Structured Anaphora to Quantifier Domains: A Unified Account of Quantificational and Modal Subordination

Adrian Brasoveanu*

Stanford University and University of California at Santa Cruz

Abstract. The paper proposes an account of the contrast (noticed in [9]) between the interpretations of the following two discourses: *Harvey courts a girl at every convention.* {*She is very pretty.* vs. *She always comes to the banquet with him.*}. The initial sentence is ambiguous between two quantifier scopings, but the first discourse as a whole allows only for the wide-scope indefinite reading, while the second allows for both. This cross-sentential interaction between quantifier scope and anaphora is captured by means of a new dynamic system couched in classical type logic, which extends Compositional DRT ([16]) with *plural information states* (modeled, following [24], as sets of variable assignments). Given the underlying type logic, compositionality at sub-clausal level follows automatically and standard techniques from Montague semantics become available. The paper also shows that modal subordination (*A wolf might come in. It would eat Harvey first*) can be analyzed in a parallel way, i.e. the system captures the anaphoric and quantificational parallels between the individual and modal domains argued for in [23]. In the process, we see that modal/individual-level quantifiers enter anaphoric connections as a matter of course, usually functioning simultaneously as both indefinites and pronouns.

1 Introduction: Quantificational Subordination

The present paper proposes an account of the contrast between the interpretations of the discourses in (1) and (2) below from [9] (the superscripts and subscripts indicate the antecedent-anaphor relations).

1. **a.** Harvey courts a^u girl at every convention. **b.** She_u is very pretty.
2. **a.** Harvey courts a_u girl at every convention. **b.** She_u always comes to the banquet with him. [**c.** The_u girl is usually also very pretty.]

Sentence (1a/2a) by itself is ambiguous between two quantifier scopings: it "can mean that, at every convention, there is some girl that Harvey courts or that there is some girl that Harvey courts at every convention. [...] Harvey always courts the same girl [...] [or] it may be a different girl each time" ([9]: 377). The contrast between the continuations in (1b) and (2b) is that the former allows only for the 'same girl' reading of sentence (1a/2a), while the latter is also compatible with the 'possibly different girls' reading.

Discourse (1) raises the following question: how can we capture the fact that a *singular anaphoric pronoun* in sentence (1b) can interact with and disambiguate *quantifier scopings*[1] in sentence (1a)?

* I am grateful to Maria Bittner, Sam Cumming, Hans Kamp, Roger Schwarzschild, Matthew Stone and three WoLLIC 2007 reviewers for their detailed comments and to Daniel Altshuler, Pranav Anand, Donka Farkas, Carlos Fasola, Michael Johnson, Ernie Lepore, Friederike Moltmann, Sarah Murray, Jessica Rett, Ken Shan, Will Starr and the Rutgers Semantics Reading Group (Nov 14, 2006) for discussion. The financial support provided by Stanford University (Humanities Fellows Program) is gratefully acknowledged. The usual disclaimers apply. To appear in the *Proceedings of WoLLIC 2007 (Lecture Notes in Computer Science)*, D. Leivant & R. de Queiroz (eds.), Springer-Verlag (LNCS online–http://www.springerlink.com/content/105633/), ©Springer-Verlag.

[1] To see that it is indeed quantifier scopings that are disambiguated, substitute *exactly* one^u *girl* for a^u *girl* in (1a); this yields two truth-conditionally independent scopings: (*i*) *exactly one girl>>every convention*, which is true in

That number morphology on the pronoun *she* is crucial is shown by the discourse in (3) below, where the (preferred) relative scoping of *every convention* and *a girl* is the opposite of the one in discourse (1).

3. **a.** Harvey courts a^u girl at every convention. **b.** $They_u$ are very pretty.

Discourse (2) raises the following questions. First, why is it that adding an adverb of quantification, i.e. *always/usually*, makes both readings of sentence (2a) available? Moreover, on the newly available reading of sentence (2a), i.e. the *every convention>>a girl* scoping, how can we capture the intuition that the singular pronoun *she* and the adverb *always* in sentence (2b) elaborate on the quantificational dependency between conventions and girls introduced in sentence (2a), i.e. how can we capture the intuition that we seem to have simultaneous anaphora to the two quantifier domains and to the quantificational dependency between them?

The phenomenon instantiated by discourses (1) and (2) is subsumed under the more general label of *quantificational subordination* (see [13]: 139, (2)), which covers a variety of phenomena involving interactions between generalized quantifiers and morphologically singular cross-sentential anaphora. The main goal of this paper is give an account of quantificational subordination couched within a new compositional dynamic system which straightforwardly generalizes to an account of modal subordination, thereby capturing the anaphoric and quantificational parallels between the individual and modal domains argued for in [23], [1], [22], [5] and [7] among others (building on [18] and [19]).

2 Plural Compositional DRT (PCDRT)

This section introduces the semantic framework in which the analysis of discourses (1) and (2) is couched. The main proposal is that (compositionally) assigning natural language expressions finer-grained semantic values (finer grained than the usual meanings assigned in static Montague semantics) enables us to capture the interaction between generalized quantifiers, singular pronouns and adverbs of quantification exhibited by the contrast between (1) and (2).

Accounting for *cross-sentential* phenomena in semantic terms (as opposed to purely/primarily pragmatic terms) requires some preliminary justification. First, the same kind of finer-grained semantic values (to be introduced presently) are independently motivated by intra-sentential phenomena (see the account of mixed weak & strong donkey sentences in [2]). Second, the phenomenon instantiated by (1) and (2) is as much intra-sentential as it is cross-sentential. Note that there are four separate components that come together to yield the contrast in interpretation between (1) and (2): (*i*) the generalized quantifier *every convention*, (*ii*) the indefinite *a girl*, (*iii*) the singular number morphology on the pronoun *she* and (*iv*) the adverb of quantification *always/usually*. To derive the intuitively correct interpretations for (1) and (2), we have to attend to both the cross-sentential connections *a girl–she* and *every convention–always/usually* and the intra-sentential interactions *every convention–a girl* and *always–she*.

I conclude that an account of the contrast between (1) and (2) that involves a revamping of semantic values has sufficient initial plausibility to make its pursuit worthwhile. To this end, I introduce a new dynamic system couched in classical (many-sorted) type logic which extends Compositional DRT (CDRT, [16]) in two ways: (*i*) with plural information states and (*ii*) with selective generalized quantification. The resulting system is dubbed Plural CDRT (PCDRT).

---

a situation in which Harvey courts more than one girl per convention, but there is exactly one (e.g. Faye Dunaway) that he never fails to court, and (*ii*) *every convention>>exactly one girl*.

2.1 Plural Information States

The main technical innovation relative to CDRT is that, just as in Dynamic Plural Logic ([24]), information states I, J etc. are modeled as *sets* of variable assignments i, j etc.; such *plural* info states can be represented as matrices with assignments (sequences) as rows, as shown below.

<table>
<tr><th>Info State I</th><th>...</th><th>u</th><th>u'</th><th>...</th></tr>
<tr><td>i_1</td><td>...</td><td>x_1 (i.e. ui_1)</td><td>y_1 (i.e. $u'i_1$)</td><td>...</td></tr>
<tr><td>i_2</td><td>...</td><td>x_2 (i.e. ui_2)</td><td>y_2 (i.e. $u'i_2$)</td><td>...</td></tr>
<tr><td>i_3</td><td>...</td><td>x_3 (i.e. ui_3)</td><td>y_3 (i.e. $u'i_3$)</td><td>...</td></tr>
<tr><td>...</td><td>...</td><td>...</td><td>...</td><td>...</td></tr>
<tr><td colspan="3">Quantifier domains (sets) are stored columnwise: $\{x_1, x_2, x_3, \ldots\}$, $\{y_1, y_2, y_3, \ldots\}$</td><td colspan="2">Quantifier dependencies (relations) are stored rowwise: $\{(x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots\}$</td></tr>
</table>

Plural info states enable us to encode discourse reference to both quantifier domains, i.e. *values*, and quantificational dependencies, i.e. *structure*. The values are the sets of objects that are stored in the columns of the matrix, e.g. a discourse referent (dref) u for individuals stores a set of individuals relative to a plural info state given that u is assigned an individual by each assignment/row. The structure is encoded in the rows of the matrix: for each assignment/row in the info state, the individual assigned to a dref u by that assignment is structurally correlated with the individual assigned to some other dref u' by the same assignment.

