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DONKEY PLURALITIES: PLURAL INFORMATION STATES VS. NON-ATOMIC INDIVIDUALS

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Abstract[1]

The paper argues that two distinct and independent notions of plurality are involved in natural language anaphora and quantification: *plural reference* (the usual non-atomic individuals) and *plural discourse reference*, i.e. reference to a quantificational dependency between sets of objects (e.g. atomic / non-atomic individuals) that is established and subsequently elaborated upon in discourse. Following van den Berg (1996), plural discourse reference is modeled as plural information states (i.e. as sets of variable assignments) in a new dynamic system couched in classical type logic that extends Compositional DRT (Muskens 1996). Given the underlying type logic, compositionality at sub-clausal level follows automatically and standard techniques from Montague semantics (e.g. type shifting) become available. The idea that plural info states are semantically necessary (over and above non-atomic individuals) is motivated by relative-clause donkey sentences with multiple instances of singular donkey anaphora. At the same time, allowing for non-atomic individuals over and above plural info states enables us to capture the intuitive parallels between singular and plural (donkey) anaphora, while deriving the incompatibility between singular donkey anaphora and collective predicates.

1 The Phenomenon and the Basic Proposal

1.1 Plural Reference and Plural Discourse Reference

The main goal of this paper is to systematically distinguish two notions of plurality involved in natural language anaphora and quantification, namely: (**i**) *plural reference*, i.e. the usual reference to non-atomic individuals, e.g. the non-atomic / plural / sum individual *megan⊕gaby* in *Megan and Gaby are deskmates* (see Link 1983 and Schwarzschild 1992 among many others); (**ii**) *plural discourse reference*, i.e. reference to a quantificational dependency between sets of objects (e.g. atomic / non-atomic individuals, but also times, eventualities, possible worlds etc.) that is established and subsequently elaborated upon in discourse, e.g. the dependency between gifts and girls introduced in the first conjunct and elaborated upon in the second conjunct of the discourse in (1) below.

(1) John bought a^{u} gift for $every^{u'}$ girl in his class and asked $their_{u'}$ deskmates to wrap $them_{u}$.

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The first conjunct in (1) introduces a quantificational dependency between the set u' of girls in John's class and the set u of gifts bought by John: each u'-girl is correlated with the u-gift(s) that John bought for her. This correlation / dependency is elaborated upon in the second conjunct: for each u'-girl, John asked her deskmate to wrap the corresponding u-gift(s).

However, morphologically plural anaphora of the kind instantiated in (1) does not provide a clear-cut argument for distinguishing plural reference and plural discourse reference: both of them / either of them could be involved in the interpretation of (1). Nor does it provide a forceful argument for a semantic (as opposed to a pragmatic) encoding of discourse-level reference to quantificational dependencies: it might be that the second conjunct in (1) is cumulatively interpreted (in the sense of Scha 1981) and that the correlation between girls and gifts (brought to salience by the first conjunct) is only pragmatically supplied.

1.2 Multiple Weak / Strong Singular Donkey Anaphora and Collective Predicates

I will therefore use sentences with multiple instances of *singular* donkey anaphora like (2) and (3) below to provide independent semantic motivation for plural discourse reference.

(2) Every^{u} person who buys a$^{u'}$ book on amazon.com and has a$^{u''}$ credit card uses it$_{u''}$ to pay for it$_{u'}$.

(3) Every^{u} boy who bought a$^{u'}$ Christmas gift for a$^{u''}$ girl in his class asked her$_{u''}$ deskmate to wrap it$_{u'}$.

Sentence (2) shows that singular donkey anaphora can refer to non-singleton sets of atomic individuals, while (3) shows that singular donkey anaphora can refer to a dependency between such sets. Let us examine them in turn. Example (2) is a mixed weak & strong donkey sentence: it asserts that, for *every* book (strong) that any credit-card owner buys on amazon.com, there is *some* credit card (weak) that s/he uses to pay for the book[2].

Intuitively, example (2) does not apply only to persons that bought exactly one book on amazon.com or that have exactly one credit card that is, morphologically singular donkey anaphora is not semantically singular[3]. Moreover, note that the credit card can vary from book to book, e.g. I can use my MasterCard to buy set theory books and my Visa to buy detective novels; that is, even the weak indefinite a$^{u''}$ credit card can introduce a non-singleton set of atoms. And, for each buyer, the two sets of atoms, i.e. all the purchased books and some of the credit cards, are correlated and the dependency between these sets (left unspecified in the restrictor) is specified in the nuclear scope: each book is correlated with the credit card that was used to pay for it. The translation of sentence (2) in classical (static) first-order logic, provided in (4) below, summarizes these observations.

(4) $\forall x(pers(x) \wedge \exists y(bk(y) \wedge buy(x, y)) \wedge \exists z(card(z) \wedge hv(x, z))$
$\rightarrow \forall y'(bk(y') \wedge buy(x, y') \rightarrow \exists z'(card(z') \wedge hv(x, z') \wedge use\_to\_pay(x, z', y'))))$

Given that (2) is intuitively interpreted as shown in (4) above, a plausible hypothesis is that singular donkey anaphora involves plural reference, i.e. non-atomic individuals (or, if you prefer, sets of atoms), as proposed in Lappin and Francez (1994) for example. Under this view, sentence (2) is analyzed as follows: the strong donkey anaphora to u'-books involves the maximal sum individual y containing all and only the books bought by a given u-person;

[2] Note that the same kind of interpretation is associated with non-generic variants of (2), e.g. *Based on last year's statistics, every person who bought a book on amazon.com and had a credit card used it to pay for it.*

[3] At least not in the sense in which singular (Russellian) definite descriptions like *the (one) book s/he buys* or *the (one) credit card s/he has* are semantically singular; see Rooth (1987), Heim (1990), Geurts (2002) and Brasoveanu (2007) (among others) for more discussion of the (wavering) uniqueness effects associated with singular donkey anaphora.

at the same time, the weak donkey anaphora to u''-credit cards involves a non-maximal individual z (possibly non-atomic) containing some of the credit cards that said u-person has[4]. Finally, the nuclear scope of (2) is cumulatively interpreted, i.e. given the maximal sum y of books and the sum z of some credit cards (and, possibly, some cover of y and z), for any part $y' \leq y$ (in the cover – if there is one), there is a part $z' \leq z$ (in the cover) such that z' was used to pay for y' and, also, for any $z' \leq z$ (in the cover), there is a $y' \leq y$ (in the cover) such that z' was used to pay for y'.

Such a plural reference approach to weak / strong donkey anaphora faces the following problem, noticed in Kanazawa (2001): if the classical strong donkey sentence *Every^u farmer who owns a$^{u'}$ donkey beats it$_{u'}$* involves reference to non-atomic individuals, we predict that singular donkey anaphora is compatible with collective predicates (at least in a situation in which all donkey-owning farmers have more than one donkey). This prediction, however, is incorrect, as shown by the infelicitous sentence in (5) below.

(5) #Every^u farmer who owns a$^{u'}$ donkey gathers it$_{u'}$ around the fire at night. (based on an example in Kanazawa 2001)

One way to maintain the plural reference approach and derive the infelicity of (5) is to assume (following a suggestion in Neale 1990) that singular donkey pronouns always distribute over the non-atomic individual they are anaphoric to. For example, the singular pronoun *it$_{u'}$* in (5) contributes a distributive operator and requires each donkey atom in the maximal sum of u'-donkeys to be gathered around the fire at night. The infelicity of (5) follows from the fact that collective predicates do not apply to atomic individuals.

