Motivating Dynamic Semantics

A sentence is not an island.

Sentences are embedded in larger discourses. They are anaphorically related to other sentences in the same discourse.

For example:

(1) John owns a donkey. He feeds it at night.
Notice the anaphoric connection between the indefinite NP 'a donkey' and the subsequent pronoun 'it'.

(2) is a good (enough) paraphrase of (1):

(2) John owns a donkey. John feeds it at night.

But neither (3) nor (4) is as good:

(3) John owns a donkey. John feeds a donkey at night.
(4) John owns Benjamin (the donkey). John feeds Benjamin at night.

Can't seem to eliminate the pronoun 'it' (bound by the indefinite 'a donkey') from (1).

This becomes a problem once we decide to regiment (1) in the notation of First-Order Logic (FOL):

(5) $\exists x (\text{donkey}(x) \land \text{owns}(\text{John}, x) \land \text{feeds}(\text{John}, x))$
(6) $\exists x ((\text{donkey}(x) \land \text{owns}(\text{John}, x)) \land \text{feeds}(\text{John}, x))$

What we want:

(7) $\exists x (\text{donkey}(x) \land \text{owns}(\text{John}, x) \land \text{feeds}(\text{John}, x))$

The problem is that, to get this meaning, we must first compose a part of the first sentence with the second sentence, and then combine what we have with the remaining part of the first sentence:

(7): [a donkey] [John owns][He feeds it]
Motivating Dynamic Semantics

If we restrict ourselves to completing sentences before we compose them with other sentences, then the best we can do is (6).

(6): [John owns][a donkey] [He feeds it]

Motivating Dynamic Semantics

Who needs it? Discourse semantics is too hard. I'm going to stick with the semantics of sentences.

But the donkey is known for its stubbornness...

(8) If John owns a donkey, he feeds it.
(9) Every farmer who owns a donkey feeds it.

Motivating Dynamic Semantics

Incorrect first-orderizations:

(10) \exists x (\text{donkey}(x) \land \text{owns}(\text{John}, x)) \rightarrow \text{feeds}(\text{John}, x)
(11) \forall y \exists x (\text{farmer}(y) \land \text{donkey}(x) \land \text{owns}(y, x)) \rightarrow \text{feeds}(\text{John}, x))

In both, the final ‘x’ is not in the scope of ‘\exists x’.

Motivating Dynamic Semantics

Correct first-orderizations:

(12) \forall x (\text{donkey}(x) \land \text{owns}(\text{John}, x) \rightarrow \text{feeds}(\text{John}, x))
(13) \forall y \forall x (\text{farmer}(y) \land \text{donkey}(x) \land \text{owns}(y, x)) \rightarrow \text{feeds}(\text{John}, x))

Moral: the limitations of FOL (on the standard semantics) can be seen even within sentences.

Nor are ‘donkey’ sentences rare animals. They are as common as the beast of burden itself.

Motivating Dynamic Semantics

A solution: 'Dynamic semantics’ [due (independently) to Kamp (1981) and Heim (1982)]

What is dynamic semantics?

Dynamic Semantics

Consider the phenomenon of context-sensitivity.

The same sentence can be true or false, depending on the context.

Dynamic Semantics

‘I am standing.’

True as uttered by Sam.
False as uttered by Herman.
Dynamic Semantics

The meaning of a sentence can be thought of as a function (cf. Kaplan (1989)), that takes in a context... ...and gives back a truth-value (T or F).

Dynamic Semantics

A parallel phenomenon.

Right now, the sentence below is false:

‘Herman said that snow is black.’

Dynamic Semantics

But now Herman says, ‘Snow is black.’

In the context arising immediately after his utterance, the earlier sentence is true:

‘Herman said that snow is black.’

Dynamic Semantics

Call the context immediately before Herman’s utterance of ‘Snow is black’, $c_1$.

And call the context immediately after Herman’s utterance, $c_2$.

Dynamic Semantics

Clearly, the sentence ‘Herman said snow is black’ is context-sensitive, since it is true in $c_2$ but not in $c_1$.

Dynamic Semantics

Equally clearly, Herman’s utterance of ‘Snow is black’ changed the context from $c_1$ to $c_2$.

($c_1$ must differ from $c_2$ since it delivers a different truth-value to the sentence above).

Dynamic Semantics

Dynamics takes the semantics of context-sensitivity one step further, to a semantics of context change.

Dynamic Semantics

According to dynamic semantics, the meaning of a sentence is an ‘update’, that takes in a context, and gives back a ... CONTEXT.

Dynamic Semantics

But hang on, what does this new view of meaning have to do with the problems with which we began?

(1) John owns a donkey. He feeds it at night.

(8) If John owns a donkey, he feeds it.

