CSE 107
Probability and Statistics for Engineers
Information for Final Exam

Mean and Variance Formulas:

$E[aX + b] = aE[X] + b$

$E[a_1X_1 + a_2X_2 + \cdots + a_nX_n] = a_1E[X_1] + a_2E[X_2] + \cdots + a_nE[X_n]$

$Var(aX + b) = a^2Var(X)$

Convolution Product:

$$(g * h)(z) = \int_{-\infty}^{\infty} g(x)h(z - x)\,dx$$

Random Variables:

Discrete Uniform on $[a, b] = \{a, a + 1, a + 2, \dots, b\}$, where $a, b \in \mathbb{Z}$

$$p_X(k) = \begin{cases} \dfrac{1}{b - a + 1} & \text{if } a \le k \le b \\ 0 & \text{otherwise} \end{cases} \qquad F_X(x) = \begin{cases} 0 & \text{if } x < a \\ \dfrac{\lfloor x \rfloor - a + 1}{b - a + 1} & a \le x < b \\ 1 & x \ge b \end{cases}$$

$$E[X] = \frac{a + b}{2} \qquad Var(X) = \frac{(b - a)(b - a + 2)}{12}$$

Bernoulli with parameter p

$$p_X(k) = \begin{cases} p & \text{if } k = 1 \\ 1 - p & \text{if } k = 0 \end{cases} \qquad F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - p & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$$

$$E[X] = p \qquad Var(X) = p(1 - p)$$

Binomial with parameters n, p

$$p_X(k) = \begin{cases} \binom{n}{k} p^k (1 - p)^{n-k} & \text{if } 0 \le k \le n \\ 0 & \text{otherwise} \end{cases} \qquad F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ \displaystyle\sum_{k=0}^{\lfloor x \rfloor} \binom{n}{k} p^k (1 - p)^{n-k} & 0 \le x < n \\ 1 & x \ge n \end{cases}$$

$$E[X] = np \qquad Var(X) = np(1 - p)$$

Geometric with parameter p

$$p_X(k) = \begin{cases} (1-p)^{k-1} \cdot p & \text{if } k = 1, 2, \ldots \\ 0 & \text{otherwise} \end{cases} \qquad F_X(x) = \begin{cases} 0 & \text{if } x < 1 \\ 1 - (1-p)^{\lfloor x \rfloor} & x \geq 1 \end{cases}$$

$$E[X] = \frac{1}{p} \qquad Var(X) = \frac{1-p}{p^2}$$

Poisson with parameter λ

$$p_X(k) = \begin{cases} e^{-\lambda} \cdot \dfrac{\lambda^k}{k!} & \text{if } k = 0, 1, \ldots \\ 0 & \text{otherwise} \end{cases} \qquad F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ \displaystyle\sum_{k=0}^{\lfloor x \rfloor} e^{-\lambda} \cdot \frac{\lambda^k}{k!} & x \geq 0 \end{cases}$$

$$E[X] = \lambda \qquad Var(X) = \lambda$$

Continuous Uniform on $[a, b]$ where $a, b \in \mathbb{R}$

$$f_X(x) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases} \qquad F_X(x) = \begin{cases} 0 & \text{if } x < a \\ \dfrac{x-a}{b-a} & a \leq x < b \\ 1 & x \geq b \end{cases}$$

$$E[X] = \frac{a+b}{2} \qquad Var(X) = \frac{(b-a)^2}{12}$$

Exponential with parameter λ

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases} \qquad F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-\lambda x} & x \geq 0 \end{cases}$$

$$E[X] = \frac{1}{\lambda} \qquad Var(X) = \frac{1}{\lambda^2}$$

Normal with mean μ and variance σ^2

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad F_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{x} e^{-\frac{(t-\mu)^2}{2\sigma^2}}\, dt$$

$$E[X] = \mu \qquad Var(X) = \sigma^2$$

Standard Normal ($\mu = 0$ and $\sigma = 1$)

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \qquad \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}}\, dt$$

$$E[X] = 0 \qquad Var(X) = 1$$

Distributions Associated with Random Processes

Pascal Distribution of order k

Let Y_k be the k^{th} arrival time in a Bernoulli process with parameter p. Then $Y_k = T_1 + T_2 + \cdots + T_k$ where T_i are independent geometric random variables with paramenter p.

$$p_{Y_k}(t) = \binom{t-1}{k-1} p^k (1-p)^{t-k} \qquad \text{for } t = k, k+1, k+2, \ldots$$

$$E[Y_k] = \frac{k}{p} \qquad\qquad Var(Y_k) = \frac{k(1-p)}{p^2}$$

Erlang Distribution of order k

Let Y_k be the k^{th} arrival time in a Poisson process with parameter λ. Then $Y_k = T_1 + T_2 + \cdots + T_k$ where T_i are independent exponential random variables with paramenter λ.

$$f_{Y_k}(t) = \frac{\lambda^k t^{k-1} e^{-\lambda t}}{(k-1)!} \qquad \text{for } t \in [0, \infty)$$

$$E[Y_k] = \frac{k}{\lambda} \qquad\qquad Var(Y_k) = \frac{k}{\lambda^2}$$

Equations Associated with Markov Chains

Consider a Markov chain model with state space $S = \{1, 2, \ldots, m\}$ and state transition probabilities p_{ij} for $i, j \in S$. Let X_n denote the state at time $n \geq 0$, and let $r_{ij}(n) = P(X_n = j \mid X_0 = i)$ be the n-step transition probabilities. Then $r_{ij}(n)$ satisfies the following recurrence.

Chapman-Kolmogorov Equations: $$r_{ij}(n) = \sum_{k=1}^{m} r_{ik}(n-1) \cdot p_{kj} \qquad \text{for all } i, j \in S \text{ and } n \geq 1$$

Suppose the Markov chain consists of one aperiodic recurrent class and possibly some transient states. Let π_j denote the steady-state probabilityes for $j \in S$. Then π_j satisfy

Balance Equations: $$\pi_j = \sum_{k=1}^{m} \pi_k \cdot p_{kj} \qquad \text{for all } j \in S$$

Normalization Equations: $$1 = \sum_{k=1}^{m} \pi_k$$

Standard Normal Cumulative Distribution Function

	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998