CSE 101
Final Review Problems

1. Determine whether the following statements are True or False. No justification is required.
   a. \( n^{\sqrt{n}} = \Omega(n^2) \)
   b. \( n^4 = O(n^3) \)
   c. \( n^2 = \Theta(9^{\log_3(n)}) \)
   d. \( n^{\sqrt{n}} = \omega(\sqrt{n}) \)
   e. \( n^2 = o(n^3) \)
   f. \( \ln(n) = o(n) \)
   g. \( 2^n = O(n^2) \)
   h. \( n^{1.5} = \omega(n^{1.45}) \)
   i. \( n \ln(n) = \Theta(\ln(\ln(n))) \)
   j. \( f(n) = \omega(f(n)) \) for any function \( f(n) \)

2. Given a Binary Search Tree based on the following C++ struct

   ```cpp
   struct Node{
     int key;
     Node* left;
     Node* right;
   };
   ```

   Complete the recursive C++ function below called `TreeWalk()` that takes as input a `Node` pointer \( R \) and a string \( s \), then returns a string consisting of all keys in the subtree rooted at \( R \), separated by spaces. The order of the keys depends on the input string \( s \), which will be either "pre", "in" or "post", indicating a pre-order, in-order or a post-order tree walk, respectively. If the input \( s \) is not one of the strings "pre", "in" or "post", then your function will return the empty string. The recursion will terminate when \( R \) has the value `nullptr`.

   ```cpp
   std::string TreeWalk(Node* R, std::string s){
     // your code starts here
     // your code ends here
   }
   ```
3. Perform Dijkstra($G, s$) on the weighted digraph below with source vertex $s = 5$. If at some point two vertices have equal minimum d-values, extract the one with smaller label first from the min Priority Queue.

![Graph Diagram]

a. Determine the order in which vertices are extracted from the min Priority Queue.

b. For each vertex $x$, determine the values $d[x]$ and $p[x]$.

**Solution:**

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d[x]$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p[x]$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4. Perform BuildHeap($A$) on the following unordered array $A$, making it into a max-heap. Observe that identical keys are accompanied by letters representing different satellite data. Thus the elements 2a and 2b have the same key, but are distinguishable elements in the max-heap.

<table>
<thead>
<tr>
<th>A</th>
<th>2a</th>
<th>4</th>
<th>7</th>
<th>1a</th>
<th>2b</th>
<th>3</th>
<th>1b</th>
<th>5a</th>
<th>2c</th>
<th>6</th>
<th>8</th>
<th>5b</th>
</tr>
</thead>
</table>

Show the state of array $A$ after the call to BuildHeap($A$).
5. Insert the keys: 5, 9, 7, 2, 6, 4, 8, 3, 1, 10 (in order) into an initially empty Binary Search Tree $T$.
(Note: use the Binary Search Tree Insert algorithm to do this.)

a. Give the keys in the order printed by a **pre-order tree walk**.

b. Give the keys in the order printed by a **post-order tree walk**.

Note: the three questions below **do not** refer in any way to the Red Black Tree Insert algorithm. Instead they ask if it is possible to assign colors in the BST $T$, which you found above, so as to satisfy the RBT properties. Be sure to include nil children when computing the black-height of $T$.

c. Is it possible to assign the colors \{Red, Black\} to the vertices of $T$ so that the Red-Black Tree properties are satisfied, and $bh(T) = 1$? If it is possible, specify all such colorings by stating, for each coloring, the set of keys belonging to red nodes.

d. Is it possible to assign the colors \{Red, Black\} to the vertices of $T$ so that the Red-Black Tree properties are satisfied, and $bh(T) = 2$? If it is possible, specify all such colorings by stating, for each coloring, the set of keys belonging to red nodes.

e. Is it possible to assign the colors \{Red, Black\} to the vertices of $T$ so that the Red-Black Tree properties are satisfied, and $bh(T) = 3$? If it is possible, specify all such colorings by stating, for each coloring, the set of keys belonging to red nodes.