2.2 The Outline of the Proposed Account

Thus, plural info states enable us to pass information about both quantifier domains and quantificational dependencies across sentential/clausal boundaries, which is exactly what we need to account for the interpretation of discourses (1) and (2). More precisely, we need the following two ingredients.

First, we need a suitable meaning for selective generalized determiners that will store two things in the input plural info state: (*i*) the restrictor and nuclear scope sets of individuals that are introduced and related by the determiner; (*ii*) the quantificational dependencies between the individuals in the restrictor/nuclear scope set and any other quantifiers/indefinites in the restrictor/nuclear scope of the quantification, e.g. between *every convention* in (1a/2a) and the indefinite *a girl* in its nuclear scope. Given that plural info states store both sets of individuals and dependencies between them, both kinds of information are available for subsequent anaphoric retrieval; for example, *always* and *she* in (2b) are simultaneously anaphoric to both the sets of conventions and girls and the dependency between these sets introduced in (2a).

The second ingredient is a suitable meaning for singular number morphology on pronouns like she_u in (1b) and (2b) above. This meaning has to derive the observed interactions between (*i*) singular pronouns, (*ii*) quantifiers and indefinites in the previous discourse, e.g. *every convention* and a^u *girl* in (1a/2a), and (*iii*) quantifiers in the same sentence, e.g. the adverb *always* in (2b). In particular, I will take singular number morphology on she_u to require the set of u-individuals stored by the current plural info state to be a singleton. The set of u-individuals is introduced by the indefinite a^u *girl* and is available for anaphoric retrieval irrespective of whether the indefinite has wide or narrow scope in sentence (1a/2a). Thus, once again, plural info states are crucial for the analysis: they enable us to store and pass on structured sets of individuals, so that we can constrain their cardinality by subsequent, non-local anaphoric elements.

If the indefinite a^u *girl* has narrow scope relative to *every convention*, the singleton requirement contributed by she_u applies to the set of all girls that are courted by Harvey at some convention or other. Requiring this set to be a singleton boils down to removing from consideration all the plural info states that would satisfy the narrow-scope indefinite reading *every convention*>>a^u *girl*, but not the wide-scope reading a^u *girl*>>*every convention*. We therefore derive the intuition that, irrespective of which quantifier scoping we assume for sentence (1a), any plural info state that we obtain after a successful update with sentence (1b) is bound to satisfy the representation in which the indefinite a^u *girl* (or a quantifier like *exactly one*^{u} *girl*) takes wide scope.

In discourse (2), however, the adverb of quantification *always* in (2b), which is anaphoric to the nuclear scope set introduced by *every convention* in (2a), can take scope either below or above the singular pronoun she_u. If *always* takes scope below she_u, we obtain the same reading as in discourse (1). If *always* takes scope above she_u, it 'breaks' the input plural info state storing all the conventions into smaller sub-states, each storing a particular convention. Consequently, the singleton requirement contributed by she_u is enforced locally, relative to each of these sub-states, and not globally, relative to the whole input info state, so we end up requiring the courted girl to be unique *per convention* and not across the board.

The remainder of this section presents the basics of the compositional dynamic system, while Section 3 introduces the PCDRT meanings for selective generalized determiners, indefinites and singular/plural pronouns.

2.3 DRS's and Conditions in PCDRT

We work with a Dynamic Ty2 logic, i.e. with the Logic of Change in [16] which reformulates dynamic semantics ([8], [12]) in Gallin's Ty2 ([6]). We have three basic types: type t (truth-values), type e (individuals; variables: x, x' etc.) and type s ('variable assignments'; variables: i, j etc.). A suitable set of axioms ensures that the entities of type s do behave as variable assignments[2].

A dref for individuals u is a function of type se from 'assignments' i_s to individuals x_e (the subscripts on terms indicate their type). Intuitively, the individual $u_{se}i_s$ is the individual that the 'assignment' i assigns to the dref u. Dynamic info states I, J etc. are plural: they are sets of 'variable assignments', i.e. terms of type st. An individual dref u stores a set of individuals with respect to a plural info state I, abbreviated as $uI := \{u_{se}i_s : i_s \in I_{st}\}$, i.e. uI is the image of the set of 'assignments' I under the function u.

A sentence is interpreted as a Discourse Representation Structure (DRS), which is a relation of type $(st)((st)t)$ between an input state I_{st} and an output state J_{st}, as shown in (4) below. A DRS requires: (*i*) the input info state I to differ from the output state J at most with respect to the **new dref's** and (*ii*) all the **conditions** to be satisfied relative to the output state J. For example, the DRS $[u_1, u_2|girl\{u_1\}, convention\{u_2\}, courted\_at\{u_1, u_2\}]$ abbreviates the term $\lambda I_{st}.\lambda J_{st}.\ I[u_1, u_2]J \wedge girl\{u_1\}J \wedge convention\{u_2\}J \wedge courted\_at\{u_1, u_2\}J$. The definition of dref introduction (a.k.a. random assignment) is given in (5) below[3].

4. $[\textbf{new dref's}\,|\,\textbf{conditions}] := \lambda I_{st}.\lambda J_{st}.\ I[\textbf{new dref's}]J \wedge \textbf{conditions}J$
5. $[u] := \lambda I_{st}.\lambda J_{st}.\ \forall i_s \in I(\exists j_s \in J(i[u]j)) \wedge \forall j_s \in J(\exists i_s \in I(i[u]j))$

[2] See [16] for more details.
[3] See [2] for its justification.

DRS's of the form [**conditions**] $:= \lambda I_{st}.\lambda J_{st}.\ I = J \wedge \mathbf{conditions}J$ are *tests*, e.g. $[girl\{u_1\}] := \lambda I_{st}.\lambda J_{st}.\ I = J \wedge girl\{u_1\}J$ tests that the input state I satisfies the condition $girl\{u_1\}$. Conditions are interpreted *distributively* relative to a plural info state, e.g. $courted\_at\{u_1, u_2\}$ is basically the term $\lambda I_{st}.\ I \neq \emptyset \wedge \forall i_s \in I(courted\_at(u_1 i, u_2 i))$ of type $(st)t$, i.e. it denotes a set of information states; see Subsect. 3.1 below for the general definition of atomic conditions.

2.4 Compositionality

Given the underlying type logic, compositionality at sub-clausal level follows automatically and standard techniques from Montague semantics (e.g. type shifting) become available. In more detail, the compositional aspect of interpretation in an extensional Fregean/Montagovian framework is largely determined by the types for the (extensions of the) 'saturated' expressions, i.e. names and sentences. Abbreviate them as **e** and **t**. An extensional static logic identifies **e** with e and **t** with t. The denotation of the noun ***girl*** is of type **et**, i.e. et: ***girl*** $\rightsquigarrow \lambda x_e.\ girl_{et}(x)$. The generalized determiner ***every*** is of type $(\mathbf{et})((\mathbf{et})\mathbf{t})$, i.e. $(et)((et)t)$.

PCDRT assigns the following dynamic types to the 'meta-types' **e** and **t**: **t** abbreviates $(st)((st)t)$, i.e. a sentence is interpreted as a DRS, and **e** abbreviates se, i.e. a name is interpreted as a dref. The denotation of the noun ***girl*** is still of type **et**, as shown in (6) below. Moreover, the determiner ***every*** is still of type $(\mathbf{et})((\mathbf{et})\mathbf{t})$ – and its definition is provided in the next section.

6. ***girl*** $\rightsquigarrow \lambda v_{\mathbf{e}}.\ [girl_{et}\{v\}]$, i.e. ***girl*** $\rightsquigarrow \lambda v_{\mathbf{e}}.\lambda I_{st}.\lambda J_{st}.\ I = J \wedge girl_{et}\{v\}J$

3 Generalized Quantification in PCDRT

We turn now to the definition of selective generalized quantification in PCDRT. The definition has to satisfy four desiderata, the first three of which are about anaphoric connections that can be established *internally* (within the generalized quantification), i.e. between antecedents in the restrictor and anaphors in the nuclear scope, and the last of which is about anaphora that can be established *externally*, i.e. between antecedents introduced by/within the quantification and anaphors that are outside the quantification.

