

# Modal subordination is neither modal nor subordinate...discuss

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## 1 Introduction

We have seen in numerous other works that indefinites introduce discourse referents that are anaphorically retrieved by syntactically non-local expressions in subsequent discourse. Roberts (1989) argues that modal expressions behave similarly, a phenomenon which she terms ‘modal subordination’.

**The phenomenon:** Roberts is concerned primarily with discourses such as (1) and (2) where the second sentence seems to be asserted relative to the modality introduced by the antecedent of the first sentence.

- (1) a. If Edna forgets to fill the birdfeeder, she will feel very bad.  
b. The birds will get hungry.
- (2) a. If John bought a book<sup>*i*</sup>, he’ll be home reading it<sub>*i*</sub> by now.  
b. It<sub>*i*</sub> will be a mystery novel.

We see in (2) that indefinites introduced in the antecedent can be retrieved in the modally subordinated sentence as well. At the same time, however, we see that this process fails in other cases such as (3).

- (3) a. If John bought a book<sup>*i*</sup>, he’ll be home reading it<sub>*i*</sub> by now.  
b. #It<sub>*i*</sub> is a mystery novel.

The puzzle, then, is to understand not only how this process works in (1)-(2), but also why it is that (3) fails.

### **The formal approach:**

The basic approach is to combine Kratzer’s theory of modality and conditionals with a DRT-style account of discourse anaphora.

- Mechanically, this involves adding our modal logic operators ( $\Diamond$  and  $\Box$ ) to DRT.
- To do this, we add a syntactic rule and a corresponding semantic rule informally described in (4).<sup>1</sup>

- (4) a. If  $\alpha, \beta$  are DRSs, then  $\beta\Diamond\alpha$  and  $\beta\Box\alpha$  are too.

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<sup>1</sup>N.B. Roberts often employs the two operators as apparently unary operators as well in cases where the modal base is the context set. This is an abbreviation only.

- b.  $\beta \Diamond \alpha$  is true relative a world  $w$  iff  $\exists w'$  such that  $w'$  is a closest accessible  $\beta$ -world from  $w$  and  $\alpha$  is embeddable in  $w'$ .
- c.  $\beta \Box \alpha$  is true relative a world  $w$  iff  $\forall w'$  such that  $w'$  is a closest accessible  $\beta$ -world from  $w$  are such that  $\alpha$  is embeddable in  $w'$ .

## 2 Kratzer's theory of modality

In terms of the treatment of modality itself, Roberts basically adopts the now standard theory espoused in a series of papers by Angelika Kratzer. To review, Kratzer's theory involves relativizing the quantificational force of modals to two sets of propositions: the **modal base** and the **ordering source**.

**Modal Base:** A contextually given set of facts, assumptions or circumstances which more or less models the accessibility relation of modal logic.

**Ordering Source:** A contextually given set of facts, assumptions or circumstances which constrain the accessibility relation to the worlds closest to  $w@$  w.r.t that set of concerns. Among other things, this allows us to capture *comparative* possibility.

In the case of epistemic modals, which Roberts considers first, the modal base is the context set (i.e.  $\cap$ c.g.).

- In Kratzer's terms, the modal base in such cases is **realistic** since it includes the actual world (but not **totally realistic** since it may include other worlds as well).
- Nonfactual moods involve a (usually unspoken) set of ideals, obligations, rules, etc.

## 3 Against the 'insertion' approach

One potential way to combine the two theories which Roberts considers and rejects is the so-called 'insertion approach'.

- Though the syn/sem mapping algorithm is not entirely clear, this approach essentially adds the information contributed by the (b) sentence into the DRS of the consequent as illustrated for (2) in (5).

$$(5) \quad [ [x, y \mid \text{John}(x) \wedge \text{book}(y)] \Box [z, w, \mathbf{r} \mid \text{reading}(z, w) \wedge z=x \wedge w=y \wedge \mathbf{murder-mystery}(\mathbf{r}) \wedge r=y] ]$$

- This approach appears to work in examples like this where the modal force in the consequent and the modally subordinated sentence are the same.
- This cannot work more generally, as we see from examples such as (6) where the modally subordinated sentence in (b) has a different quantificational force than the clause introducing the modality.

