CSE 107
Final Exam Review Problems

1. You are in a class with 200 other students. Let X be the number of other students in the class who share your birthday. For simplicity, exclude birthdays on February 29, and assume all birthdays are equally likely.

   a. Find the probability that *exactly one* other student in the class shares your birthday.

   b. Find the probability that *at least one* other student in the class shares your birthday.

2. Let X be uniform on $[0, 2]$, Y be uniform on $[1, 4]$, and let $Z = X + Y$.

   a. Determine the PDF of the random variable Z.

   b. Determine $P(4 \le Z \le 6)$.

3. Let X be a continuous random variable with PDF $f_X(x)$, and let Y be a discrete random variable with PMF

$$p_Y(y) = \begin{cases} 1/3 & \text{if } y = 1 \\ 2/3 & \text{if } y = 2 \\ 0 & \text{otherwise} \end{cases}$$

   Suppose X and Y are independent, and let $Z = XY$.

   a. Determine the CDF $F_Z(z)$ in terms of the CDF of X.

   b. Use your answer in part (a) to deduce that Z is a continuous random variable, and find its PDF $f_Z(z)$ in terms of the PDF of X.

4. Let X be a discrete random variable with $\text{range}(X) = \{1, 2, 3\}$ (i.e. think of X as a 3-sided die.) Let A denote a Bernoulli process with parameter p (i.e. think of A as a bit stream in which 1 is considered an "arrival"). Assume the process A is independent of X. Suppose further that the stream A is split into three streams B_1, B_2 and B_3 according to the following rule. Whenever an arrival occurs in A, we sample X (i.e. throw the die) and if $X = i$, then an arrival occurs in B_i, for $i = 1, 2, 3$. You may assume that the streams B_1, B_2 and B_3 are themselves Bernoulli processes.

   a. Let $p_X(1) = q$, $p_X(2) = r$ and $p_X(3) = s$. Determine the parameters of the processes B_1, B_2 and B_3 in terms of p, q, r and s.

   b. Suppose B_1, B_2 and B_3 have parameters 0.1, 0.1 and 0.2 respectively. Determine p.

   c. Use your answers to parts (a) and (b) to determine the PMF $p_X(x)$.

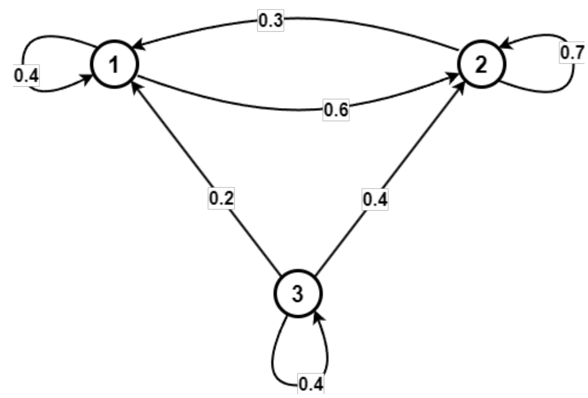
5. During morning rush hour, accidents on a certain 40 mile stretch of Highway 101 occur as a Poisson process with rate $\lambda = 1.5/\text{hour}$. Assume that the accidents occurring in the northbound lanes are independent of those occurring in the southbound lanes, and that these two groups of accidents themselves comprise two Poisson processes with arrival rates λ_N and λ_S, respectively. (We assume that no accident occurs on *both* sides.)

   a. Suppose $\lambda_N = 1.2/\text{hour}$. Determine λ_S.

   b. For any particular accident occurring on this stretch of Highway101, what is the probability that it occurs in the southbound lanes.

   c. What is the expected time from the beginning of rush hour to the 3rd northbound accident.

6. Consider a 3-state Markov chain model with the following state transition diagram.



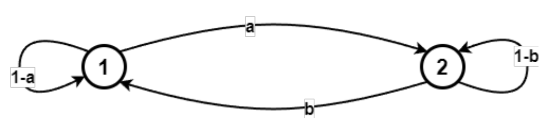
   a. Write the state transition matrix R.

   b. Find $P(X_2 = 2 \mid X_0 = 3)$.

   c. Find $P(X_3 = 2 \mid X_1 = 3)$.

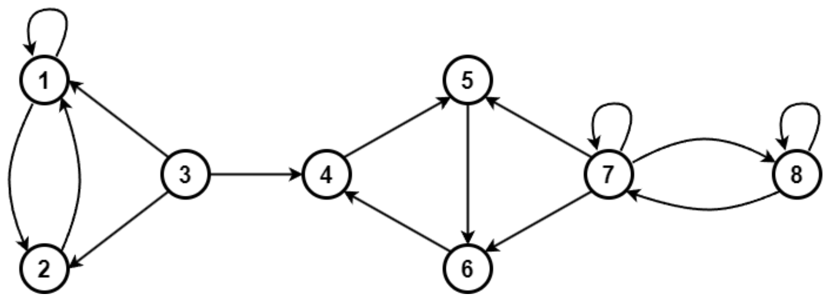
   d. Find $P(X_{101} = 2 \mid X_{100} = 1)$.

7. Let $0 < a < 1$ and $0 < b < 1$ and consider the 2-state Markov chain model with the following state transition diagram.



   Determine the steady-state probabilities of this Markov process.

8. Consider the Markov chain model with the following state transition diagram.



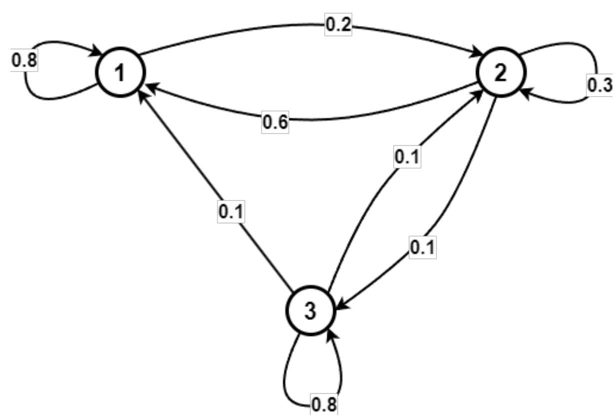
a. Determine $A(i)$ for each state $i \in \{1, 2, 3, 4, 5, 6, 7, 8\}$.

b. Determine which states are recurrent and which are transient.

c. Determine any periodic recurrent classes, and state the period.

d. Determine any aperiodic recurrent classes.

9. Consider the Markov chain model with the following state transition diagram.



Determine the steady-state probabilities of this Markov process.