But this domain-level (as opposed to discourse-level) distributivity strategy will not help us with respect to (3) above. Sentence (3) contains two instances of strong donkey anaphora: we are considering *every* Christmas gift and *every* girl. Moreover, the restrictor of the quantification in (3) introduces a dependency between the set of gifts and the set of girls: each gift is correlated with the girl it was bought for. Finally, the nuclear scope retrieves not only the two sets of objects, but also the dependency between (i.e. the structure associated with) them: each gift was wrapped by the deskmate of the girl that the gift was bought for. Thus, we have here donkey anaphora to structure in addition to donkey anaphora to values / objects.

Importantly, the structure associated with the two sets of atoms, i.e. the dependency between gifts and girls that is introduced in the restrictor and elaborated upon in the nuclear scope of the quantification, is *semantically* encoded and not pragmatically inferred. That is, the nuclear scope of the quantification in (3) is not interpreted cumulatively and the correlation between the sets of gifts and girls is not left vague / underspecified and subsequently made precise based on various extra-linguistic factors. To see that the structure is semantically encoded, consider the following situation: suppose that John buys two gifts, one for Megan and the other for Gaby; moreover, the two girls are deskmates. Intuitively, sentence (3) is true if John asked Megan to wrap Gaby's gift and Gaby to wrap Megan's gift and it is false if John asked each girl to wrap her own gift. But if the relation between gifts and girls were semantically vague / underspecified and only pragmatically supplied, we would predict that sentence (3) would be intuitively true even in the second situation.

In sum, we need: (**i**) to account for singular weak / strong donkey anaphora to structured (non-singleton) sets of individuals (see (2) and (3) above) and (**ii**) to derive the incompatibility between singular donkey anaphora and collective predicates (see (5) above).

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[4] This is basically the E-type approach to weak / strong donkey ambiguities in Lappin and Francez (1994).

1.3 The Basic Proposal: Plural Discourse Reference as Plural Information States

The notion of plural discourse reference (i.e. discourse-level plurality) as distinct and independent from plural reference (i.e. domain-level plurality) is the central component of the analysis. Following the proposal in van den Berg (1994, 1996) (which can be traced back to Barwise 1987 and Rooth 1987), I model plural discourse reference as plural information states in a new dynamic system couched in classical (many-sorted) type logic that extends Compositional DRT (CDRT, Muskens 1996). More precisely, I extend CDRT with plural information states that are modeled as *sets of variable assignments* I, J etc. (as opposed to single assignments i, j etc.) and that can be can be represented as matrices with assignments (sequences) as rows, as shown in (6) below.

A matrix (i.e. a plural info state – or: discourse-level plurality) is two-dimensional and encodes two kinds of discourse information: values and structure. The values are the sets of objects that are stored in the columns of the matrix, e.g. a discourse referent (dref) u stores a set of individuals relative to a plural info state since u is assigned an individual by each assignment (i.e. row). These individuals can be non-atomic, i.e. plural at the domain-level. The structure (quantificational dependency) is *distributively* encoded in the rows of the matrix: for each assignment / row in the plural info state, the individual assigned to a dref u by that assignment is structurally correlated with the individual assigned to some other dref u' by the same assignment. The resulting system is dubbed Plural CDRT (PCDRT).

(6)

Info State I	…	u		u'		…
i_1	…	x_1	(i.e. ui_1)	y_1	(i.e. $u'i_1$)	…
i_2	…	x_2	(i.e. ui_2)	y_2	(i.e. $u'i_2$)	…
i_3	…	x_3	(i.e. ui_3)	y_3	(i.e. $u'i_3$)	…
…	…	…		…		…

Values – sets of objects (e.g. atomic / non-atomic individuals): $\{x_1, x_2, x_3, \ldots\}$, $\{y_1, y_2, y_3, \ldots\}$ etc.

Structure (plural discourse reference) – *n*-ary relations between objects: $\{<x_1, y_1>, <x_2, y_2>, <x_3, y_3>, \ldots\}$ etc.

Plural info states enable us to capture the non-uniqueness intuitions associated with singular donkey anaphora and to give a compositional account of mixed weak & strong donkey sentences like (2) above by locating the weak / strong donkey ambiguity at the level of the indefinite articles. A weak indefinite article stores in a plural info state *some* of the individuals that satisfy its restrictor and nuclear scope (i.e. a non-maximal witness set), while a strong indefinite article stores in a plural info state *all* the individuals that satisfy its restrictor and nuclear scope (i.e. its maximal witness set). Moreover, plural info states enable us to store and pass on anaphoric information about both values and structure, thereby enabling us to account for the simultaneous donkey anaphora to values and structure in sentence (3) above.

Finally, we account for the incompatibility between singular donkey anaphora and collective predicates (see (5) above) by taking *singular* donkey indefinites and pronouns to be: (**i**) distributive at the discourse level, i.e. they require predicates to be satisfied relative to each individual assignment i in a plural info state I; (**ii**) singular, i.e. atomic, at the domain level, i.e. for each $i \in I$, ui is atomic. Collective predicates, however, apply only to non-atomic individuals – that is, they are felicitous if either (**i**) the individuals stored by each variable assignment are non-atomic, i.e. we have domain-level plurality, e.g. for each $i \in I$, ui is non-atomic and ui was gathered around the fire, or (**ii**) they are interpreted collectively at the discourse level, e.g. we sum all the individuals stored in the plural info state $I=\{i_1, \ldots, i_n, \ldots\}$ and require the resulting sum individual $ui_1 \oplus \ldots \oplus ui_n \oplus \ldots$ to be gathered around the fire.

1.4 The Parallel Account of Singular and Plural Anaphora in Plural CDRT (PCDRT)

Allowing for non-atomic individuals in the domain, i.e. allowing for plural reference over and above plural discourse reference, enables us to give an account of multiple (i.e. structured) plural donkey anaphora that is parallel to the account of singular donkey anaphora. For example, the PCDRT analysis of the plural donkey sentence in (7) below is parallel to the analysis of sentence (3) above. Note that the collective predicate *fight (each other)* in (7) is felicitous because, in contrast to example (6), we have domain-level non-atomicity introduced by the plural cardinal indefinite *two*$^{u''}$ *boys*.

(7) Every^{u} parent who gives a$^{u'}$ balloon / three$^{u'}$ balloons to two$^{u''}$ boys expects them$_{u''}$ to end up fighting (each other) for *it*$_{u'}$ / *them*$_{u'}$.[5]

Moreover, we can give a parallel account of the singular and plural sage plant examples in (8) and (9) below. The only difference between the PCDRT analyses of these two examples is that, after we process the restrictor, each assignment in the output plural info state stores a sage plant atom for (8) and a non-atomic sage plant individual with two atoms for (9). In both cases, we are able to derive the entailment that each customer bought nine sage plants.