Let us begin with internal anaphora. First, we want our definition to be able to account for the fact that anaphoric connections between the restrictor and the nuclear scope of the quantification can in fact be established, i.e. we want to account for donkey anaphora (*Every*^{u} *farmer who owns a*$^{u'}$ *donkey beats it*$_{u'}$).

Second, we want to account for such anaphoric connections while avoiding the proportion problem that *unselective* quantification (in the sense of [15]) runs into. That is, we need generalized determiners to relate sets of individuals (i.e. sets of objects of type e) and not sets of 'assignments' (i.e. sets of objects of type s). The sentence *Most*^{u} *farmers who own a*$^{u'}$ *donkey beat it*$_{u'}$ provides a typical instance of the proportion problem: intuitively, this sentence is false in a situation in which there are ten farmers, nine have a single donkey each and they do not beat it, while the tenth has twenty donkeys and he is busy beating them all. The unselective interpretation of the *most*-quantification, however, incorrectly predicts that the sentence is true in this situation because more than half of the $(farmer, donkey)$ pairs (twenty out of twenty-nine) are such that the farmer beats the donkey.

The third desideratum is that the definition of selective generalized quantification be compatible with both strong and weak donkey readings: we want to allow for the different interpretations associated with the donkey anaphora in (7) (from [13]) and (8) (from [20]) below. Sentence (7) is interpreted as asserting that most slave-owners were such that, for *every* (strong reading) slave they owned, they also owned his offspring. Sentence (8) is interpreted as asserting that every dime-owner puts *some* (weak reading) dime of her/his in the meter.

7. Most^{u} people that owned a$^{u'}$ slave also owned his$_{u'}$ offspring.
8. Every^{u} person who has a$^{u'}$ dime will put it$_{u'}$ in the meter.

The fourth desideratum is concerned with external anaphora – and this brings us back to the discourses in (1) and (2). These discourses indicate that we need to make the restrictor and nuclear scope sets of individuals related by generalized determiners available for subsequent anaphora – and we also need to make available for anaphoric take-up the quantificational dependencies between different quantifiers and/or indefinites. In particular, generalized quantification supports anaphora to two sets: (*i*) the maximal set of individuals satisfying the restrictor DRS, i.e. the *restrictor set*, and (*ii*) the maximal set of individuals satisfying the restrictor and nuclear scope DRS's, i.e. the *nuclear scope set*[4]. Note that the latter set is the nuclear scope that emerges as a consequence of the conservativity of natural language quantification – and, as [24] (among others) observes, we need to build conservativity into the definition of dynamic quantification to account for the fact that the nuclear scope DRS can contain anaphors dependent on antecedents in the restrictor.

The discourse in (9) below exemplifies anaphora to nuclear scope sets: sentence (9b) is interpreted as asserting that the people that went to the beach are the students that left the party after 5 a.m. (which, in addition, formed a majority of the students at the party). The discourses in (10) and (11) below exemplify anaphora to the restrictor sets contributed by the downward monotonic quantifiers *no*^{u} *student* and *very few*^{u} *people* respectively. Consider (10) first: any successful update with a *no*^{u} quantification ensures that the nuclear scope set is empty and anaphora to it is therefore infelicitous; the only possible anaphora in (10) is restrictor set anaphora. Restrictor set anaphora is the only possible one in (11) also, because nuclear scope anaphora yields a contradictory interpretation for (11b): most of the people with a rich uncle that inherit his fortune don't inherit his fortune.

9. **a.** Most^{u} students left the party after 5 a.m. **b.** They$_{u}$ went directly to the beach.
10. **a.** No^{u} student left the party later than 10 pm. **b.** They$_{u}$ had classes early in the morning.
11. **a.** Very few^{u} people with a rich uncle inherit his fortune. **b.** Most of them$_{u}$ don't.

Thus, a selective generalized determiner receives the translation in (12) below, which is in the spirit – but fairly far from the letter – of [24][5].

12. $\mathtt{det}^{u,u' \sqsubseteq u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^{u}(_{\langle u \rangle}(P(u)));\ \mathbf{max}^{u' \sqsubseteq u}(_{\langle u' \rangle}(P'(u')));\ [\mathbf{DET}\{u, u'\}]$

As expected, $\mathtt{det}^{u,u' \sqsubseteq u}$ relates a restrictor dynamic property $P_{\mathbf{et}}$ and a nuclear scope dynamic property $P'_{\mathbf{et}}$. When these dynamic properties are applied to individual dref's, i.e. $P(u)$ and $P'(u')$, we obtain a restrictor DRS $P(u)$ and a nuclear scope DRS $P'(u')$ of type $\mathbf{t}$. Moreover, a generalized determiner introduces two individual dref's: u stores the restrictor set and u' the nuclear scope set.

[4] Throughout the paper, I will ignore anaphora to complement sets, i.e. sets obtained by taking the complement of the nuclear scope relative to the restrictor, e.g. *Very few students were paying attention in class. They were hungover.*

[5] Cf. Definition (4.1), [24]: 149.

These two dref's and the two dynamic properties P and P' are the basic building blocks of the three separate updates in (12).

The first update, namely $\mathbf{max}^u({}_{\langle u\rangle}(P(u)))$, ensures that the restrictor set u is the maximal set of individuals, i.e. $\mathbf{max}^u(\dots)$, such that, when we take each u-individual separately, i.e. ${}_{\langle u\rangle}(\dots)$, this individual satisfies the restrictor dynamic property, i.e. $P(u)$. The second update, namely $\mathbf{max}^{u'\sqsubseteq u}({}_{\langle u'\rangle}(P'(u')))$, ensures that the nuclear scope set u' is obtained in much the same way as the restrictor set u, except for the requirement that u' is the maximal structured subset of u, i.e. $\mathbf{max}^{u'\sqsubseteq u}(\dots)$. Finally, the third update, namely $[\mathbf{DET}\{u, u'\}]$, is a test: we test that the restrictor set u and the nuclear scope set u' stand in the relation denoted by the corresponding static determiner **DET**.

The three distinct updates in (12) are conjoined and, as (13) below shows, dynamic conjunction ";" is interpreted as relation composition. Note the difference between dynamic conjunction, which is an abbreviation, and the official, classical, static conjunction "$\wedge$".

13. $D; D' := \lambda I_{st}.\lambda J_{st}.\ \exists H_{st}(DIH \wedge D'HJ)$, where D, D' are DRS's (type $\mathbf{t}$).

To formally spell out the PCDRT meaning for generalized determiners in (12) above and the meanings for indefinites and pronouns, we need: (i) two operators over plural info states, namely a selective maximization operator $\mathbf{max}^u(\dots)$ and a selective distributivity operator ${}_{\langle u\rangle}(\dots)$ and (ii) a notion of structured inclusion $u' \sqsubseteq u$ that requires the subset to preserve the quantificational dependencies, i.e. the structure, associated with the individuals in the superset.

3.1 Structured Inclusion

Let us start with the notion of structured subset. Recall that plural info states store both values (in the columns of the matrix) and structure (in the rows of the matrix). Requiring one dref u_3 to simply be a value-subset of another dref u_1 relative to an info state I is defined as shown in (14) below; for example, the leftmost u_3 column in the table below satisfies the condition $u_3 \subseteq u_1$ because $u_3I = \{x_1, x_2, x_3\} \subseteq u_1I = \{x_1, x_2, x_3, x_4\}$. Condition (14) requires only *value inclusion* and disregards structure completely. The correlation between the u_1-individuals and the u_2-individuals, i.e. the relation $\{(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)\}$, is lost in going from the u_1-superset to the u_3-subset: as far as u_3 and u_2 are concerned, x_1 is still correlated with y_1, but it is now also correlated with y_3; moreover, x_2 is now correlated with y_4 (not y_2) and x_3 with y_2 (not y_3).

