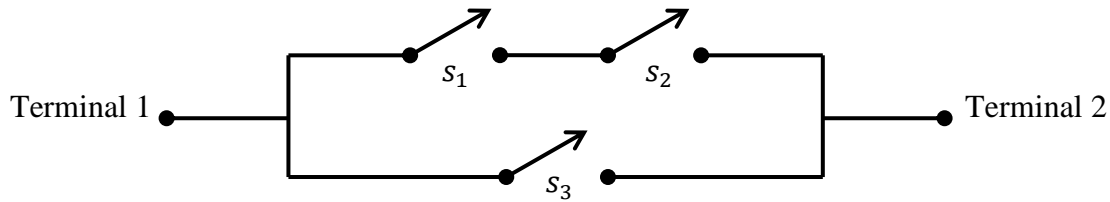


CSE 107

Midterm 1 Review Problems

1. In the circuit diagram below, switches s_1 , s_2 and s_3 are randomly and independently set in the open or closed state. Let A_i be the event that s_i is **open** (for $i = 1, 2, 3$), and let A be the event that there is a **closed path** from terminal 1 to terminal 2.



Suppose that $P(A_i) = p_i$, for $i = 1, 2, 3$. Determine $P(A)$ in terms of p_1 , p_2 and p_3 .

2. A system consists of n identical components, each of which is operational with probability p , independent of other components. The system is operational if at least m out of the n components are operational. What is the probability that the system is operational?
3. Alice and Bob have a chess match in which the first player to win a game wins the match. Each game has one of 3 possible outcomes: Bob wins, Alice wins, or the game is a draw. One game is played each day until someone wins, so the match is of potentially unlimited duration. The prize money starts at \$100 and goes up by \$100 each day a match is played. Alice wins with probability 0.4, Bob wins with probability 0.3, and a draw occurs with probability 0.3.
 - a. What is the probability that Alice wins the match?
 - b. Determine the mean and standard deviation of the total prize money.

4. A 3-sided die and a coin, which are neither fair nor independent, are rolled and tossed, respectively. The die has faces $\{1, 2, 3\}$ and the coin has sides labeled $\{1, 2\}$. Let X be the outcome of the die, and Y the outcome of the coin. The *conditional* PMF $p_{X|Y}(x|y)$ is given by the following table.

y	1	$2/8$	$5/8$	$1/8$
	2	$1/8$	$3/8$	$4/8$
		1	2	3
		x		

Also, the *marginal* PMF $p_Y(y)$ is given by the following table.

y	1	$1/3$
	2	$2/3$

- a. Fill in the following table giving the *joint* PMF $p_{X,Y}(x, y)$.

y	1			
	2			
		1	2	3
		x		

- b. Fill in the following table giving the *marginal* PMF $p_X(x)$.

	1	2	3
	x		

- c. Fill in the following table giving the conditional PMF $p_{Y|X}(y|x)$.

y	1			
	2			
		1	2	3
		x		

- d. Given that the coin flip is 2, what is the probability that the die roll is 3?

5. The number X of phone calls received by a call center within a certain time period is a Poisson random variable with parameter λ . Determine the smallest positive number λ such that the probability of receiving at least one call is at least $1/2$.