(8) Everybody^{u} who bought a$^{u'}$ sage plant here bought eight$^{u''}$ others along with it$_{u'}$. (Heim 1982)

(9) Everybody^{u} who bought two$^{u'}$ sage plants here bought seven$^{u''}$ others along with them$_{u'}$.[6]

Finally, the PCDRT account of weak / strong plural donkey readings is parallel to the account of weak / strong singular donkey readings. For example, cardinal indefinites like *two*$^{u'}$ can be either (**i**) strong, e.g. *two*$^{u''}$ *boys* in (7) above, or (**ii**) weak, e.g. *two*$^{u''}$ *dimes* in (11) below, where (11) is a minimal variation on the classical weak donkey example in (10)[7].

(10) Every^{u} driver who had a$^{u'}$ dime put it$_{u'}$ in the meter. (based on Pelletier and Schubert 1989)

(11) Every^{u} driver who had two$^{u'}$ dimes put them$_{u'}$ in the meter.

2 Compositional DRT with Plural Info States and Non-Atomic Individuals: PCDRT

We work with a Dynamic Ty2 logic, i.e. basically with Muskens' Logic of Change (Muskens 1996), which is based on Gallin's Ty2 (Gallin 1975). There are three basic types: type *t* (truth-

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[5] Based on an example due to Maria Bittner (p.c.).

[6] Based on an example in Lapin and Francez (1994), modified in Kanazawa (2001).

[7] In contrast to cardinal indefinites, *some*-based plural donkey anaphora seems to always be maximal, as shown by the intuitive interpretation of (i) below: every driver put *every* dime s/he had in the meter. Thus, the difference in interpretation between (11) and (i) indicates that the maximality associated with *some* anaphora (also instantiated by the Evans example *Harry bought some*^{u} *sheep. Bill vaccinated them*$_{u}$) is not a consequence of the fact that the anaphora is plural, but it should be attributed to the determiner *some*. That is, contrary to what seems to be the received wisdom, plural (donkey) anaphora is not necessarily maximal (at least, not necessarily maximal at the discourse level). The two independent notions of plurality argued for in PCDRT open a way to account for this observation: I think that *some* anaphora (and, perhaps, plural anaphora in general) involves a form of (local, maxima-based) *domain-level* maximality (*a* maximal sum individual such that… – see (ii) below), while the weak / strong donkey ambiguity is captured in terms of (global, supremum-based) *discourse-level* maximality (*the* maximal plural info state such that… – see (29) below). Throughout this paper, I will ignore domain-level maximality, which might in fact prove to be part and parcel of both *some*-based and cardinal-based plural (donkey) anaphora. See sections **2** and **3** of the paper for the notation used in (ii) and (iii).

(i) Every^{u} driver who had some$^{u'}$ dimes put them$_{u'}$ in the meter.

(ii) $\mathbf{max\_individual}_u(D) := \lambda I_{st}.\lambda J_{st}.\ DIJ \wedge \neg\exists K_{st}(([u];\ D)IK \wedge \oplus uJ \leq \oplus uK \wedge \oplus uJ \neq \oplus uK)$, where u is of type $\mathbf{e} := se$ and D is of type $\mathbf{t} := (st)((st)t)$.

(iii) $some^{\mathbf{wk}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [u];\ \mathbf{dist}(\mathbf{max\_individual}_u(P(u);\ P'(u)))$
$some^{\mathbf{str}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^u(\mathbf{dist}(\mathbf{max\_individual}_u(P(u);\ P'(u))))$

values), type e (atomic and non-atomic individuals; variables: x, x' etc.) and type s ('variable assignments'; variables: i, j etc.). A suitable set of axioms ensures that the entities of type s behave as variable assignments[8].

Following Link (1983) and Schwarzschild (1992) (among others), I take the domain of type e to be the power set a given non-empty set **IN** of entities, i.e. $\wp^{+}(\mathbf{IN}) := \wp(\mathbf{IN})\setminus\{\varnothing\}$. The sum of two individuals $x_e \oplus y_e$ (subscripts on terms indicate their type) is the union of the sets x and y, e.g. $\{megan\}\oplus\{gaby\}=\{megan, gaby\}$. For a set of atomic / non-atomic individuals X_{et}, the sum of the individuals in X (i.e. their union) is $\oplus X$, e.g. $\oplus\{\{megan, gaby\}, \{gaby\}, \{john\}\}=\{megan, gaby, john\}$. The part-of relation over individuals $x \leq y$ (x is a part of y) is the partial order induced by inclusion $\subseteq$ over the set $\wp^{+}(\mathbf{IN})$. The atomic individuals are the singleton subsets of **IN**, identified by means of the predicate $\mathbf{atom}(x) := \forall y \leq x(y=x)$.

A dref for individuals u is a function of type se from 'assignments' i_s to individuals x_e. Intuitively, the individual $u_{se}i_s$ is the individual that the 'assignment' i assigns to the dref u. Dynamic info states I, J etc. are plural: they are sets of 'variable assignments', i.e. they are terms of type st. As shown in matrix (6) above, an individual dref u stores a set of atomic and / or non-atomic individuals with respect to a plural info state I, abbreviated as $uI := \{u_{se}i_s: i_s \in I_{st}\}$, i.e. uI is the image of the set of 'assignments' I under the function u.

The resulting Plural Compositional DRT (PCDRT) system pushes further the research program in Muskens (1996) of constructing theories and formal systems that unify different frameworks (e.g. Montague semantics and dynamic semantics): PCDRT unifies in classical type logic the static, compositional analysis of generalized quantification in Montague semantics, Link's static analysis of plurality and van den Berg's Dynamic Plural Logic. Moreover, PCDRT can be extended in the usual way with additional sorts for eventualities, times and possible worlds, which enables us to account for temporal and modal anaphora and quantification in a way that is *parallel* to the account of individual-level anaphora and quantification; see Brasoveanu (2007) and references therein for more discussion and for an account of quantificational and modal subordination that extends the present account of multiple singular and plural donkey anaphora.

2.1 DRS's, Atomic Conditions, New Dref's and the Definition of Truth in PCDRT

A sentence is interpreted as a Discourse Representation Structure (DRS), i.e. as a relation of type $(st)((st)t)$ between an input info state I_{st} and an output info state J_{st}. As shown in (12) below, a DRS is represented as a [**new dref's** | **conditions**] pair, which abbreviates a term of type $(st)((st)t)$ that places two kinds of constraints on the output info state J: (**i**) J differs from the input info state I at most with respect to the **new dref's** and (**ii**) J satisfies all the **conditions**. An example is provided in (13) below.

(12) $[\textbf{new dref's} \mid \textbf{conditions}] := \lambda I_{st}.\lambda J_{st}.\ I[\textbf{new dref's}]J \wedge \textbf{conditions}J$

(13) $[u, u' \mid person\{u\}, book\{u'\}, buy\{u, u'\}] :=$
$\lambda I_{st}.\lambda J_{st}.\ I[u, u']J \wedge person\{u\}J \wedge book\{u'\}J \wedge buy\{u, u'\}J$

DRS's of the form [**conditions**] that do not introduce new dref's are *tests* and they abbreviate terms of the form $\lambda I_{st}.\lambda J_{st}.\ I=J \wedge \textbf{conditions}J$, e.g. $[book\{u'\}] := \lambda I_{st}.\lambda J_{st}.\ I=J \wedge book\{u'\}J$.