14. $u_3 \subseteq u_1 := \lambda I_{st}.\ u_3I \subseteq u_1I$
15. $u_3 \Subset u_1 := \lambda I_{st}.\ \forall i_s \in I(u_3i = u_1i \vee u_3i = \#)$

Info State I	u_1	u_2	u_3 ($u_3 \subseteq u_1, u_3 \not\Subset u_1$)	u_3 ($u_3 \Subset u_1$)
i_1	x_1	y_1	x_1	x_1
i_2	x_2	y_2	x_3	x_2
i_3	x_3	y_3	x_1	$\#$
i_4	x_4	y_4	x_2	x_4

If we use the notion of value-only subset in (14), we make incorrect empirical predictions. Consider, for example, the discourse in (16) below, where u_1 stores the set of conventions[6] and u_2

[6] In the case of a successful *every*-quantification, the restrictor and nuclear scope sets are identical with respect to both value and structure, so we can safely identify them.

stores the set of corresponding girls; furthermore, assume that *every*^{u_1} *convention* takes scope over a^{u_2} *girl* and that the correlation between u_1-conventions and courted u_2-girls is the one represented in the table above. Intuitively, the adverb *usually* in (16b) is anaphoric to the set of conventions introduced in (16a) and (16b) is interpreted as asserting that, at most conventions, the girl courted by Harvey *at that convention* comes to the banquet with him. The leftmost dref u_3 in the table above does store most u_1-conventions (three out of four), but it does not preserve the correlation between u_1-conventions and u_2-girls established in sentence (16a).

We obtain similarly incorrect results for donkey sentences like the one in (17) below: the restrictor of the quantification introduces a dependency between all the donkey-owning u_1-farmers and the u_2-donkeys that they own; the nuclear scope set u_3 needs to contain most u_1-farmers, but in such a way that the correlated u_2-donkeys remain the same. That is, the nuclear scope set contains a *most*-subset of donkey-owning farmers that beat *their respective donkey(s)*. The notion of value-only inclusion in (14) is, yet again, inadequate.

16. **a.** Harvey courts a^{u_2} girl at every^{u_1} convention. **b.** She$_{u_2}$ usually$^{u_3 \subseteq u_1}$ comes to the banquet with him.
17. Most$^{u_1, u_3 \subseteq u_1}$ farmers who own a^{u_2} donkey beat it$_{u_2}$.

Thus, to capture the intra- and cross-sentential interaction between anaphora and quantification, we need the notion of *structured inclusion* defined in (15) above, whereby we go from a superset to a subset by discarding rows in the matrix. We are therefore guaranteed that the subset will contain *only* the dependencies associated with the superset (but not necessarily *all* dependencies – see below). To implement this, I follow [24] and introduce a dummy/exception individual # that is used as a tag for the cells in the matrix that should be discarded in order to obtain a structured subset u_3 of a superset u_1 – as shown by the rightmost u_3 column in the table above.

Unlike [24], I do not take the introduction of the dummy individual # to require making the underlying logic partial, i.e. I will not take a lexical relation that has # as one of its arguments, e.g. $girl(\#)$ or $courted\_at(\#, x_1)$, to be undefined. I will just require the dummy individual # to make any lexical relation false[7]. This allows us to keep the underlying type logic classical while making sure that we do not accidentally introduce # and inadvertently discard a cell when we evaluate another lexical relation later on. Thus, lexical relations (i.e. atomic conditions) are interpreted *distributively* relative to the non-dummy sub-state of the input plural info state I, as shown in (19) below.

18. $I_{u_1 \neq \#, \ldots, u_n \neq \#} := \{i_s \in I : u_1 i \neq \# \wedge \ldots \wedge u_n i \neq \#\}$
19. $R\{u_1, \ldots, u_n\} := \lambda I_{st}.\ I_{u_1 \neq \#, \ldots, u_n \neq \#} \neq \emptyset \wedge \forall i_s \in I_{u_1 \neq \#, \ldots, u_n \neq \#}(R(u_1 i, \ldots, u_n i))$

The notion of structured inclusion $\Subset$ in (15) above ensures that the subset inherits *only* the superset structure – but we also need it to inherit *all* the superset structure, which we achieve by means of the second conjunct in definition (20) below. This conjunct is needed (among others) to account for the donkey sentence in (7) above, which is interpreted as talking about *every* slave owned by any given person, i.e. the nuclear scope set, which is a *most*-subset of the restrictor set, needs to inherit *all* the superset structure (each slave owner in the nuclear scope set needs to be associated with *every* slave that s/he owned).

20. $u' \sqsubseteq u := \lambda I_{st}.\ (u' \Subset u)I \wedge \forall i_s \in I(ui \in u' I_{u' \neq \#} \rightarrow ui = u'i)$

[7] We ensure that any lexical relation R of arity n (i.e. of type $e^n t$, defined recursively as in [16]: 157-158, i.e. as $e^0 t := t$ and $e^{m+1} t := e(e^m t)$) yields falsity whenever # is one of its arguments by letting $R \subseteq (D_e^{\mathfrak{M}} \setminus \{\#\})^n$.

3.2 Maximization and Distributivity

We turn now to the maximization and distributivity operators $\mathbf{max}^u$ and $\mathbf{dist}_u$, which are defined in the spirit – but not the letter – of the corresponding operators in [24]. Selective maximization and selective distributivity together enable us to dynamize λ-abstraction over both values (individuals, i.e. quantifier domains) and structure (quantificational dependencies); that is, $\mathbf{max}^u$ and $\mathbf{dist}_u$ enable us to extract and store the restrictor and nuclear scope structured sets needed to define dynamic generalized quantification.

Consider the definition of $\mathbf{max}^u$ in (21) below first: the first conjunct introduces u as a new dref, i.e. $[u]$, and makes sure that each individual in uJ satisfies D, i.e. we store *only* individuals that satisfy D. The second conjunct enforces the maximality requirement: any other set uK obtained by a similar procedure, i.e. any other set of individuals that satisfies D, is included in uJ – that is, uJ stores *all* individuals that satisfy D.

21. $\mathbf{max}^u(D) := \lambda I_{st}.\lambda J_{st}.\ ([u];D)IJ \wedge \forall K_{st}(([u];D)IK \rightarrow uK_{u\neq\#} \subseteq uJ_{u\neq\#})$
22. $\mathbf{max}^{u'\sqsubseteq u}(D) := \mathbf{max}^{u'}([u' \sqsubseteq u];D)$
23. $I_{u=x} = \{i_s \in I : ui = x\}$
24. $\mathbf{dist}_u(D) := \lambda I_{st}.\lambda J_{st}.\ uI = uJ \wedge \forall x_e \in uI(DI_{u=x}J_{u=x})$ [8]

Definition (24) states that updating an info state I with a DRS D *distributively* over a dref u means: (*i*) generating the u-partition of I, i.e. $\{I_{u=x} : x \in uI\}$, (*ii*) updating each cell $I_{u=x}$ in the partition with the DRS D and (*iii*) taking the union of the resulting output info states. The first conjunct in (24) is required to ensure that there is a bijection between the partition induced by the dref u over the input state I and the one induced over the output state J; without this requirement, we could introduce arbitrary new values for u in the output state J, i.e. arbitrary new partition cells[9]. The second conjunct is the one that actually defines the distributive update: the DRS D relates every partition cell in the input state I to the corresponding partition cell in the output state J.

3.3 Generalized Quantifiers and Indefinites

The PCDRT meanings for generalized determiners and weak/strong indefinites are provided in (28), (29) and (30) below[10].

25. ${}_u(D) := \lambda I_{st}.\lambda J_{st}.\ I_{u=\#} = J_{u=\#} \wedge I_{u\neq\#} \neq \emptyset \wedge \mathbf{dist}_u(D)I_{u\neq\#}J_{u\neq\#}$
26. ${}_{\langle u\rangle}(D) := \lambda I_{st}.\lambda J_{st}.\ I_{u=\#} = J_{u=\#} \wedge (I_{u\neq\#} = \emptyset \rightarrow I = J) \wedge (I_{u\neq\#} \neq \emptyset \rightarrow \mathbf{dist}_u(D)I_{u\neq\#}J_{u\neq\#})$
27. $\mathbf{DET}\{u,u'\} := \lambda I_{st}.\ \mathbf{DET}(uI_{u\neq\#}, u'I_{u'\neq\#})$, where $\mathbf{DET}$ is a static det.
28. $\mathtt{det}^{u,u'\sqsubseteq u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^u({}_{\langle u\rangle}(P(u)));\ \mathbf{max}^{u'\sqsubseteq u}({}_{\langle u'\rangle}(P'(u')));\ [\mathbf{DET}\{u,u'\}]$
29. $\mathtt{a}^{\mathbf{wk}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [u];\ {}_u(P(u));\ {}_u(P'(u))$
30. $\mathtt{a}^{\mathbf{str}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^u({}_u(P(u));\ {}_u(P'(u)))$

[8] In general, $\mathbf{dist}_{u_1,\ldots,u_n}(D)$ is defined as: $\lambda I_{st}.\lambda J_{st}.\ \forall x_1 \ldots \forall x_n(I_{u_1=x_1,\ldots,u_n=x_n} \neq \emptyset \leftrightarrow J_{u_1=x_1,\ldots,u_n=x_n} \neq \emptyset) \wedge \forall x_1 \ldots \forall x_n(I_{u_1=x_1,\ldots,u_n=x_n} \neq \emptyset \rightarrow DI_{u_1=x_1,\ldots,u_n=x_n}J_{u_1=x_1,\ldots,u_n=x_n})$.