- (6)
- a. A thief<sup>*i*</sup> might break into the house.
  - b. He<sub>*i*</sub> would take the silver.

## 4 Accommodation of the missing antecedent

What we need is a mechanism to provide the ‘subordinated’ modal with the same set of worlds described by the sentence with the initial modal.

- In other words, modals introduce discourse referents which later ‘subordinated’ modals can retrieve in subsequent discourse.
- Applying this intuition to (6), we get truth conditions paraphrasable as follows: “It’s possible that a thief will break into the house, and *if he does*, he will undoubtedly take the silver.”
- Formally, we need to end up with a representation something like (7).

$$(7) \quad [ \Diamond[x \mid \text{thief}(x) \wedge \text{break-into-house}(x)]; [x \mid \text{thief}(x) \wedge \text{break-into-house}(x)] \Box[y \mid \text{take-silver}(y) \wedge y=x] ]$$

- The puzzle is determining when and how we can copy the restrictions on the subordinated modal.
- The basic approach is to say that the relativization of the second involves *accommodation* similar to that of presuppositions.
- One worry in the use of accommodation is that it is a very powerful tool and one that needs to be *constrained* in principled ways in order to match the data. In particular, it needs to be constrained so as to derive the difference between (8) and (9) (repeated from above).

- (8)    a. If John bought a book<sup>*i*</sup>, he’ll be home reading it<sub>*i*</sub> by now.  
          b. It<sub>*i*</sub> will be a mystery novel.
- (9)    a. If John bought a book<sup>*i*</sup>, he’ll be home reading it<sub>*i*</sub> by now.  
          b. #It<sub>*i*</sub> is a mystery novel.

**Constraint 1:** Accommodation of the antecedent requires non-factual mood in the subordinated sentence.

- This can be extended straightforwardly for so-called ‘Generalized Discourse Subordination’ where *usually*, *always*, or a generic operator is present in the subordinated sentence. In these cases, we require roughly ‘non-episodic’ morphology rather than non-factual morphology.
- It is unclear, however, how to extend this restriction to the telescoping cases Roberts discusses later such as (10) where there does not seem to be any sort of special morphosyntax of any sort.

- (10)    a. Each degree candidate<sup>*j*</sup> walked to the stage.  
          b. He<sub>*j*</sub> took his<sub>*j*</sub> diploma from the Dean and returned to his<sub>*j*</sub> seat.

**Constraint 2:** “It must be plausible that the modally subordinated utterance has a hypothetical common ground suggested by the immediately preceding context”

- Essentially, the accommodation process is constrained by the fact that hearers try to interpret sentences are being related to previous sentences in particular ways, i.e. to interpret discourses in a **coherent** fashion.

- While Roberts does not spell out the details of the theory of discourse coherence being assumed, the theory being suggested seems to be along the lines of the coherence relations of Kehler (2002).
- In this sort of theory, (adjacent) sentences are related to one another by one some sort of coherence relation (e.g. Exemplification, Elaboration, Temporal Sequence, Occasion, Explanation, Cause-Effect, Resemblance, and others).
- In the examples we’ve seen so far, the discourse relation that is relevant is Elaboration or perhaps Temporal Sequence, but this need not be so.

For example, Roberts argues that the accommodation in Partee’s bathroom sentence, in (11), can be accounted for by similar means.

(11) Either there’s no bathroom in this house or it’s in a funny place.

(12)  $[ \neg [ x \mid \text{bathroom}(x) \wedge \text{in-house}(x) ] ] \vee [ [ x \mid \text{bathroom}(x) \wedge \text{in-house}(x) ] \Box [ y \mid \text{funny-place}(y) \wedge y=x ] ] ]$

- The anaphoric connection here passes Constraint 1 since disjunction conveys non-factuality of each individual disjunct.
- And it passes Constraint 2 since a disjunction presents “alternative answers to the same topic of discussion”.

Why, then, is cross-disjunct anaphora not licensed in the seemingly similar (13)?

(13) #Either every bathroom does not belong to this house, or it’s in a funny place.

- The difference then is that the intended antecedent would have to be inferred rather than having been linguistically realized. This leads us to Constraint 3.