Atomic conditions, e.g. lexical relations like $book\{u'\}$ or $buy\{u, u'\}$, are sets of plural info states, i.e. they are terms of type $(st)t$. As shown in (14) below, they are interpreted collectively at the discourse level, e.g. the condition $book\{u'\}$ requires the *sum* of all the

[8] See Muskens (1996) and chapter **3** in Brasoveanu (2007) for more details.

individuals in uI, i.e. $\oplus uI$, to be in the set denoted by the static property *book* of type et[9]. The main empirical argument for the default collective interpretation of atomic conditions is provided by discourses like (15) below. Informally, the first sentence introduces the set of u-purses *distributively* relative to the set of u'-girls, i.e. the output plural info state I is such that $u'I$ is the set of all girl-atoms and, for each 'assignment' $i \in I$, ui is the purse-atom that John bought for the corresponding girl-atom $u'i$. The second sentence in (15) *collectively* elaborates on the set of purchased purses, which is felicitous under the proposed interpretation of atomic conditions, namely $identical\_except\_for\_color\{u\} := \lambda I_{st}.\ identical\_except\_for\_color(\oplus uI)$.

(14) **a.** $book\{u'\} := \lambda I_{st}.\ book(\oplus u'I)$ **b.** $buy\{u, u'\} := \lambda I_{st}.\ buy(\oplus uI, \oplus u'I)$

(15) John bought an^{u} alligator purse for every$^{u'}$ girl in his class. They$_{u}$ were identical except for the color.

Thus, the discourse-level distributivity associated with the interpretation of singular donkey indefinites and singular pronouns is contributed by the singular number morphology (as shown in section **2.4** below) and not by lexical relations (i.e. atomic conditions).

Let us turn now to the PCDRT definition of new dref introduction. Consider first the simpler CDRT notion of new dref introduction, i.e. of random assignment of value to a dref u relative to *single* 'variable assignments', symbolized as $[u]$ and defined as shown in (16) below (for more discussion, see Muskens 1996 and chapter **3** in Brasoveanu 2007). Informally, $i[u]j$ means that the 'assignments' i and j differ at most with respect to the value they assign to the dref u. The PCDRT definition of new dref introduction, provided in (17) below, is the pointwise generalization of the relation in (16) between single 'assignments' i_s and j_s to a relation between sets of 'assignments' (i.e. plural info states) I_{st} and J_{st}.

(16) $[u] := \lambda i_s.\lambda j_s.\ \forall v_{se}(\mathbf{udref}(v) \wedge v{\neq}u \rightarrow vi{=}vj)$

(17) **New Dref's in PCDRT:** $[u] := \lambda I_{st}.\lambda J_{st}.\ \forall i_s{\in}I(\exists j_s{\in}J(i[u]j)) \wedge \forall j_s{\in}J(\exists i_s{\in}I(i[u]j))$

Informally, $I[u]J$ means that each input 'assignment' i has a $[u]$-successor output 'assignment' j and, vice-versa, each output 'assignment' j has a $[u]$-predecessor input 'assignment' i. This ensures that we preserve the values and structure associated with the previously introduced dref's u', u'' etc. The definition in (17) treats the structure and value components of a plural info state in parallel – we non-deterministically introduce both (**i**) some new (random) values for u and (**ii**) some new (random) structure associating the u-values and the values of any other (previously introduced) dref's u', u'' etc. The fact that the PCDRT definition in (17) treats the dynamics of value and structure in parallel distinguishes it from most dynamic systems based on plural info states, including van den Berg (1996), Krifka (1996) and Nouwen (2003), which only introduce values non-deterministically, while any newly introduced set of values is *deterministically* associated with a particular structure[10].

The PCDRT distinction between the two informational components of an info state, i.e. values and structure, and their parallel treatment is motivated both empirically and formally.

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[9] For simplicity, I assume that all static properties are closed under sums (i.e. they are cumulative), e.g. $\forall x_e \forall y_e(book(x) \wedge book(y) \rightarrow book(x \oplus y))$. I also assume that certain static lexical relations are distributive at the domain level, e.g. *book* is domain-level distributive in the sense that, if an individual x is a book, then its atomic parts are also books, i.e. $\forall x_e(book(x) \rightarrow \forall y_e{\leq}x(\mathbf{atom}(y) \rightarrow book(y)))$. Whether (and to what extent) these assumptions are correct can only be determined by investigating the range of possible interactions between distributivity, cumulativity and collectivity at the domain level (i.e. relative to non-atomic individuals) and distributivity, cumulativity and collectivity at the discourse level (i.e. relative to plural info states).

[10] See chapter **5** in Brasoveanu (2007) for a detailed comparison between definition (17), which is equivalent to the definition of random assignment in van den Berg (1994), and the definition of random assignment in van den Berg (1996) (see also Krifka 1996 and Nouwen 2003), which treats structure deterministically.

Empirically, the definition in (17) enables us to account for mixed reading donkey sentences like (2) above. Recall that, intuitively, we want to allow the credit cards to vary from book to book; that is, we want the restrictor of the *every*-quantification in (2) to non-deterministically introduce some set of u''-cards and non-deterministically associate them with the u'-books and let the nuclear scope filter the non-deterministically assigned values and structure by requiring each u''-card to be used to pay for the corresponding u'-book. Formally, the PCDRT definition in (17) is the natural generalization of the CDRT definition in (16) insofar as it preserves its properties: just as (16) defines $[u]$ as an equivalence relation of type $s(st)$ between 'assignments', (17) defines $[u]$ as an equivalence relation of type $(st)((st)t)$ between sets of 'assignments' (i.e. between plural info states).

The PCDRT definition of truth (which has the expected format) is provided in (18) below.

(18) **Truth:** A DRS D (of type $(st)((st)t)$) is *true* with respect to an input info state I_{st} iff $\exists J_{st}(DIJ)$.

With the basic dynamic system now in place, we can turn to the compositional interpretation of generalized quantification, pronouns and indefinites.

2.2 Compositionality

Given the underlying type logic, compositionality at sub-clausal level follows automatically and standard techniques from Montague semantics (e.g. type shifting) become available.

In more detail, the compositional aspect of interpretation in an extensional Fregean/Montagovian framework is largely determined by the types for the (extensions of the) 'saturated' expressions, i.e. names and sentences. Let us abbreviate them as **e** and **t**.

An extensional static logic with domain-level plurality identifies **e** with e (atomic and non-atomic individuals) and **t** with t (truth-values). The denotation of the noun *book* is of type **et**, i.e. et: book $\rightsquigarrow \lambda x_e.\ book_{et}(x)$. The generalized determiner *every* is of type (**et**)((**et**)**t**), i.e. $(et)((et)t)$. We go dynamic with respect to both value and structure by making the 'meta-types' **e** and **t** more complex, i.e. by assigning finer-grained meanings to names and sentences. More precisely, PCDRT assigns the following dynamic types to the 'meta-types' **e** and **t**: **t** abbreviates $(st)((st)t)$, i.e. a sentence is interpreted as a DRS, and **e** abbreviates se, i.e. a name is interpreted as a dref for individuals. The denotation of the noun *book* is still of type **et**, as shown in (19) below. The denotations of generalized determiners, indefinite articles and pronouns are introduced in sections **2.3** and **2.4** below; determiners and articles have denotations of the expected type, i.e. (**et**)((**et**)**t**), while pronouns anaphoric to a dref u are interpreted as the Montagovian quantifier-lift of the dref u (of type **e**), i.e. their type is (**et**)**t**.