[9] [17]: 87 was the first to observe that we need to add the first conjunct in (24) to the original definition of distributivity in (18), [24]: 145.

[10] See [2] for the justification of the account of weak/strong donkey ambiguities in terms of weak/strong indefinite articles and see [4] for a detailed investigation of various kinds of indefinites within a related dynamic framework.

The **max**-based definition of generalized quantification correctly predicts that anaphora to restrictor/nuclear scope sets is always anaphora to *maximal* sets, i.e. E-type anaphora[11]. That is, the maximality of anaphora to quantifier sets is an automatic consequence of the fact that we independently need **max**-operators to formulate truth-conditionally correct dynamic meanings for quantifiers. This is one of the major results in [24], preserved in PCDRT.

The existential commitment associated with dref introduction is built into (*i*) the definition of lexical relations in (19) above (i.e. $I_{u_1 \neq \#, \ldots, u_n \neq \#} \neq \emptyset$) and (*ii*) the definition of the operator $_u(\ldots)$ in (25) above (i.e. $I_{u \neq \#} \neq \emptyset$)[12].

There is, however, no such existential commitment in the definition of generalized determiners $\mathbf{det}^{u,u' \sqsubseteq u}$, which employs the distributivity operator $_{\langle u \rangle}(\ldots)$ defined in (26) above. The use of $_{\langle u \rangle}(\ldots)$ enables us to capture the meaning of both upward and downward monotonic quantifiers by means of the same definition. The problem posed by downward monotonic quantifiers is that their nuclear scope set can or has to be empty; for example, after a successful update with a $\mathbf{no}^{u,u' \sqsubseteq u}$ quantification, the nuclear scope set u' is necessarily empty, i.e. the dref u' will always store only the dummy individual # relative to the output info state; this, in turn, entails that no lexical relation in the nuclear scope DRS that has u' as an argument can be satisfied. The second conjunct in the definition of $_{\langle u \rangle}(\ldots)$, i.e. $I_{u \neq \#} = \emptyset \rightarrow I = J$, enables us to resolve the conflict between the emptiness requirement enforced by a **no**-quantification and the non-emptiness requirement enforced by lexical relations[13],[14].

3.4 Singular Number Morphology

Let us turn now to the last component needed for the account of discourses (1) and (2), namely the representation of singular pronouns. Their PCDRT translation, provided in (32) below, has the expected Montagovian form: it is the distributive type-lift of the dref u they are anaphoric to, with the addition of the condition $\mathbf{unique}\{u\}$. The condition is contributed by singular number morphology and requires uniqueness of the non-dummy value of the dref u relative to the current plural info state I. In contrast, plural pronouns do not require uniqueness, as shown in (33) below. The meanings for singular and plural anaphoric definite articles in (34) and (35) below (we need them to interpret the anaphoric DP *the girl* in (2c) above among others) exhibit the same kind of unique/non-unique contrast as the meanings for singular and plural pronouns.

The uniqueness enforced by the condition $\mathbf{unique}\{u\}$ is *weak* in the sense that it is relativized to the current plural info state. However, we can require *strong* uniqueness, i.e. uniqueness relative

[11] Recall the Evans examples *Few senators admire Kennedy and they are very junior.* and *Harry bought some sheep. Bill vaccinated them.* and (9), (10) and (11) above.

[12] We need these non-emptiness requirements because the pair $(\emptyset_{st}, \emptyset_{st})$ belongs, on the one hand, to the denotation of $[u]$ for any dref u (see (5) above) and, on the other hand, to the denotation of $\mathbf{dist}_u(D)$ for any dref u and DRS D (see (24) above).

[13] Even if definition (28) allows for empty restrictor and nuclear scope sets, we can still capture the fact that subsequent anaphora to such empty sets is infelicitous (e.g. anaphora to the nuclear scope sets in (10) and (11) above) because pronominal meanings are defined in terms of the operator $_u(\ldots)$ – see, for example, the PCDRT translations for **she** and **they** in (32) and (33) below.

[14] The fact that the second conjunct in (26) requires the identity of the input and output states I and J correctly predicts that anaphora to both empty restrictor/nuclear scope sets and indefinites in restrictor/nuclear scope DRS's associated with such empty sets is infelicitous. For example, the nuclear scope DRS of a successful $\mathbf{no}^{u,u' \sqsubseteq u}$-quantification, i.e. $\mathbf{max}^{u' \sqsubseteq u}(_{\langle u' \rangle}(P'(u')))$, will always be a test; hence, we correctly predict that anaphora to any indefinites in the nuclear scope of a **no**-quantification is infelicitous, e.g. *Harvey courts* $a^{u''}$ *girl at* $no^{u,u' \sqsubseteq u}$ *convention.* $\#She_{u''}/They_{u''}$ *is/are very pretty.*

to the entire model, by combining the $\mathbf{max}^u$ operator and the $\mathbf{unique}\{u\}$ condition, as shown by the Russellian, non-anaphoric meaning for definite descriptions provided in (36) below. This alternative meaning for definite articles, which requires existence and strong uniqueness, is needed to interpret the DP *the banquet* in (2b) above.

31. $\mathbf{unique}\{u\} := \lambda I_{st}.\ I_{u\neq\#} \neq \emptyset \wedge \forall i_s \in I_{u\neq\#} \forall i'_s \in I_{u\neq\#}(ui = ui')$
32. $\mathit{she}_u \rightsquigarrow \lambda P_{\mathbf{et}}.\ [\mathbf{unique}\{u\}];\ _u(P(u))$
33. $\mathit{they}_u \rightsquigarrow \lambda P_{\mathbf{et}}.\ _u(P(u))$
34. $\mathit{the\_sg}_u \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [\mathbf{unique}\{u\}];\ _u(P(u));\ _u(P'(u))$
35. $\mathit{the\_pl}_u \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ _u(P(u));\ _u(P'(u))$
36. $\mathit{the\_sg}^u \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^u(_u(P(u)));\ [\mathbf{unique}\{u\}];\ _u(P'(u))$

The PCDRT translation for proper names and the definitions of dynamic negation and truth are provided in (37), (38) and (39) below. I take the default context of interpretation for all discourses, i.e. the default input info state relative to which a DRS is true/false, to be the singleton info state $\{i_\#\}$, where $i_\#$ is the 'assignment' that stores the dummy individual # relative to all individual dref's. Finally, the abbreviations in (40) and (41) below and the equivalences in (42) and (43) enable us to simplify – and, therefore, enhance the readability of – some very common PCDRT representations.

