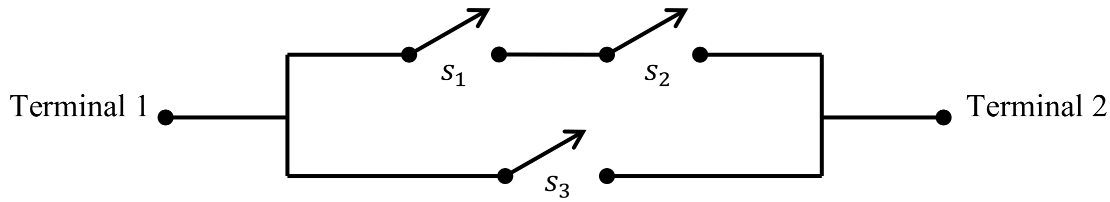
# CSE 107
# Midterm 1 Review Problems

1. In the circuit diagram below, switches $s_1$, $s_2$ and $s_3$ are randomly and independently set in the open or closed state. Let $A_i$ be the event that $s_i$ is **open** (for $i = 1, 2, 3$), and let $B$ be the event that there is a **closed path** from terminal 1 to terminal 2.




   Suppose that $P(A_i) = p_i$, for $i = 1, 2, 3$. Determine $P(B)$ in terms of $p_1$, $p_2$ and $p_3$.

2. A system consists of $n$ identical components, each of which is operational with probability $p$, independent of other components. The system is operational if at least $m$ out of the $n$ components are operational. What is the probability that the system is operational?

3. Alice and Bob have a chess match in which the first player to win a game wins the match. Each game has one of 3 possible outcomes: Bob wins, Alice wins, or the game is a draw. One game is played each day until someone wins, so the match is of potentially unlimited duration. The prize money starts at \$100 and goes up by \$100 each day a match is played. Alice wins with probability 0.4, Bob wins with probability 0.3, and a draw occurs with probability 0.3.

   a. What is the probability that Alice wins the match?

   b. Determine the mean and standard deviation of the total prize money.

4. A 3-sided die and a coin, which are neither fair nor independent, are rolled and tossed, respectively. The die has faces $\{1, 2, 3\}$ and the coin has sides labeled $\{1, 2\}$. Let $X$ be the outcome of the die, and $Y$ the outcome of the coin. The *conditional* PMF $p_{X|Y}(x|y)$ is given by the following table.

| | | | | |
|---|---|---|---|---|
| $y$ | 1 | $2/8$ | $5/8$ | $1/8$ |
| | 2 | $1/8$ | $3/8$ | $4/8$ |
| | | 1 | 2 | 3 |
| | | | $x$ | |

Also, the *marginal* PMF $p_Y(y)$ is given by the following table.

| | | |
|---|---|---|
| $y$ | 1 | $1/3$ |
| | 2 | $2/3$ |

a. Fill in the following table giving the *joint* PMF $p_{X,Y}(x, y)$.

| | | | | |
|---|---|---|---|---|
| $y$ | 1 | | | |
| | 2 | | | |
| | | 1 | 2 | 3 |
| | | | $x$ | |

b. Fill in the following table giving the *marginal* PMF $p_X(x)$.

| | | |
|---|---|---|
| | | |
| 1 | 2 | 3 |
| | $x$ | |

c. Fill in the following table giving the conditional PMF $p_{Y|X}(y|x)$.

| | | | | |
|---|---|---|---|---|
| $y$ | 1 | | | |
| | 2 | | | |
| | | 1 | 2 | 3 |
| | | | $x$ | |

d. Given that the coin flip is 2, what is the probability that the die roll is 3?

5. The number $X$ of phone calls received by a call center within a certain time period is a Poisson random variable with parameter $\lambda$. Determine the smallest positive number $\lambda$ such that the probability of receiving at least one call is at least $1/2$.