(19) book $\rightsquigarrow \lambda v_{\mathbf{e}}.\ [book\{v\}]$, i.e. book $\rightsquigarrow \lambda v_{\mathbf{e}}.\lambda I_{st}.\lambda J_{st}.\ I=J \wedge book\{v\}J$

See Brasoveanu (2007) for the complete definition of (**i**) the syntax of a fragment of English containing the multiple donkey sentences in (2) and (3) above and (**ii**) its corresponding PCDRT semantics defined in terms of type-driven translation.

2.3 Generalized Quantification

Selective generalized determiners are relations between two dynamic properties $P_{\mathbf{et}}$ (the restrictor) and $P'_{\mathbf{et}}$ (the nuclear scope), i.e. their denotations are of type (**et**)((**et**)**t**). The dynamic definition of selective generalized determiners has to be formulated in such a way that: (**i**) we capture the fact that anaphors in the nuclear scope can have antecedents in the restrictor, (**ii**) we avoid the proportion problem, i.e. the generalized determiner relates sets of individuals and not sets of 'assignments' and (**iii**) we can account for mixed reading (weak & strong) donkey sentences. Thus, the main problem posed by the dynamic definition of generalized quantification is to find a suitable way to extract the restrictor and nuclear scope sets of individuals based on the restrictor and the nuclear scope dynamic properties.

The proposed ways to solve this problem fall into two broad classes. The first class of solutions employs a dynamic framework based on singular info states (e.g. classical DRT/FCS/DPL) and analyzes generalized quantification as internally dynamic and externally static. The main idea is that the restrictor set of individuals is extracted based on the restrictor dynamic property, while the nuclear scope set of individuals is extracted based on both the restrictor and the nuclear scope dynamic property, so that the anaphoric connections between them are captured. The second class of solutions employs a dynamic framework based on plural information states and analyzes generalized quantification as both internally and externally dynamic (see van den Berg 1994, 1996 – but also Krifka 1996 and Nouwen 2003 among others). The main idea is that the restrictor set of individuals is extracted based on the restrictor dynamic property and the nuclear scope set of individuals is the maximal *structured subset* of the restrictor set of individuals that satisfies the nuclear scope dynamic property.

Given that the notion of a dref being a structured subset of another dref required for the van den Berg-style definition involves non-trivial complexities that are (more or less) orthogonal to the issues at hand, I will define selective generalized quantification following the format of the DRT/FCS/DPL-style definition. However, since PCDRT is a system based on *plural* info states and formulated in classical type logic, the definition of selective generalized determiners I provide in (20) and (21) below is novel. This definition is intermediate between the two ways of defining dynamic quantification mentioned above and, as such, it is useful in formally exhibiting the commonalities and differences between them; see Brasoveanu (2007) for more discussion and a more detailed comparison of the two definitions.

(20) **Selective Generalized Determiners in PCDRT – The Basic Meaning:**

$\textbf{\textit{det}}^{u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [\mathbf{det}_u(P(u), P'(u))]$, where $\mathbf{e} := se$ and $\mathbf{t} := (st)((st)t)$.

(21) **Selective Generalized Determiners in PCDRT – The Dynamic Condition:**

$\mathbf{det}_u(D, D') := \lambda I_{st}.\ \mathbf{DET}(u[DI],\ u[(D; D')I])$,

where $u[DI] := \{\oplus uJ\text{: } ([u \mid \mathbf{atom}\{u\}]; D)IJ\}$

and $\mathbf{atom}\{u\} := \lambda I_{st}.\ \mathbf{atom}(\oplus uI)$

and **DET** is the corresponding static determiner.

(22) **Dynamic Conjunction:** $D; D' := \lambda I_{st}.\lambda J_{st}.\ \exists H_{st}(DIH \wedge D'HJ)$

The generalized quantifiers we will be considering throughout this paper are domain-level and discourse-level distributive in the sense that they relate two sets of *atomic* individuals (i.e. domain-level distributivity) and these sets of atomic individuals are required to satisfy the restrictor and nuclear scope dynamic properties *one individual at a time* (i.e. discourse-level distributivity). We enforce both kinds of distributivity by means of the dynamic condition $\mathbf{atom}\{u\}$; this condition is collectively interpreted relative to a plural info state I, which ensures two things: (**i**) any two 'assignments' i and i' in the info state I assign the same individual x to u, i.e. $\forall i_s{\in}I \forall i'_s{\in}I(ui{=}ui')$; (**ii**) moreover, the individual x assigned to u throughout the info state I is an atomic individual, i.e. $\forall i_s{\in}I(\mathbf{atom}(ui))$.

The condition $\mathbf{det}_u$ defined in (21) above tests that the static determiner **DET** relates two sets of atomic individuals, namely the restrictor set $u[DI]$ and the nuclear scope set $u[(D; D')I]$. The restrictor set $u[DI]$ is the set of atomic individuals that can be assigned to the individual dref u and that satisfy the restrictor DRS $D_{\mathbf{t}}$, i.e. $P_{\mathbf{et}}(u_{\mathbf{e}})$. The semicolon ';' stands for dynamic conjunction, interpreted as shown in (22) above, i.e. as relation composition. The nuclear scope set $u[(D; D')I]$ is the set of atomic individuals that can be assigned to the individual dref u and that satisfy the dynamically conjoined restrictor DRS $D_{\mathbf{t}}$ (i.e. $P_{\mathbf{et}}(u_{\mathbf{e}})$) and nuclear scope DRS $D'_{\mathbf{t}}$ (i.e. $P'_{\mathbf{et}}(u_{\mathbf{e}})$). Dynamically conjoining D and D' ensures that the donkey pronouns in the nuclear scope DRS can be successfully linked to their antecedents in the restrictor DRS.

The definition of generalized quantification in (20)-(21) above is basically neutral with respect to the weak / strong ambiguity exhibited by donkey anaphora: the plural info state H_{st} that is the output of the restrictor DRS D and the input of the nuclear scope DRS D' can store either *every* (strong) or only *some* (weak) of the relevant individuals. As the following section argues, the maximality / non-maximality choice should be attributed to the indefinites.

2.4 Pronouns and Indefinites

I take the number morphology on pronouns and indefinites to contribute the discourse-level unselective distributivity operator **dist** defined in (23) below. Distributively updating an input info state I with a DRS D of type $\mathbf{t} := (st)((st)t)$ means that we update each assignment i in I with the DRS D and then take the union of the resulting output info states. The **dist** operator is discourse-level because it distributes over plural info states and is unselective in the sense of Lewis (1975): we update one *case*, i.e. one 'assignment' i in I, at a time[11].

(23) $\mathbf{dist}(D) := \lambda I_{st}.\lambda J_{st}.\ \exists R_{s((st)t)}(I{=}\mathbf{Dom}(R) \wedge J{=}\cup\mathbf{Ran}(R) \wedge \forall{<}k_s,L_{st}{>}{\in}R(D\{k\}L))$,
where $\mathbf{Dom}(R) := \{k_s\colon \exists L_{st}(RkL)\}$ and $\mathbf{Ran}(R) := \{L_{st}\colon \exists k_s(RkL)\}$.