37. $\mathit{Harvey}^u \rightsquigarrow \lambda P_{\mathbf{et}}.\ [u|u \Subset Harvey];\ _u(P(u))$,
    where $Harvey := \lambda i_s.\ harvey_e$ (i.e. $Harvey$ is a 'rigid' individual dref).
38. $\sim D := \lambda I_{st}.\ I \neq \emptyset\ \wedge\ \forall H_{st} \neq \emptyset(H \subseteq I \rightarrow \neg\exists K_{st}(DHK))$
39. A DRS D of type $\mathbf{t}$ is *true* with respect to an input info state I_{st} iff $\exists J_{st}(DIJ)$.
40. $_u(C) := \lambda I_{st}.\ I_{u\neq\#} \neq \emptyset \wedge \forall x_e \in uI_{u\neq\#}(CI_{u=x})$, where C is a condition (type $(st)t$).
41. $_u(u_1, \ldots, u_n) := \lambda I_{st}.\lambda J_{st}.\ I_{u=\#} = J_{u=\#} \wedge I_{u\neq\#}[u_1, \ldots, u_n]J_{u\neq\#}$,
    where $u \notin \{u_1, \ldots, u_n\}$ and $[u_1, \ldots, u_n] := [u_1]; \ldots ; [u_n]$.
42. $_u([C_1, \ldots, C_m]) = [_u(C_1), \ldots, {}_u(C_m)]$
43. $_u([u_1, \ldots, u_n\,|\,C_1, \ldots, C_m]) = [_u(u_1, \ldots, u_n)\,|\,{}_u(C_1), \ldots, {}_u(C_m)]$

4 Quantificational Subordination in PCDRT

This section presents the PCDRT analysis of discourses (1) and (2). We start with the two possible quantifier scopings for the discourse-initial sentence (1a/2a). For simplicity, I will assume that the two scopings are due to the two different lexical entries for the ditransitive verb *court_at*, provided in (44) and (45) below[15]: *court_at*^{1} assigns the indefinite *a girl* wide scope relative to *every convention*, while *court_at*^{2} assigns it narrow scope. I assume that the basic syntactic structure of the sentence is the one given in (46).

44. $\mathit{court\_at}^1 \rightsquigarrow \lambda Q'_{(\mathbf{et})\mathbf{t}}.\lambda Q''_{(\mathbf{et})\mathbf{t}}.\lambda v_{\mathbf{e}}.\ Q'(\lambda v'_{\mathbf{e}}.\ Q''(\lambda v''_{\mathbf{e}}.\ [court\_at\{v, v', v''\}]))$
45. $\mathit{court\_at}^2 \rightsquigarrow \lambda Q'_{(\mathbf{et})\mathbf{t}}.\lambda Q''_{(\mathbf{et})\mathbf{t}}.\lambda v_{\mathbf{e}}.\ Q''(\lambda v''_{\mathbf{e}}.\ Q'(\lambda v'_{\mathbf{e}}.\ [court\_at\{v, v', v''\}]))$
46. *Harvey [[court_at*$^{1/2}$ *[a girl]] [every convention]]*

[15] But it should be clear that PCDRT is compatible with any of the quantifier scoping mechanisms proposed in the literature; for a version of PCDRT that incorporates Quantifying-In / Quantifier Raising, see [2].

Turning to the meaning of the quantifier *every convention*, note that we can safely identify the restrictor and nuclear scope dref's u and u' of any $\mathit{every}^{u,u' \sqsubseteq u}$-quantification: the definition in (28) above entails that, if J is an arbitrary output state of a successful $\mathit{every}^{u,u' \sqsubseteq u}$-quantification, u and u' have to be identical with respect to both value and structure, i.e. $\forall j_s \in J(uj = u'j)$. We can therefore assume that *every* contributes only one dref, as shown in (47) below. I will also assume that the restrictor set of the every^{u_1}-quantification is non-empty, so we can safely replace the operator $_{\langle u_1 \rangle}(\dots)$ with the operator $_{u_1}(\dots)$. The PCDRT representations of the two quantifier scopings for sentence (1a/2a) are provided in (50) and (51) below. For simplicity, I take the translation of the proper name *Harvey* to be $\lambda P_{\mathbf{et}}.\ P(Harvey)$ instead of the more complex one in (37) above[16].

47. $\mathit{every}^{u_1} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^{u_1}(_{u_1}(P(u_1)));\ _{u_1}(P'(u_1))$
48. $\mathit{every}^{u_1}\ \mathit{convention} \rightsquigarrow \lambda P_{\mathbf{et}}.\ \mathbf{max}^{u_1}([convention\{u_1\}]);\ _{u_1}(P(u_1))$
49. $a^{\mathbf{wk}:u_2}\ \mathit{girl} \rightsquigarrow \lambda P_{\mathbf{et}}.\ [u_2 \,|\, girl\{u_2\}];\ _{u_2}(P(u_2))$
50. $a^{\mathbf{wk}:u_2}\ \mathit{girl} >> \mathit{every}^{u_1}\ \mathit{convention} \rightsquigarrow$
$[u_2 \,|\, girl\{u_2\}];\ _{u_2}(\mathbf{max}^{u_1}([convention\{u_1\}]));\ [_{u_2}(court\_at\{Harvey, u_2, u_1\})]$
51. $\mathit{every}^{u_1}\ \mathit{convention} >> a^{\mathbf{wk}:u_2}\ \mathit{girl} \rightsquigarrow$
$\mathbf{max}^{u_1}([convention\{u_1\}]);\ [_{u_1}(u_2) \,|\, _{u_1}(girl\{u_2\}),\ _{u_1}(court\_at\{Harvey, u_2, u_1\})]$

The representation in (50) updates the default input info state $\{i_\#\}$ as follows. First, we introduce some non-empty (i.e. non-dummy) set of individuals relative to the dref u_2. Then, we test that each u_2-individual is a girl. Then, relative to each u_2-individual, we introduce the set of all conventions and store it in the dref u_1. Finally, we test that, for each u_2-girl and for each of the corresponding u_1-conventions (i.e., in this case: for every convention), Harvey courted her at the convention. The output info state obtained after updating with (50) contains a non-empty set of u_2-girls that where courted by Harvey at every convention and, relative to each u_2-girl, u_1 stores the set of all conventions.

The representation in (51) updates the default input info state $\{i_\#\}$ as follows. First, we introduce the set of all conventions relative to the dref u_1. Then, for each u_1-convention, we introduce a u_2-set of individuals. Finally we test that, for each u_1-convention, each of the corresponding u_2-individuals are girls and are such that Harvey courted them at the convention under consideration. The output info state obtained after updating with (51) stores the set of all conventions under the dref u_1 and, relative to each u_1-convention, the dref u_2 stores a non-empty set of girls (possibly different from convention to convention) that Harvey courted at that particular convention.

We can now see how sentence (1b) – in particular, the singular number morphology on the pronoun she_{u_2} – forces the 'wide-scope indefinite' reading: the condition $\mathbf{unique}\{u_2\}$ (see (52) and (53) below) effectively conflates the two scopings by requiring the set of u_2-girls obtained after updating with (50) or (51) to be a singleton. This requirement leaves the truth-conditions derived on the basis of (50) untouched, but makes the truth-conditions associated with (51) strictly stronger.

52. $\mathit{she}_{u_2} \rightsquigarrow \lambda P_{\mathbf{et}}.\ [\mathbf{unique}\{u_2\}];\ _{u_2}(P(u_2))$
53. $[\mathbf{unique}\{u_2\}, very\_pretty\{u_2\}]$

In contrast, sentence (2b) contains the adverb of quantification always_{u_1}, which can take scope above or below the singular pronoun she_{u_2}. In the former case, the u_2-uniqueness requirement is

[16] The reader can check that this simplification does not affect the PCDRT truth-conditions for the discourses under consideration.

weakened (i.e., in a sense, neutralized) by being relativized to u_1-conventions. As shown in (54) below, I take the meaning of always_{u_1} to be a universal quantification over an anaphorically retrieved restrictor, i.e. over the nuclear scope set introduced by the quantifier every^{u_1} *convention* in the preceding sentence. Since *always* is basically interpreted as *every* (modulo the anaphorically retrieved restrictor), its translation is parallel to the translation for *every* in (47) above. The general format for the interpretation of quantifiers that anaphorically retrieve their restrictor set is provided in (55).

54. $\mathit{always}_{u_1} \rightsquigarrow \lambda P_{\mathbf{et}}.\ _{u_1}(P(u_1))$
55. $\mathit{det}_u^{u' \sqsubseteq u} \rightsquigarrow \lambda P_{\mathbf{et}}.\ \mathbf{max}^{u' \sqsubseteq u}(_{\langle u' \rangle}(P(u')));\ [\mathbf{DET}\{u, u'\}]$

The definite description *the banquet* in (2b) is intuitively a Russellian definite description (see (36) above), which contributes existence and a relativized (i.e. anaphoric) form of uniqueness: we are talking about a *unique* banquet *per convention*. For simplicity, however, I will assume that sentence (2b) contributes a transitive predication of the form *come_with_Harvey_to_the_banquet_of* relating girls and conventions[17], which, as shown in (56) and (57) below, can be translated in two different ways corresponding to the two possible relative scopes of she_{u_2} and always_{u_1} (that is, the scoping technique is the same as in (44) and (45) above). The translation in (56) gives the pronoun she_{u_2} wide scope over the adverb always_{u_1}, while the translation in (57) gives the pronoun narrow scope relative to the adverb. The corresponding PCDRT representations, obtained on the basis of the syntactic structure in (58), are provided in (59) and (60) below.