**Constraint 3:** Accommodation “requires the explicit prior representation of potential antecedent discourse referents”

- Constraint 3, of course, is not specific to modal subordination as seen by Partee’s famous marble example in (14).

(14) a. Nine of the ten marbles are in the bag.  
b. #It’s under the couch. (it=the tenth marble)

- This also accounts for the difference between the following two examples:

(15) First square  $19^i$  and then cube it<sub>i</sub>.

(16) First take [the square of  $19$ ]<sup>i</sup> and then cube it<sub>i</sub>.

- The addition of Constraint 3 and Constraint 2, however, make it clear that rather than being somehow exceptional (as Roberts’ copying with *accommodation*-based account would suggest), Modal Subordination is an entirely ordinary instance of anaphora.
- This point is argued by Stone (1999) who also presents the following challenge to Roberts (1989)’s DRS copying approach.

- (17) a. A wolf might walk in.  
 b. We would be safe because John has a gun.  
 c. He would use it to shoot it

- Crucially, (17-b) introduces new information from the real world (John’s gun-totingness).
- The modally subordinated sentences are evaluated with respect not to the modal base introduced by (17-a), but rather to that base *plus* the information in (17-b).
- If the mechanism for modal subordination is DRS-copying, as in Roberts’ account, we should not be able to accommodate the modified base since it is present in prior representation.

Roberts’ account treats modal subordination as being somehow exceptional, a ‘last resort’ sort of accommodation applying to rescue otherwise aberrant discourses. What we seem to be finding, however, is that modal subordination is not at all aberrant, but rather is a relatively ordinary instance of anaphora, subject to the same constraints as anaphora in general.

## 5 Non-epistemic modals

Thus far, we have discussed only cases with epistemic modals, but the phenomenon in question is broader and extends to cases of deontic and other modals as well:

- (18) a. If I had brought a book<sup>*i*</sup> with me to Georgia, I could have read it<sub>*i*</sub> on the plane.  
 b. I would probably have finished it<sub>*i*</sub> by now.
- (19) a. You should eat a bagel<sup>*i*</sup>.  
 b. It<sub>*i*</sub> would fill you up.

From a dynamic perspective, however, there is nothing different happening here than in the epistemic cases. We need to develop an interpretation algorithm for deontic modals in non-subordination cases, but the accommodation process should be the same as in epistemic modals seen above:

1. Add the propositions of the antecedent (often unspoken) to the modal base. In counterfactuals, we take the modal base to be empty initially.  $m^+(w) := m(w) \cup p$
2. Take the intersection of  $m^+(w)$  to get the derived context set.  $\cap m^+(w)$
3. Order  $\cap m^+(w)$  by the propositions in  $o(w)$  and take the closest world(s). The subset of  $o$ -closest worlds in  $\cap m^+(w)$  is the domain for modal quantification.

## 6 Formal system

Outside of modals, the formal system Roberts uses is fairly straightforward and familiar. To DRT, she adds two syntactic rules and two semantic rules (one for  $\Box$  and  $\Diamond$  respectively).

### Syntax

- (20) a. If  $K_i$  and  $K_j$  are DRSEs, then  $K_i \Box K_j$  is a condition.  
 b. If  $K_i$  and  $K_j$  are DRSEs, then  $K_i \Diamond K_j$  is a condition.

### Semantics (epistemic version w/o ordering source)

- (21) a.  $\langle w, f \rangle \Vdash_M (K_i \Box_m K_j)$  iff  
 $\forall u, g [g \langle X_{ki} \rangle f \wedge u \in \cap [m(w) \cup \{ \{v : \langle v, g \rangle \models_M K_i \} \}]] \rightarrow$   
 $\exists h (h \langle X_{kj} \rangle g \wedge \langle u, h \rangle \models_M K_j)$   
 b.  $\langle w, f \rangle \Vdash_M (K_i \Diamond_m K_j)$  iff  
 it is not the case that  $\langle w, f \rangle \Vdash_M (K_i \Box_m \neg K_j)$
- (22) Given a model M,  $K_i \Box_m K_j$  denotes the set of all world-assignment pairs such that all world-assignment pairs  $\langle u, g \rangle$  such that:  
 (1) the assignment differs from the input assignment at most w.r.t. the value assigned to the drefs introduced in  $K_i$  and  
 (2) the world is in the derived context set  $(\cap [m(w) \cup \{ \{v : \langle v, g \rangle \models_M K_i \} \}])$   
 Are also such that:  
 (1) there is an assignment ( $\exists$  deriving from our dynamic definition of truth) differing from  $g$  at most w.r.t. the drefs in  $K_j$  and  
 the conditions in  $K_j$  are met.

