CSE 107
Midterm 2 Review Problems

1. Bob throws a dart at a circular target of radius r. He hits the target with certainty, but is equally likely to hit any point within the target. Let Z be the distance from Bob's dart to the center of the target.

   a. Find the CDF $F_Z(z)$ and the PDF $f_Z(z)$.
   b. Find the mean $E[Z]$.
   c. Find the variance $Var(Z)$.

2. A city's temperature in degrees Celsius is modeled as a normal random variable X with mean 10 and standard deviation 10. Let Y be its temperature in Fahrenheit, where X and Y are related by

$$X = \frac{5(Y-32)}{9}.$$

   What is the probability that the temperature is above 77 degrees Fahrenheit?

3. Let X and Y be jointly continuous random variables, and suppose

$$f_{X|Y}(x|y) = \begin{cases} \frac{1}{y} & \text{if } 0 < y \le 1 \text{ and } 1 - y \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

   and

$$f_Y(y) = \begin{cases} 2y & \text{if } 0 < y \le 1 \\ 0 & \text{otherwise} \end{cases}.$$

   Hint: Before you do the following problems, draw a picture of the region defined by the inequalities $0 < y \le 1$ and $1 - y \le x \le 1$.

   a. Determine the joint PDF $f_{X,Y}(x,y)$.
   b. Determine the marginal PDF $f_X(x)$.
   c. Determine the expected value $E[X]$.
   d. Determine the conditional expectation $E[X|Y = y]$.

4. Let X be an exponential random variable with parameter λ, and let $Y = X + 1$. Determine the PDF $f_Y(y)$.

5. Alice is at the casino again, with a choice of two games. The first returns winnings (positive or negative) that are normally distributed with parameters $\mu = 1$ and $\sigma = 2$. The second is uniformly distributed with winnings in the range -1 to 2. (All amounts are in dollars.) She flips a coin with $P(\text{head}) = p$ to decide which game to play. If heads, she plays the first game, and if tails, she plays the second. Determine her expected winnings, in terms of p.

6. Let Y be a normal random variable with variance 1, and with mean another random variable X. Suppose X is continuous uniform on the interval $[1,3]$.

   a. Find the PDF $f_Y(y)$.

   b. Find the conditional PDF $f_{X|Y}(x|y)$.

   c. Suppose we sample Y and get $Y = 3$. What is the probability that $X \leq 2$?

   d. Find $E[Y]$.

7. Let X and Y be independent, jointly continuous random variables, where X is uniform on $[a, b]$ and Y is uniform on $[c, d]$.

   a. Write the PDFs $f_X(x)$ and $f_Y(y)$.

   b. Write the CDFs $F_X(x)$ and $F_Y(y)$.

   c. Let $Z = \max(X, Y)$. Determine the PDF $f_Z(z)$. (Assume $a \leq d$ and $c \leq b$, so that the two intervals overlap.)

   d. Let $Z = X + Y$. Determe the PDF $f_Z(z)$.