daughters of C in that order (i.e. C $\rightarrow$ A B) and α and β are of the same type τ of the form $\mathbf{t}$ or $(\sigma\mathbf{t})$ for some $\sigma\in\mathbf{Typ}$, then C $\rightsquigarrow \alpha; \beta$ if $\tau=\mathbf{t}$ or C $\rightsquigarrow \lambda v_{\sigma}.\ \alpha(v); \beta(v)$, if $\tau=(\sigma\mathbf{t})$, provided that this is a well-formed term.
15. **TR 4 – Quantifying-In (QIn).** If $DP^{v} \rightsquigarrow \alpha$, B $\rightsquigarrow \beta$ and DP^{v} and B are daughters of C, then C $\rightsquigarrow \alpha(\lambda v.\ \beta)$, provided that this is a well-formed term.
16. **TR 5 – Generalized Coordination (GCo).** If $A_{1} \rightsquigarrow \alpha_{1}$, …, $A_{n} \rightsquigarrow \alpha_{n}$, Conj $\rightsquigarrow \beta$, $A_{n+1} \rightsquigarrow \alpha_{n+1}$ and A_{1}, …, A_{n}, Conj and A_{n+1} are the only daughters of A in that order (i.e. A

[86] Generalized sequencing is just generalized dynamic conjunction in the sense of (11) above. This rule translates the meaning of complex texts (Txt) that are formed out of a text (Txt) and a sentence (CP) – see **PS1** in (6) above. In this sense, it is a generalization of the Sequencing rule in Muskens (1996). But it also handles predicate modification in general, e.g. it translates the meaning of an NP that is formed out of a common noun N and a relative clause CP – see **PS11** in (6) above. In this sense, it is a generalization of the Predicate Modification rule in Heim & Kratzer (1998).

$\to A_1 \dots A_n$ Conj A_{n+1}), then A $\rightsquigarrow \beta(\alpha_1)\dots(\alpha_n)(\alpha_{n+1})$, provided this a well-formed term and has the same type as $\alpha_1, \dots, \alpha_n, \alpha_{n+1}$.

The translation procedure, i.e. the relation 'tree Θ *translates as* term α', is formally defined as the smallest relation $\rightsquigarrow$ between trees and Dynamic Ty2 terms that is conform to **TR0**-**TR5** and is closed under Dynamic Ty2 equivalence, e.g. if tree Θ *translates as* term α and term β is such that $\|\alpha\|^{\mathit{M},\theta} = \|\beta\|^{\mathit{M},\theta}$, then Θ *translates as* β.

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