Singular number morphology contributes the function **sg** of type (**et**)(**et**) defined in (24) below, while plural number morphology contributes the function **δ** of the same type. The function **sg** contributes discourse-level distributivity and domain-level singularity / atomicity, i.e. it takes a dynamic property P (of type **et**) and it returns the 'singularized' version of this property, which is the sub-property of P that applies only to atomic individuals together with the closure of this sub-property under arbitrary unions (the closure is due to the **dist** operator). The function **δ** contributes only discourse-level distributivity (i.e. closure of P under arbitrary unions), while being compatible with domain-level plurality (i.e. non-atomicity).

(24) $\mathbf{sg} := \lambda P_{\mathbf{et}}.\lambda v_{\mathbf{e}}.\ \mathbf{dist}([\mathbf{atom}\{v\}];\ P(v))$ $\quad\quad$ $\boldsymbol{\delta} := \lambda P_{\mathbf{et}}.\lambda v_{\mathbf{e}}.\ \mathbf{dist}(P(v))$

I assume that singular and plural number morphology on pronouns contributes the **sg** and **δ** functions respectively, as shown in (25) below. A pronoun anaphoric to a dref u is interpreted as the Montagovian quantifier-lift of the dref u (of type **e**), i.e. its type is (**et**)**t**. The number morphology on the pronoun specifies if the dref u is required to have the 'singularized' version of the dynamic property P, i.e. $_{\mathbf{sg}}P$, or the 'plural distributive' version, i.e. $_{\boldsymbol{\delta}}P$[12]. The only difference between the two properties is the atomicity requirement contributed by $_{\mathbf{sg}}P$, but not by $_{\boldsymbol{\delta}}P$; this enables us to derive the incompatibility between collective predicates and singular pronouns (see (5) above), while allowing for collectives with plural pronouns (see (7) above).

(25) $\mathtt{he}_u \rightsquigarrow \lambda P_{\mathbf{et}}.\ _{\mathbf{sg}}P(u)$, where $_{\mathbf{sg}}P := \mathbf{sg}(P)$ $\quad\quad$ $\mathtt{they}_u \rightsquigarrow \lambda P_{\mathbf{et}}.\ _{\boldsymbol{\delta}}P(u)$, where $_{\boldsymbol{\delta}}P := \boldsymbol{\delta}(P)$[13]

Let us turn now to the interpretation of indefinites. PCDRT enables us to provide a unitary account for the weak / strong donkey ambiguity as it is exhibited by both singular indefinite articles (see (2) above) and cardinal determiners (see (7) above for the strong reading of `two` and (11) for its weak reading). The only difference between weak and strong indefinites (of

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[11] See Brasoveanu (2007): chapter **6** for the corresponding PCDRT notion of *selective* discourse-level distributivity, needed for the van den Berg-style definition of dynamic generalized quantification.

[12] We also need a plural non-distributive / collective meaning for plural pronouns, e.g. $\mathtt{they}_u \rightsquigarrow \lambda P_{\mathbf{et}}.\ P(u)$, to be able to account for discourse (15) above. Since the analysis of (15) also requires a notion of dynamic quantification that is externally dynamic (see chapter **6** in Brasoveanu 2007 for the PCDRT formulation of such a notion), I will not address this problem here.

[13] Anaphoric definite articles receive similar translations, namely $\mathtt{the}_{sg:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ _{\mathbf{sg}}P(u);\ _{\mathbf{sg}}P'(u)$ and $\mathtt{the}_{pl:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ _{\boldsymbol{\delta}}P(u);\ _{\boldsymbol{\delta}}P'(u)$.

both kinds) is the absence vs. presence of a maximization operator $\mathbf{max}^u$ taking scope over both the restrictor and the nuclear scope of the indefinites, as shown in (26) and (27) below.

(26) $a^{\mathtt{wk}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [u];\ _{\mathbf{sg}}P(u);\ _{\mathbf{sg}}P'(u)$ $\qquad a^{\mathtt{str}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^u(_{\mathbf{sg}}P(u);\ _{\mathbf{sg}}P'(u))$

(27) $two^{\mathtt{wk}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ [u];\ _{\mathbf{2}}P(u);\ _{\mathbf{2}}P'(u)$ $\qquad two^{\mathtt{str}:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{max}^u(_{\mathbf{2}}P(u);\ _{\mathbf{2}}P'(u))$

Just as in the case of pronouns, singular number morphology on indefinite articles contributes two **sg** functions modifying the restrictor and nuclear scope dynamic properties P and P'. Cardinal indefinites contribute similar functions – the only difference is that, at the domain-level, these functions require the newly introduced individuals to have a particular number of atoms. For example, in the case of `two`, we have the function **2** of type (**et**)(**et**) defined in (28) below, which requires each individual to contain exactly two atoms. Just as in the case of **sg** and **δ** above, the 'two'-ized dynamic property $\mathbf{2}(P)$ is abbreviated as $_{\mathbf{2}}P$ (i.e. $_{\mathbf{2}}P := \mathbf{2}(P)$).

(28) $\mathbf{2} := \lambda P_{\mathbf{et}}.\lambda v_{\mathbf{e}}.\ \mathbf{dist}([\mathbf{2\_atoms}\{v\}];\ P(v))$,
where $\mathbf{2\_atoms}\{u\} := \lambda I_{st}.\ \mathbf{2\_atoms}(\oplus uI)$ and $\mathbf{2\_atoms}(x_e) := |\{y_e\!: y \leq x \wedge \mathbf{atom}(y)\}| = 2$.

Attributing the weak / strong ambiguity to the indefinites enables us to give a compositional account of the mixed reading sentence in (2) above because we *locally* decide for each indefinite whether it receives a weak or a strong reading[14]. The $\mathbf{max}^u$ operator, defined in (29) below, ensures that, after we process a strong indefinite, the output plural info state stores with respect to the dref u the *maximal* set of individuals satisfying both the restrictor dynamic property P and the nuclear scope dynamic property P'. In contrast, a weak indefinite will non-deterministically store *some* set of individuals satisfying its restrictor and nuclear scope.

(29) $\mathbf{max}^u(D) := \lambda I_{st}.\lambda J_{st}.\ ([u];\ D)IJ \wedge \forall K_{st}(([u];\ D)IK \rightarrow uK \subseteq uJ)$, where D is of type **t**.

The first conjunct in (29) introduces u as a new dref and makes sure that each individual in uJ 'satisfies' D, i.e. uJ stores *only* individuals that 'satisfy' D. The second conjunct enforces the maximality requirement: any other set uK obtained by a similar procedure (i.e. any other set of individuals that 'satisfies' D) is included in uJ, i.e. uJ stores *all* the individuals that satisfy D. The DRS $\mathbf{max}^u(D)$ can be thought of as dynamic λ-abstraction over individuals: the 'abstracted variable' is the dref u, the 'scope' is the DRS D and the result of the 'abstraction' is a set of individuals uJ containing all and only the individuals that 'satisfy' D. Thus, the $\mathbf{max}^u$ operator together with plural info states and the **dist** operators introduced above enable us to 'dynamize' λ-abstraction over both values and structure.

We can now turn to the PCDRT representations for donkey sentences introduced in section **1**.

3 Multiple Donkey Anaphora and Collective Predicates in PCDRT

The compositionally obtained PCDRT representation (simplified based on various PCDRT equivalences) for the mixed reading donkey sentence in (2) is given in (30) below; based on this representation, we derive the intuitively correct truth-conditions, provided in (31).