56. $\mathit{come\_to\_banquet\_of}^{1} \rightsquigarrow \lambda Q_{(\mathbf{et})\mathbf{t}}.\lambda Q'_{(\mathbf{et})\mathbf{t}}.\ Q'(\lambda v'_{\mathbf{e}}.\ Q(\lambda v_{\mathbf{e}}.\ [c.t.b.of\{v', v\}]))$
57. $\mathit{come\_to\_banquet\_of}^{2} \rightsquigarrow \lambda Q_{(\mathbf{et})\mathbf{t}}.\lambda Q'_{(\mathbf{et})\mathbf{t}}.\ Q(\lambda v_{\mathbf{e}}.\ Q'(\lambda v'_{\mathbf{e}}.\ [c.t.b.of\{v', v\}]))$
58. *she [[always] come_to_banquet_of*$^{1/2}$*]*
59. $\mathit{she}_{u_2} >> \mathit{always}_{u_1} \rightsquigarrow [\mathbf{unique}\{u_2\},\ _{u_2}(c.t.b.of\{u_2, u_1\})]$
60. $\mathit{always}_{u_1} >> \mathit{she}_{u_2} \rightsquigarrow [_{u_1}(\mathbf{unique}\{u_2\}),\ _{u_1}(c.t.b.of\{u_2, u_1\})]$

Thus, there are two possible PCDRT representations for sentence (2a) and two possible representations for sentence (2b). Out of the four combinations, three end up requiring the indefinite $a^{\mathbf{wk}:u_2}$ *girl* to have wide scope relative to every^{u_1} *convention*. The fourth combination (51+60), provided in (61) below, encodes the 'narrow-scope indefinite' reading that is intuitively available for discourse (2), but not for (1). The PCDRT representation in (61) updates the default input info state $\{i_{\#}\}$ as follows: first, we introduce the set of all conventions relative to the dref u_1, followed by the introduction of a non-empty set of u_2-individuals relative to each u_1-convention; the remainder of the representation tests that, for each u_1-convention, the corresponding u_2-set is a singleton set whose sole member is a girl that is courted by Harvey at the u_1-convention under consideration and that comes with him to the banquet of that convention.

61. $\mathbf{max}^{u_1}([convention\{u_1\}]);$
$[_{u_1}(u_2) \mid _{u_1}(girl\{u_2\}),\ _{u_1}(court\_at\{Harvey, u_2, u_1\}),\ _{u_1}(\mathbf{unique}\{u_2\}),\ _{u_1}(c.t.b.of\{u_2, u_1\})]$

[17] The relevant PCDRT translation for the definite article is $\mathit{the}_{u_1}^{u_3} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ _{u_1}(\mathbf{max}^{u_3}(_{u_3}(P(u_3)));$ $[\mathbf{unique}\{u_3\}];\ _{u_3}(P'(u_3)))$, which, together with a relational interpretation of the noun *banquet* as *banquet of* $\mathit{it}_{u_1} \rightsquigarrow \lambda v_{\mathbf{e}}.\ [banquet\{v\}, of\{v, u_1\}]$, yields the following PCDRT translation for our Russellian definite description with relativized uniqueness: $\mathit{the}_{u_1}^{u_3}$ *banquet of* $\mathit{it}_{u_1} \rightsquigarrow \lambda P_{\mathbf{et}}.\ _{u_1}(\mathbf{max}^{u_3}([banquet\{u_3\}, of\{u_3, u_1\}]);$ $[\mathbf{unique}\{u_3\}];\ _{u_3}(P(u_3)))$.

Summarizing, PCDRT enables us to formulate a compositional dynamic account of the intra- and cross-sentential interaction between generalized quantifiers, anaphora and number morphology exhibited by the quantificational subordination discourses in (1) and (2) above. The main proposal is that plural info states together with a suitable dynamic reformulation of the independently motivated denotations for generalized determiners and number morphology in static Montague semantics enable us to account for quantificational subordination in terms of structured anaphora to quantifier domains.

5 A Parallel Account of Modal Subordination

In this section, I will briefly indicate how PCDRT can be extended to give a compositional account of modal subordination discourse like the one in (62) below (based on [21]).

62. **a.** A^u wolf might come in. **b.** It$_u$ would attack Harvey first.

Under its most salient interpretation, (62) asserts that, for all the speaker knows, it is possible that a wolf comes in. Moreover, in *any* such epistemic possibility, the wolf attacks Harvey first. Discourse (62) is parallel to discourse (2) above: the interaction between the indefinite $\mathtt{a}^u$ `wolf` and the modal `might` on the one hand and the singular pronoun $\mathtt{it}_u$ and the modal `would` on the other hand is parallel to the interaction between $\mathtt{a}^u$ `girl-every convention` and $\mathtt{she}_u$`-always`.

The addition of another basic type $\mathbf{w}$ for possible worlds together with dref's p, p' etc. of type $s\mathbf{w}$ is almost everything that is needed to account for discourse (62). In the resulting Intensional PCDRT (IP-CDRT) system, the dref's p, p' etc. store sets of possible worlds, i.e. *propositions*, relative to a plural info state, e.g. $pI := \{p_{s\mathbf{w}}i_s : i_s \in I_{st}\}$, i.e. pI is the image of the set of 'assignments' I under the function p. The basic IP-CDRT system is very much parallel to the PCDRT system introduced in the previous sections, so I provide only some of the relevant definitions. In particular, the definition of structured inclusion for sets of worlds in (65) below employs a dummy/exception world $\#_{\mathbf{w}}$[18] which makes every lexical relation false just as the dummy/exception individual $\#_e$ does.

63. $R_p\{u_1, \ldots, u_n\} := \lambda I_{st}.\ I_{p\neq\#, u_1\neq\#, \ldots, u_n\neq\#} \neq \emptyset \wedge \forall i_s \in I_{p\neq\#, u_1\neq\#, \ldots, u_n\neq\#}(R_{pi}(u_1 i, \ldots, u_n i))$[19]
64. $[p] := \lambda I_{st}.\lambda J_{st}.\ \forall i_s \in I(\exists j_s \in J(i[p]j)) \wedge \forall j_s \in J(\exists i_s \in I(i[p]j))$
65. $p' \sqsubseteq p := \lambda I_{st}.\ (p' \Subset p)I \wedge \forall i_s \in I(pi \in p'I_{p'\neq\#} \rightarrow pi = p'i)$,
where $p' \Subset p := \lambda I_{st}.\ \forall i_s \in I(p'i = pi \vee p'i = \#)$.

In an intensional Fregean/Montagovian framework, the compositional aspect of interpretation is largely determined by the types for the extensions of the 'saturated' expressions, i.e. names and sentences, plus the type that enables us to build intensions out of these extensions. Let us abbreviate them as $\mathbf{e}$, $\mathbf{t}$ and $\mathbf{s}$ respectively. We preserve the dynamic types that PCDRT assigns to the 'meta-types' $\mathbf{e}$ and $\mathbf{t}$, i.e. $\mathbf{t} := (st)((st)t)$ and $\mathbf{e} := se$; predictably, IP-CDRT uses possible-word dref's to build intensions, i.e. $\mathbf{s} := s\mathbf{w}$. Just as generalized determiners in PCDRT relate dynamic properties P, P' etc. of type $\mathbf{et}$ (see (28) above), modal verbs relate dynamic propositions $\mathbb{P}$, $\mathbb{P}'$

[18] We can take the dummy world $\#_{\mathbf{w}}$ to be the world where no individual whatsoever exists, hence all the lexical relations are false because a relation between certain individuals obtains at a particular world w only if those individuals exist in w.