(9) Every farmer who owns a donkey feeds it.
Dynamic Semantics

Take the first case:

(1) John owns a donkey. He feeds it at night.

We want it to translate into the FOL:

(7) \( \exists x (\text{donkey}(x) \land \text{owns}(\text{John}, x) \land \text{feeds}(\text{John}, x)) \)

But the best we can do (compositionally) is:

(6) \( \exists x (\text{donkey}(x) \land \text{owns}(\text{John}, x)) \land \text{feeds}(\text{John}, x) \)

What if I told you that, on a dynamic semantics for FOL, the following equivalence holds:

\( \exists x (\phi) \land \psi \iff DS \exists x (\phi \land \psi) \)

Since (6) and (7) fit the schema on the left and right hand sides, respectively, they are equivalent on dynamic semantics:

\( \exists x (\text{donkey}(x) \land \text{owns}(\text{John}, x)) \land \text{feeds}(\text{John}, x) \iff DS \exists x (\text{donkey}(x) \land \text{owns}(\text{John}, x) \land \text{feeds}(\text{John}, x)) \)

The equivalence means that indefinites can bind indefinitely rightwards across \( \land \)'s:

\( \exists x (\phi) \land \psi \land \xi \land \chi \iff DS \exists x (\phi \land \psi \land \xi \land \chi) \)

\( \exists x (\phi \land \psi \land \xi) \land \chi \iff DS \exists x (\phi \land \psi \land \xi \land \chi) \)

And what about the other cases?

(8) If John owns a donkey, he feeds it.

(9) Every farmer who owns a donkey feeds it.

For these the equivalence below will suffice:

\( \exists x (\phi) \rightarrow \psi \iff DS \forall x (\phi \rightarrow \psi) \)

(Only sans the usual restriction to cases where ‘\( \psi \)’ doesn’t contain ‘\( x \)' free.)

The 2nd equivalence allows us to turn existentials in the antecedent of a conditional into universals taking scope over the whole conditional (but no further).

\( \exists x (\text{donkey}(x) \land \text{owns}(\text{John}, x)) \rightarrow \text{feeds}(\text{John}, x) \iff DS \forall x (\text{donkey}(x) \land \text{owns}(\text{John}, x) \rightarrow \text{feeds}(\text{John}, x)) \)

\( \forall y (\exists x (\text{farmer}(y) \land \text{donkey}(x) \land \text{owns}(y, x)) \rightarrow \text{feeds}(y, x)) \iff DS \forall y \forall x (\text{farmer}(y) \land \text{donkey}(x) \land \text{owns}(y, x) \rightarrow \text{feeds}(y, x)) \)
Dynamic Semantics

We will now proceed to show you how to construct a dynamic semantics for FOL on which these hold:

\[ \exists x (\phi) \wedge \psi \leftrightarrow_{DS} \exists x (\phi \wedge \psi) \]

\[ \exists x (\phi) \rightarrow \psi \leftrightarrow_{DS} \forall x (\phi \rightarrow \psi) \]

Dynamic Predicate Logic (DPL)

The particular version of dynamic semantics we will look at is Dynamic Predicate Logic (DPL – Groenendijk & Stokhof 1991).

DPL: The Plan.

- semantic values in DPL vs. FOL
- definition of DPL semantics
- relations between DPL connectives
- formula equivalence in DPL:
  \[ \exists x (\phi) \wedge \psi \leftrightarrow \exists x (\phi \wedge \psi) \]
  \[ \exists x (\phi) \rightarrow \psi \leftrightarrow \forall x (\phi \rightarrow \psi) \]
- Discourse Representation Structures (DRS’s) in DPL

DPL: Semantics.

Why binary relations between assignments?

For our narrow purposes (i.e. cross-sentential and 'donkey' anaphora), a variable assignment is an effective model of a context. All we ask from a context here is that it keep track of anaphoric relations – hence assignments.

DPL: Semantics.

Why a binary relation between assignments?

Dynamic semantics associates a sentence with the manner in which it updates any context (i.e. its context change potential).

The update is modeled as a relation (not a function) because it is non-deterministic: updating from a context \( c_1 \) has different possible outcomes.

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DPL: Semantics.

The definition of the DPL interpretation function \( \| \phi \|_{DPL}^M \) relative to a standard first-order model \( M = \langle D^M, I^M \rangle \), where:

\( D \) is the domain of entities
\( I \) is the interpretation function which assigns to each \( n \)-place relation \( R \) a subset of \( D^n \):
DPL: Semantics.