(30) $[\mathbf{every}_u([person\{u\}];\ \mathbf{max}^{u'}(\mathbf{dist}([\mathbf{atom}\{u'\},\ book\{u'\},\ buy\{u,u'\}]));$
$[u''];\ \mathbf{dist}([\mathbf{atom}\{u''\},\ c.card\{u''\},\ have\{u,u''\}]),$
$\mathbf{dist}([use\_to\_pay\{u,u',u''\}]))]$

[14] Moreover, since the only difference between weak / strong indefinites is the absence / presence of the $\mathbf{max}^u$ operator, we can think of indefinites as *underspecified* with respect to the presence / absence of this operator: the decision to introduce it or not is made online depending on the discourse and utterance context – much like aspectual coercion (e.g. the iterative interpretation of *John sent a letter to the company for years* or *The light is flashing*) or the selection of a particular type for the denotation of an expression (e.g. proper names are type-lifted when they are conjoined with generalized quantifiers) re context-driven online processes.

(31) $\lambda I_{st}.\ \forall x_e \forall y_e(\mathbf{atom}(x) \wedge \mathit{person}(x) \wedge \mathbf{atom}(y) \wedge \mathit{book}(y) \wedge \mathit{buy}(x, y) \wedge$
$\exists z_e(\mathbf{atom}(z) \wedge \mathit{c.card}(z) \wedge \mathit{have}(x, z))$
$\rightarrow \exists z_e(\mathbf{atom}(z) \wedge \mathit{c.card}(z) \wedge \mathit{have}(x, z) \wedge \mathit{use\_to\_pay}(x, y, z)))$

Informally, the update in (30) can be described as follows. After the input info state is updated with the restrictor of the quantification in (2), we are in a plural info state that stores, for each atomic *u*-person that is a book buyer and a card owner: (**i**) the maximal set of purchased book atoms, stored relative to the dref *u'* (since the indefinite `a`$^{\mathbf{str}:u'}$ `book` is strong), (**ii**) some non-deterministically introduced set of credit card atoms, stored relative to the dref *u''* (since the indefinite `a`$^{\mathbf{wk}:u''}$ `credit card` is weak) and, finally, (**iii**) some non-deterministically introduced structure correlating the *u'*-atoms and the *u''*-atoms.

The nuclear scope of the quantification in (2) is anaphoric to both values (in this case, atomic individuals) and structure: we test that the non-deterministically introduced values for *u''* and the non-deterministically introduced structure associating *u''* and *u'* (the structure is tested by means of the **dist** operator) satisfy the nuclear scope condition, i.e. we test that, for each 'assignment' in the info state, the *u''*-card stored in that 'assignment' is used to pay for the *u'*-book stored in the same 'assignment'. That is, the nuclear scope elaborates on the structure (i.e. the dependency between *u''* and *u'*) non-deterministically introduced in the restrictor.

The pseudo-scopal relation between the strong indefinite `a`$^{\mathbf{str}:u'}$ `book` and the weak indefinite `a`$^{\mathbf{wk}:u''}$ `credit card` ("pseudo" because, by the Coordinate Structure Constraint, the strong indefinite cannot syntactically take scope over the weak indefinite) emerges as a consequence of the fact that PCDRT uses plural information states, which store and pass on information about both the objects and the dependencies between them.

As (32) below shows, the PCDRT analysis of sentence (3) is largely parallel to the analysis of sentence (2), except for the fact that both indefinites (`a`$^{\mathbf{str}:u'}$ `Christmas gift` and `a`$^{\mathbf{str}:u''}$ `girl`) are strong. After the input info state is updated with the restrictor of the quantification, we are in a plural info state that, for a particular *u*-boy atom, stores (**i**) relative to *u'*: the maximal set of gift atoms that said *u*-boy bought for some girl, (**ii**) relative to *u''*: the maximal set of girl atoms for which said *u*-boy bought a gift and (**iii**) the structure associating the *u'*-atoms and the *u''*-atoms (this is due to the two **dist** operators in the restrictor), i.e., for each 'assignment', the *u'*-gift stored in it was bought for the *u''*-girl stored in it.

(32) $[\mathbf{every}_u([\mathit{boy}\{u\}];\ \mathbf{max}^{u'}(\mathbf{dist}([\mathbf{atom}\{u'\}, [\mathit{gift}\{u'\}];$
$\mathbf{max}^{u''}(\mathbf{dist}([\mathbf{atom}\{u''\}, \mathit{girl}\{u''\}, \mathit{buy\_for}\{u, u', u''\}])))),$
$\mathbf{dist}(\mathbf{max}^{u'''}([\mathit{d.mate}\{u'''\}, \mathit{of}\{u''', u''\}]); [\mathbf{atom}\{u'''\}]; [\mathit{a.t.w}\{u, u''', u'\}]))]$

(33) $\lambda I_{st}.\ \forall x_e \forall R_{e(et)}{\neq}\emptyset(\mathbf{atom}(x) \wedge \mathit{boy}(x) \wedge$
$\mathbf{Dom}(R) = \{y_e: \mathbf{atom}(y) \wedge \mathit{gift}(y) \wedge \exists z_e(\mathbf{atom}(z) \wedge \mathit{girl}(z) \wedge \mathit{buy\_for}(x, y, z))\} \wedge$
$\forall y_e{\in}\mathbf{Dom}(R)(\forall z_e(Ryz \leftrightarrow \mathbf{atom}(z) \wedge \mathit{girl}(z) \wedge \mathit{buy\_for}(x, y, z)))$
$\rightarrow \forall y_e \forall z_e(Ryz \rightarrow \exists z'_e(\mathbf{atom}(z') \wedge \forall z''_e(\mathit{d.mate}(z'') \wedge \mathit{of}(z'',z) \leftrightarrow z''{=}z') \wedge \mathit{a.t.w}(x,z',y))))$

Yet again, the nuclear scope of the quantification is anaphoric to both values and structure: we require each 'assignment' in the plural info state to be such that the deskmate of the *u''*-girl in that 'assignment' was asked to wrap the *u'*-gift in the same 'assignment'[15]. Thus, just as in the

[15] The possessive `her`$_{u''}{}^{u'''}$ `deskmate` in (3) is analyzed as a Russellian definite description that contributes both existence (since we introduce the dref *u'''*) and uniqueness (relativized to *u''*-girls), as shown in (i) and (ii) below. The *u'''*-uniqueness is a consequence of combining the **max** operator (with scope only over the restrictor – cf. the scope of **max** in strong indefinites) and the **atom** condition. See Brasoveanu (2007) for more details.

(i) $\mathit{the}^{sg:u} \rightsquigarrow \lambda P_{\mathbf{et}}.\ \lambda P'_{\mathbf{et}}.\ \mathbf{max}^{u}(P(u)); [\mathbf{atom}\{u\}]; P'(u)$
(ii) $\mathit{her}_u{}^{u'} \rightsquigarrow \lambda P_{\mathbf{et}}.\lambda P'_{\mathbf{et}}.\ \mathbf{dist}([\mathbf{atom}\{u\}]; \mathbf{max}^{u'}(P(u'); [\mathit{of}\{u', u\}]); [\mathbf{atom}\{u'\}]; P'(u'))$

previous example, the nuclear scope elaborates on the structured dependency between the two sets of atoms (gifts and girls) introduced in the restrictor. As (33) shows, the dynamics of structure is truth-conditionally captured as quantification over the relation variable $R_{e(et)}$.