[19] The definition of atomic conditions in (63) assumes static lexical relations $R_w(x_1, \ldots, x_n)$ of the expected intensional type $e^n(\mathbf{w}t)$, where $e^n\tau$ (for any type τ) is defined as: $e^0\tau := \tau$ and $e^{m+1}\tau := e(e^m\tau)$.

etc. of type **st**, as shown in (66) below. Moreover, just as a pronoun anaphorically retrieves an individual dref and makes sure that a dynamic property holds of that dref (see the meaning for *she* in (32) above), the indicative verbal mood anaphorically retrieves p^*, which is the designated dref for the actual world, and makes sure that a dynamic proposition holds of p^*, as shown in (67) below.

Finally, just as the quantifier *always* in (2b) is anaphoric to the nuclear scope set introduced by *every convention* in (2a), the modal quantifier *would* in (62b) is anaphoric to the nuclear scope set introduced by *might* in (62a). The general format for the translation of anaphoric modal quantifiers is provided in (68) below (cf. the translation of anaphoric determiners in (55) above).

66. if^{p} + $\mathit{modal}^{p'\sqsubseteq p}_{\mu,\omega} \rightsquigarrow$
$\lambda\mathbb{P}_{\mathbf{st}}.\lambda\mathbb{P}'_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^{p}({}_{\langle p\rangle}(\mathbb{P}(p)));\ \mathbf{max}^{p'\sqsubseteq p}({}_{\langle p'\rangle}(\mathbb{P}'(p')));\ [\mathbf{MODAL}_{q,\mu,\omega}\{p,p'\}]$
67. $\mathit{indicative}_{p^*} \rightsquigarrow \lambda\mathbb{P}_{\mathbf{st}}.\ [\mathbf{unique}\{p^*\}];\ {}_{p^*}(\mathbb{P}(p^*))$
68. $\mathit{modal}^{p'\sqsubseteq p}_{\mu,\omega} \rightsquigarrow \lambda\mathbb{P}_{\mathbf{st}}.\lambda q_{\mathbf{s}}.\ \mathbf{max}^{p'\sqsubseteq p}({}_{\langle p'\rangle}(\mathbb{P}(p')));\ [\mathbf{MODAL}_{q,\mu,\omega}\{p,p'\}]$

This concludes our brief survey of IP-CDRT. It is hopefully clear by now that IP-CDRT enables us to provide an analysis of the modal subordination discourse in (62) above that is parallel to the analysis of the quantificational subordination discourse in (2); see [2] for a detailed account showing that the parallels between anaphora and quantification in the individual and modal domains are systematically captured in IP-CDRT.

6 Comparison with Previous Approaches

PCDRT differs from most previous dynamic approaches in at least three respects. The first difference is conceptual: PCDRT captures the idea that reference to structure is as important as reference to value and that the two should be treated in parallel. This is primarily encoded in the definition of new dref introduction in (5) above, which differs from the corresponding definitions in [24], [11] and [17] (among others) with respect to the treatment of discourse reference to structure.

The second difference is empirical: the motivation for plural information states is provided by several distinct kinds of phenomena, including singular intra- and cross-sentential individual-level anaphora and modal anaphora and subordination, in contrast to the previous literature (e.g. [24], [11] and [17]), which relies mostly on plural individual-level anaphora (but see [25] for an analysis of questions and modal subordination in a related dynamic system). Consequently, the empirical coverage of (Intensional) PCDRT is correspondingly broader.

Finally, from a formal point of view, PCDRT accomplishes two non-trivial goals for the first time. On the one hand, it is not obvious how to recast van den Berg's Dynamic Plural Logic in classical type logic, given that the former logic is partial and conflates discourse-level plurality (i.e. the use of plural information states) and domain-level plurality (i.e. non-atomic individuals)[20].

On the other hand, Intensional PCDRT – which builds on and unifies [14]/[10], [16], [24] and [23] – is, to my knowledge, the first dynamic framework that systematically and explicitly captures the anaphoric and quantificational parallels between the individual and modal domains while, at the same time, keeping the underlying logic classical and preserving the Montagovian approach to compositionality[21].

[20] See [3] for more discussion of discourse-level vs. domain-level plurality.
[21] See [2] for a more detailed comparison with alternative accounts, e.g. Skolem function based approaches and dynamic approaches that employ discourse relations (e.g. [26]).

References

1. Bittner, M. (2001). Topical Referents for Individuals and Possibilities. In *Proceedings of SALT 11*, Hastings, R. et al (eds.), CLC, Cornell University, 36-55.
2. Brasoveanu, A. (2007a). *Structured Nominal and Modal Reference*. PhD dissertation, Rutgers University.
3. Brasoveanu, A. (2007b). Donkey Pluralities: Plural Information States vs. Non-atomic Individuals. To appear in the *Proceedings of Sinn und Bedeutung 11*.
4. Farkas, D.F. (2002). Varieties of Indefinites. In the *Proceedings of SALT 12*, Jackson, B. (ed.), CLC, Cornell University, 59-84.
5. Frank, A. (1996). *Context Dependence in Modal Constructions*. PhD dissertation, Stuttgart University.
6. Gallin, D. (1975). *Intensional and Higher-Order Modal Logic with Applications to Montague Semantics*. North-Holland Mathematics Studies.
7. Geurts, B. (1999). *Presuppositions and Pronouns*, Amsterdam, Elsevier.
8. Kamp, H. (1981). A theory of truth and semantic representation. In *Formal Methods in the Study of Language. Part 1*, Groenendijk, J., T. Janssen & M. Stokhof (eds.), Mathematical Center, Amsterdam, 277-322.
9. Karttunen, L. (1976). Discourse Referents. In *Syntax and Semantics Vol. 7*, J.D. McCawley (ed.), New York: Academic Press, 363-385.
10. Kratzer, A. (1981). The Notional Category of Modality. In *Words, Worlds, and Contexts*, Eikmeyer, H.J. & H. Rieser (eds.), Walter de Gruyter, Berlin, 38-74.
11. Krifka, M. (1996). Parametric Sum Individuals for Plural Anaphora. In *Linguistics and Philosophy* 19, 555-598.
12. Heim, I. (1982).*The Semantics of Definite and Indefinite Noun Phrases*. PhD dissertation, UMass Amherst.
13. Heim, I. (1990). E-Type Pronouns and Donkey Anaphora. In *Linguistics and Philosophy* 13, 137-177.
14. Lewis, D. (1973). *Counterfactuals*. Harvard University Press.
15. Lewis, D. (1975). Adverbs of Quantification. In *Formal Semantics of Natural Language*, E. Keenan (ed.), Cambridge: Cambridge University Press, 3-15.
16. Muskens, R. (1996). Combining Montague Semantics and Discourse Representation. In *Linguistics and Philosophy* 19, 143-186.
17. Nouwen, R. (2003). *Plural Pronominal Anaphora in Context*. PhD Dissertation, University of Utrecht.
18. Partee, B. (1973). Some Structural Analogies between Tenses and Pronouns in English. In *Journal of Philosophy* 70, 601-609.
19. Partee, B. (1984). Nominal and Temporal Anaphora. In *Linguistics and Philosophy* 7, 243-286.
20. Pelletier, F.J. & L.K. Schubert (1989). *Generically Speaking or Using Discourse Representation Theory to Interpret Generics*. In *Properties, Types, and Meanings, Vol. 2*, G. Chierchia, B.H. Partee & R. Turner (eds.), Dordrecht: Kluwer, 193-268.
21. Roberts, C. (1987).*Modal Subordination, Anaphora and Distributivity*. PhD dissertation, UMass Amherst.
22. Schlenker, P. (2005). Ontological Symmetry in Language: A Brief Manifesto. To appear in *Mind & Language*.
23. Stone, M. (1999). *Reference to Possible Worlds*. RuCCS Report 49, Rutgers University, New Brunswick.
24. van den Berg, M. (1996). *Some Aspects of the Internal Structure of Discourse*. PhD dissertation, University of Amsterdam.
25. van Rooy, R. (1998). Modal subordination in Questions, in the *Proceedings of Twendial 1998*, J. Hulstijn & A. Nijholt (eds.), 237-248.
26. Wang, L., E. McCready & N. Asher (2006). Information Dependency in Quantificational Subordination. In *Where Semantics Meets Pragmatics*, K. Turner & K. von Heusinger (eds.), Elsevier (forthcoming).