1. In the circuit diagram below, switches $s_1$, $s_2$ and $s_3$ are randomly and independently set in the open or closed state. Let $A_i$ be the event that $s_i$ is open (for $i = 1, 2, 3$), and let $B$ be the event that there is a closed path from terminal 1 to terminal 2.

Suppose that $P(A_i) = p_i$, for $i = 1, 2, 3$. Determine $P(B)$ in terms of $p_1$, $p_2$ and $p_3$.

2. A system consists of $n$ identical components, each of which is operational with probability $p$, independent of other components. The system is operational if at least $m$ out of the $n$ components are operational. What is the probability that the system is operational?

3. Alice and Bob have a chess match in which the first player to win a game wins the match. Each game has one of 3 possible outcomes: Bob wins, Alice wins, or the game is a draw. One game is played each day until someone wins, so the match is of potentially unlimited duration. The prize money starts at $100 and goes up by $100 each day a match is played. Alice wins with probability 0.4, Bob wins with probability 0.3, and a draw occurs with probability 0.3.

a. What is the probability that Alice wins the match?

b. Determine the mean and standard deviation of the total prize money.
4. A 3-sided die and a coin, which are neither fair nor independent, are rolled and tossed, respectively. The die has faces \{1, 2, 3\} and the coin has sides labeled \{1, 2\}. Let \(X\) be the outcome of the die, and \(Y\) the outcome of the coin. The conditional PMF \(p_{X|Y}(x|y)\) is given by the following table.

\[
\begin{array}{ccc}
    y & 1 & 2/8 \\
    2 & 1/8 & 3/8 \\
    1 & 2/8 & 3/8 \\
\end{array}
\]

Also, the marginal PMF \(p_Y(y)\) is given by the following table.

\[
\begin{array}{c}
    y \\
    1 & 1/3 \\
    2 & 2/3 \\
\end{array}
\]

a. Fill in the following table giving the joint PMF \(p_{X,Y}(x,y)\).

\[
\begin{array}{ccc}
    y & 1 & 2/8 \\
    2 & 1/8 & 3/8 \\
    1 & 2/8 & 3/8 \\
\end{array}
\]

b. Fill in the following table giving the marginal PMF \(p_X(x)\).

\[
\begin{array}{ccc}
    x & 1 & 2 \\
    1 & 2/8 & 3/8 \\
    2 & 1/8 & 3/8 \\
    3/8 & 3/8 & 3/8 \\
\end{array}
\]

c. Fill in the following table giving the conditional PMF \(p_{Y|X}(y|x)\).

\[
\begin{array}{ccc}
    y & 1 & 2/8 \\
    2 & 1/8 & 3/8 \\
    1 & 2/8 & 3/8 \\
\end{array}
\]

d. Given that the coin flip is 2, what is the probability that the die roll is 3?

5. The number \(X\) of phone calls received by a call center within a certain time period is a Poisson random variable with parameter \(\lambda\). Determine the smallest positive number \(\lambda\) such that the probability of receiving at least one call is at least \(1/2\).