The PCDRT analysis of the plural donkey example in (7) above is completely parallel to the analysis of (3); similarly, the singular and plural sage plant examples in (8) and (9) above and the singular and plural weak donkey sentences in (10) and (11) receive parallel analyses.

In PCDRT, the incompatibility between singular donkey anaphora and collective predicates exemplified in (5) above is a consequence of the fact that the singular number morphology on the donkey pronoun $it_{u'}$ contributes a discourse-level distributivity operator **dist** and a domain-level atomicity requirement **atom**{*u'*} which, together, require the collective (i.e. non-atomic) gather[16] in (5) to be predicated of atomic individuals.

The present version of PCDRT does not account for (1) because, for simplicity, section **2.3** above defined selective generalized determiners as externally static; see Brasoveanu (2007), chapter **6** for a version of PCDRT with externally dynamic quantification that can account (1).

4 Comparison with Alternative Approaches

PCDRT differs from previous dynamic and static approaches to singular / plural donkey anaphora in a couple of respects. The first difference is conceptual: PCDRT explicitly encodes the idea that reference to structure is as important as reference to value and that the two should be treated in parallel (in contrast to van den Berg 1996, Krifka 1996 and Nouwen 2003; see the definition of dref introduction in section **2.1** above).

The PCDRT analysis of reference to structure as *discourse* reference to structure, i.e. in terms of plural discourse reference / plural info states, contrasts with the analysis of reference to structure by means of (dref's for) choice and / or Skolem functions. Although such functions could be used to capture (donkey) anaphora to structure, they would have variable arity depending on how many simultaneous anaphoric connections there are. That is, the arity of the functions is determined by the discourse context. It is therefore preferable to encode this context dependency in the database that stores discourse information, i.e. in the info state, and not in the representation of a lexical item, i.e. in the pronoun and / or its antecedent.

The second difference is empirical: the motivation for plural information states is provided by *singular* and *intra-sentential* donkey anaphora, in contrast to much of the previous literature (van den Berg 1996, Krifka 1996, Nouwen 2003 and Asher and Wang 2003 among others) which relies on plural and cross-sentential anaphora of the kind instantiated by (1) above.

Intra-sentential donkey anaphora to structure provides a much stronger argument for the idea that plural info states are *semantically* necessary. To see this, consider anaphora to value first: a pragmatic account is plausible for cases of cross-sentential anaphora, e.g. in *A man came in. He sat down*, the pronoun *he* can be taken to refer to whatever man is pragmatically brought to salience by the use of an indefinite in the first sentence, but less plausible for cases of intra-sentential donkey anaphora: no particular donkey is brought to salience in *Every farmer who owns a donkey beats it*. Similarly, a pragmatic account of anaphora to structure is plausible for cases of cross-sentential anaphora like (1) above: the first conjunct in (1) correlates each girl with the gift(s) that John bought for her and the second conjunct elaborates on this

[16] The PCDRT translation for the verb gather is provided in (ii) below; the collectivity requirement is explicitly formalized by means of the condition ~[**atom**{*v'*}], modeled, for simplicity, as an assertion and not as a presupposition. Dynamic negation '~' is defined following the usual DRT/FCS/DPL format.

(i) gather ⇝ $\lambda Q_{\mathbf{(et)t}}.\lambda v_{\mathbf{e}}.\ Q(\lambda v'_{\mathbf{e}}.\ [\sim[\mathbf{atom}\{v'\}], gather\{v, v'\}])$, where $\sim D := \lambda I_{st}.\ \neg\exists K_{st}(DIK)$ (with D of type $\mathbf{t}$).

correlation – for each girl, John asked her deskmate to wrap the corresponding gift(s). That is, the wrapping structure is the same as the buying structure, but the identity of structure might be a pragmatic addition to semantic values that are unspecified for structure (e.g. the second conjunct could be interpreted cumulatively). A pragmatic approach, however, is less plausible for cases of intra-sentential donkey anaphora to structure instantiated by (3) and (7) above.

Third, PCDRT differs from the previous dynamic approaches to plural anaphora insofar as it models plural reference and plural discourse reference as two distinct and independent notions. The previous dynamic approaches basically fall into two classes based on the way in which they conflate these two notions. The approaches in the first class (van den Berg 1994, 1996, Nouwen 2003, Asher and Wang 2003 among others) make plural reference dependent on plural discourse reference, i.e. they allow the variable assignments to store only atomic individuals and non-atomic individuals can be accessed in discourse only by summing over plural info states. These dynamic approaches (much like the E-type approach in Neale 1990) find it difficult to capture the intuitively correct truth-conditions of plural sage plant examples like the one in (9) above (see, for example, the relevant discussions in van den Berg 1996 and Kanazawa 2001) and, to they extent they can derive the correct truth-conditions, they fail to capture the intuitive parallels between singular and plural (donkey) anaphora, e.g. between (9) and the singular sage plant example in (8) or between (3) and (7) above.

The approaches in the second class (e.g. Krifka 1996, building on Barwise 1987 and Rooth 1987) make plural discourse reference dependent on plural reference, i.e. the central notion of *parametrized sum individual* associates each atom that is part of a non-atomic individual with a variable assignment that 'parametrizes' / is dependent on that atom, e.g. the non-atomic / sum individual under discussion might contain all and only the farmer atoms that are donkey owners and each farmer atom is associated with a variable assignment that stores (relative to a new dref) a donkey atom that the farmer owns. Besides the fact that these approaches, just like the previous ones, have difficulties with plural sage plant examples and with the parallels between singular and plural (donkey) anaphora, they predict that we cannot access the dependent individuals (e.g. the donkeys owned by some farmer or other) *directly*, but only via anaphora to and / or quantification over the sum individuals they depend on. As sentence (15) above shows, this prediction is incorrect: the second sentence in (15) anaphorically retrieves the alligator purses directly and not as a function of the girls they are dependent on.

Moreover, such approaches require an independent notion of *cover* (see, for example, Krifka 1996) to account for the codistributivity effects associated with the interpretation of discourses like *Three*^{u} *soldiers aimed at five*$^{u'}$ *targets. They*$_{u}$ */ The*$_{u}$ *soldiers hit them*$_{u'}$ */ the*$_{u'}$ *targets* (Kamp and Reyle 1993 and Winter 2000 discuss the first and the second sentence respectively). The fact that PCDRT countenances both notions of plurality, i.e. both non-atomic individuals and plural info states, enables it to encode covers by letting each assignment i in a plural info state I be such that the sum of soldier atoms ui aimed at and hit the sum of target atoms $u'i$.

For a detailed comparison of the PCDRT account of weak / strong donkey ambiguities with other static and dynamic approaches, see Brasoveanu (2007), chapter **5**. I will only mention that mixed reading DP-conjunction donkey sentences, e.g. (the newspaper claims that, based on the most recent statistics) *Every*^{u} *company that hired a*$^{\mathbf{str}:u'}$ *Moldavian man, but no*$^{u''}$ *company that hired a*$^{\mathbf{wk}:u'}$ *Transylvanian man promoted him*$_{u'}$ *within two weeks of hiring*, are used to argue against approaches that locate the weak / strong ambiguity in the donkey pronouns (e.g. the E-type approach in Lappin and Francez 1994) and not in the indefinites.

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