1. For any pair of $M$-variable assignments $<g, h>$:
   a. Atomic formulas ('lexical' relations and identity):
   \[
   \begin{align*}
   \|R(x_1, \ldots, x_n)\| <g, h> &= T \\
   &\text{iff } g = h \text{ and } g(x_1), \ldots, g(x_n) \in I(R)
   \\
   \|x_1 = x_2\| <g, h> &= T \text{ iff } g = h \text{ and } g(x_1) = g(x_2)
   \end{align*}
   \]
   b. Connectives:
   - Dynamic Conjunction
   \[
   \|\phi \land \psi\| <g, h> = T \text{ iff there is a } k \text{ s.t. } \|\phi\| <g, k> = T \text{ and } \|\psi\| <k, h> = T
   \]
   - Dynamic Negation
   \[
   \|\neg\phi\| <g, h> = T \text{ iff } g = h \text{ and there is no } k \text{ s.t. } \|\phi\| <g, k> = T
   \]
   i.e. \[
   \|\neg\phi\| <g, h> = T \text{ iff } g = h \text{ and } g \notin \text{Dom}(\|\phi\|),
   \]
   where:
   \[
   \text{Dom}(\|\phi\|) := \{ g: \text{ there is an } h \text{ s.t. } \|\phi\| <g, h> = T \}.
   \]
   c. Existential Quantifier:
   \[
   \|\exists x (\phi)\| <g, h> = T \text{ iff there is a } k \text{ s.t. } g[x]k \text{ and } \|\phi\| <g, k> = T
   \]
   where $g[x]k$ means that $k$ differs from $g$ at most with respect to the value it assigns to $x$.
   i.e. for any variable $u$, if $\forall u$ then $g(u) = k(u)$.
   d. Truth:
   A formula $\phi$ is true with respect to an input assignment $g$ iff there is an output assignment $h$ s.t. $\|\phi\| <g, h> = T$.
   i.e. $\phi$ is true with respect to $g$ iff $g \in \text{Dom}(\|\phi\|)$.
   NB: Dynamic meanings are more fine-grained than truth-conditions.

DPL: The Plan.

- semantic values in DPL vs. FOL
- definition of DPL semantics
- relations between DPL connectives
  - formula equivalence in DPL:
  \[
  \exists x (\phi) \land \psi \Leftrightarrow \exists x (\phi \land \psi)
  \]
  \[
  \exists x (\phi) \rightarrow \psi \Leftrightarrow \forall x (\phi \rightarrow \psi)
  \]
  - Discourse Representation Structures (DRS's) in DPL

DPL: Abbreviations.

2.a. Abbreviations – connectives:
- Anaphoric closure:
   \[
   !\phi := \{ <g, h>: g = h \text{ and } g \in \text{Dom}(\|\phi\|) \}
   \]
   Exercise: Prove this.
DPL: Abbreviations.

2.a. Abbreviations – connectives:

Disjunction: φ ∨ ψ := ~(~φ ∧ ~ψ)

i.e. \( \| φ ∨ ψ \| = \{ <g, h> : g=h \text{ and for any } k \text{ s.t. } g\|φ\| <k, h> = T, \text{ there is an } l \text{ s.t. } \|ψ\| <k, l> = T \} \)

Exercise: Prove this.

Implication: φ → ψ := ~((φ ∧ ~ψ)

i.e. \( \| φ → ψ \| = \{ <g, h> : g=h \text{ and for any } k \text{ s.t. } g\|φ\| <k, h> = T \} \)

Exercise: Prove this.

DPL: The Plan.

√ semantic values in DPL vs. FOL
√ definition of DPL semantics
√ relations between DPL connectives

\[ ∃x(φ) ∧ ψ ⇔ ∃x(φ ∧ ψ) \]
\[ ∃x(φ) → ψ ⇔ ∀x(φ → ψ) \]

Discourse Representation Structures (DRS’s) in DPL

DPL: Equivalence.

Let’s return to the general equivalences we wanted to prove.

Equivalence:
Two formulas are DPL-equivalent, symbolized as ‘\( ⇔ \)’, iff they denote the same set of pairs of variable assignments.

i.e. iff they denote the same binary relation over assignments.

That is:
\( φ ⇔ ψ \) iff \( \| φ \|_{DPL} = \| ψ \|_{DPL} \)

More explicitly:
\( φ ⇔ ψ \) iff for any pair of assignments \( <g, h> \):
\( g\|φ\| = h\|ψ\| \)

i.e. both \( φ\| <g, h> \) and \( ψ\| <g, h> \) are T or both are F

Since DPL denotations determine truth-conditions, two DPL-equivalent formulas will have the same truth-conditions.

Recall that:
\( φ \) is true with respect to \( g \) iff \( g\in \text{Dom}(\| φ \|) \).

Thus:
Suppose \( φ ⇔ ψ \). Then \( \| φ \| = \| ψ \| \).
Then \( \text{Dom}(\| φ \|) = \text{Dom}(\| ψ \|) \).
DPL: $\exists x(\phi) \land \psi \iff \exists x(\phi \land \psi)$.

