

## CSE 107

### Midterm 2 Review Problems

- Bob throws a dart at a circular target of radius  $r$ . He hits the target with certainty, but is equally likely to hit any point within the target. Let  $Z$  be the distance from Bob's dart to the center of the target.
  - Find the CDF  $F_Z(z)$  and the PDF  $f_Z(z)$ .
  - Find the mean  $E[Z]$ .
  - Find the variance  $Var(Z)$ .

- A city's temperature in degrees Celsius is modeled as a normal random variable  $X$  with mean 10 and standard deviation 10. Let  $Y$  be its temperature in Fahrenheit, where  $X$  and  $Y$  are related by

$$X = \frac{5(Y - 32)}{9}.$$

What is the probability that the temperature is above 77 degrees Fahrenheit?

- Let  $X$  and  $Y$  be jointly continuous random variables, and suppose

$$f_{X|Y}(x|y) = \begin{cases} \frac{1}{y} & \text{if } 0 < y \leq 1 \text{ and } 1 - y \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

and

$$f_Y(y) = \begin{cases} 2y & \text{if } 0 < y \leq 1 \\ 0 & \text{otherwise} \end{cases}.$$

Hint: Before you do the following problems, draw a picture of the region defined by the inequalities  $0 < y \leq 1$  and  $1 - y \leq x \leq 1$ .

- Determine the joint PDF  $f_{X,Y}(x,y)$ .
  - Determine the marginal PDF  $f_X(x)$ .
  - Determine the expected value  $E[X]$ .
  - Determine the conditional expectation  $E[X|Y = y]$ .
- Let  $X$  be an exponential random variable with parameter  $\lambda$ , and let  $Y = X + 1$ . Determine the PDF  $f_Y(y)$ .
  - Alice is at the casino again, with a choice of two games. The first returns winnings (positive or negative) that are normally distributed with parameters  $\mu = 1$  and  $\sigma = 2$ . The second is uniformly distributed with winnings in the range  $-1$  to  $2$ . (All amounts are in dollars.) She flips a coin with  $P(\text{head}) = p$  to decide which game to play. If heads, she plays the first game, and if tails, she plays the second. Determine her expected winnings, in terms of  $p$ .

6. Let  $Y$  be a normal random variable with variance 1, and with mean another random variable  $X$ . Suppose  $X$  is continuous uniform on the interval  $[1, 3]$ .
- Find the PDF  $f_Y(y)$ .
  - Find the conditional PDF  $f_{X|Y}(x|y)$ .
  - Suppose we sample  $Y$  and get  $Y = 3$ . What is the probability that  $X \leq 2$  ?
  - Find  $E[Y]$ .