The more complicated semantics in Roberts' (2e) are like (21-a), differing in that they incorporate Kratzer's ordering source as well as the assumption that, given the ordering semantics assumed, there will not always be a closest set of worlds. I leave the verification that the formula in fact does this as an exercise for the reader.

## 7 Non-modal modal subordination

The final section of the paper briefly considers two non-modal phenomena which resemble modal-subordination broadly speaking.

### 7.1 Generalized Subordination in Discourse

- The first, which Roberts terms "Generalized Subordination in Discourse" is exemplified by the discourse in (23).

- (23) a. Harvey courts a girl<sup>*i*</sup> at every convention.  
 b. She<sub>*i*</sub> always comes to the banquet with him.  
 c. The girl<sub>*i*</sub> is usually very pretty.

Note, we are not interested in the wide-scope indefinite which could be continued by (24).

- (24) She<sub>*i*</sub> is getting very tired of his unwanted advances.

Following Lewis, we take (23-a) to instantiate quantification over cases which are then retrieved in (23-b) and (23-c) to restrict the domains of *always* and *usually* respectively.

- Just as we saw that modal subordination requires non-factual mood in the subordinated sentence, here we require some sort of non-episodic morphology (quantificational adverbs like *usually* or *always*, generics, etc.)
- Otherwise, it seems that we can account for these in the same framework, just replacing Kratzer's modal base with a corresponding situation/case/info state notion.

## 7.2 Telescoping

- The other case Roberts briefly discusses, what she terms ‘telescoping’, is seen in (25)-(26).
- (25)    a.    Each degree candidate<sup>*i*</sup> walked to the stage.  
           b.    He<sub>*i*</sub> took his<sub>*i*</sub> diploma from the Dean and returned to his<sub>*i*</sub> seat.
- (26)    a.    Every chess set comes with a spare pawn<sup>*i*</sup>.  
           b.    It<sub>*i*</sub> is taped to the top of the box.
- From a discussion of the general case, we “zoom in” on a particular instantiation.
  - These cases are reminiscent to modal subordination since they (1) do not involve syntactic subordination, (2) involve anaphoric retrieval of a dref which the naive theory would not expect to be available for subsequent retrieval, and (3) are subject to the same (or at least similar) constraints such as narrative continuity.
  - That said, something more needs to be said about the asymmetry between (27) and (26). Why does retrieval of the modally subordinated dref in (27) require particular morphosyntax, whereas (26) has no such requirement?
- (27)    a.    If John bought a book<sup>*i*</sup>, he’ll be home reading it<sub>*i*</sub> by now.  
           b.    #It<sub>*i*</sub> is a mystery novel.

## 8 Conclusion

Modal subordination involve anaphoric retrieval of a dref introduced within the scope of a prior modal which is surprisingly available for subsequent retrieval in sentences which are syntactically not subordinated.

- To account for such cases, Roberts combines Kratzer’s theory of modality with Kamp’s DRT.
- In modal subordination, the antecedent essentially has to be accommodated into the DRS of the subordinated modal.
- This combination is fairly straightforward, but on its own overgenerates and predicts that drefs introduced under modals should always be accommodatable.
- Accommodation, however, is subject to several constraints: (1) the presence of non-factual morphosyntax in the subordinated clause is required, (2) the hearer must be able to interpret the subordinated sentence into a coherent discourse with the antecedent, (3) the antecedent must be previously represented overtly (no tenth marbles).

## References

- Kehler, Andrew (2002) *Coherence, reference, and the theory of grammar*. CSLI.
- Roberts, Craige (1989) Modal Subordination and Pronominal Anaphora in Discourse. *Linguistics and Philosophy* : 683–721.
- Stone, Matthew (1999) Reference to Possible Worlds, ruCCS Report 49.