1. h.s. denotes:

\[
\{<g, h> : \text{there is an individual } a \text{ s.t.} \\
\phi(a) \land \psi \land <a, l> = T \}
\]

And now the second equivalence:

DPL: $\exists x(\phi) \rightarrow \psi \iff \forall x(\phi \rightarrow \psi)$.

1. h.s. denotes:

\[
\{<g, h> : g=h \text{ and for any } k \text{ s.t.} \\
\exists x(\phi) \land \psi \land <k, l> = T, \text{ there is an } l \text{ s.t.} \psi \land <k, l> = T \}
\]
DPL: The Plan.

- semantic values in DPL vs. FOL
- definition of DPL semantics
- relations between DPL connectives
- formula equivalence in DPL:
  \[
  \exists x (\phi) \land \psi \leftrightarrow \exists x (\phi \land \psi)
  \]
  \[
  \exists x (\phi) \rightarrow \psi \leftrightarrow \forall x (\phi \rightarrow \psi)
  \]

Discourse Representation Structures (DRS's) in DPL

DPL: Representing DRS's.

To represent Discourse Representation Structures (DRS's), i.e. 'boxes', in DPL, we first need to define:

- the semantic notion of test
  \[
  \exists x (\phi) \land \psi \leftrightarrow \exists x (\phi \land \psi)
  \]
  \[
  \exists x (\phi) \rightarrow \psi \leftrightarrow \forall x (\phi \rightarrow \psi)
  \]

- the syntactic notion of condition.

Tests: A wff \( \phi \) is a test iff \( \| \phi \| \subseteq \{ <g, g> : g \in G \} \), where \( G \) is the set of all \( M \)-variable assignments, and \( G \) is the set of all \( M \)-variable assignments.

Conditions: The set of conditions is the smallest set of wff's:
- containing atomic formulas and negative formulas (i.e. negation '~' is the main connective)
- and closed under dynamic conjunction.

Tests / Conditions are externally static – they do not pass on bindings to conjuncts yet to come:

(14) Every donkey is in the corral. #It is happy.
(15) It is not true that John owns a donkey. #He feeds it at night.

Negative formulas include:
- \( \sim \phi \)
- anaphoric closure, since \( ! \phi := \sim \phi \)
- disjunctions, since \( \phi \lor \psi := (\sim \phi \land \sim \psi) \)
- implications, since \( \phi \rightarrow \psi := (\sim \phi \land \sim \psi) \)
- universal quantifications, since \( \forall x (\phi) := \sim \exists x (\sim \phi) \)
Conjunctions and existential quantifiers are externally dynamic – they pass on bindings to conjuncts yet to come:

(16) A farmer owns a donkey. He feeds it at night.

But test / conditions can be internally dynamic, i.e. they can pass bindings between sub-formulas:

(17) Every farmer who owns a donkey feeds it at night.

We indicate that a formula is a condition by placing square brackets around it, e.g. $[\phi]$ is a wff iff $\phi$ is a condition and $||[\phi]|| = ||\phi||$

That is, square brackets are just a graphical way of showing that a formula is a condition.

Exercise: Prove that conjunction is commutative over conditions, i.e. $||[x_1] \land [x_2]|| = ||[x_2] \land [x_1]||$.

Exercise: Prove that conjunction is idempotent over conditions, i.e. $||[x]|| = ||[x] \land [x]||$.

Exercise: Prove that conjunction is commutative and idempotent over random assignments, i.e.:

$$||[x_1] \land [x_2]|| = ||[x_2] \land [x_1]||$$

$||[x]|| = ||[x] \land [x]||$.

Exercise: Prove that $[x_1, \ldots, x_n | \phi_1, \ldots, \phi_m] := [x_1] \land \ldots \land [x_n] \land [\phi_1, \ldots, \phi_m]$

where:

$||[x]|| = \{<g, h>: \text{for any variable } \nu, \text{ if } \nu \neq x \text{ then } g(\nu) = h(\nu)\}$

$x$ is called a random assignment of value $x$.

Exercise: Prove that $[x_1, \ldots, x_n | \phi_1, \ldots, \phi_m] \Leftrightarrow \exists x_1 \ldots \exists x_n (\phi_1, \ldots, \phi_m)$

The existential and universal quantifiers are partly duals:

$\neg \exists x(\phi) \Leftrightarrow \forall x(\neg \phi)$

(Exercise: Prove this.)

Clearly, $\exists x(\neg \phi) \Leftrightarrow \neg \forall x(\phi)$ doesn't hold:

$\neg \forall x(\phi)$ is a test, while $\exists x(\neg \phi)$ isn't.