Finite-Sample Critical Values of the KPSS Test:

A Response Surface Approach

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ABSTRACT

In examining the size property of the KPSS test, this study shows that finite-sample critical values are a function of both the sample size and the lag truncation parameter. Response surface estimates of the finite-sample critical values are provided.

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I. Introduction

Results from standard unit-root tests, such as the augmented Dickey-Fuller test, indicate that economic time series are typically characterized by unit-root persistence. The prevalence of the findings is met with a growing concern about the ability of these tests to find stationarity even when a series actually has no unit root. Stock (1991), for example, show that the ADF test has little power to discriminate between the unit-root null hypothesis and local stationary alternatives. The underlying problem is that, since unit-root nonstationarity is often maintained as the null hypothesis, one cannot reject it unless statistical evidence strongly suggests otherwise.

Kwiakowski, Phillips, Schmidt, and Shin (1992) recently propose a procedure, called the KPSS test, for testing for a unit root. This procedure tests the stationary null hypothesis against the unit-root alternative. The KPSS test allows examination of the persistence issue from a perspective complementary to standard unit-root tests. The stationary hypothesis has the benefit of a doubt in the former case, whereas the unit-root hypothesis enjoys such benefit in the latter case. Because of this basic difference in the test design, the KPSS test and conventional unit-root tests may yield additional useful information on data persistence when their results are interpreted together. For instance, if the ADF test fails to reject the unit-root null and the KPSS test rejects its stationary null, the results will represent strong evidence for a unit-root process. If the two types of tests both fail to reject their respective null hypotheses, on the other hand, the results will indicate that the data are not sufficiently informative regarding the presence or absence of a unit root. These two testing approaches thus complement one another; together they can provide more informative inferences than either test can do alone. Recent studies employing both stationarity and unit tests were, for example, Cheung, Chinn and Tran (1995) and Cheung and Chinn (forthcoming).

Kwiakowski *et al.* (1992) derive the asymptotic distributions for two KPSS statistics — one has a level-stationary null and the other has a trend-stationary null. By approximating the asymptotic distributions in simulation, asymptotic critical values for the two KPSS statistics are provided in Table 1 of their paper. Empirical applications of the tests encounter two potential problems, however. One is related to the relevance and accuracy of the asymptotic critical values, given that empirical applications necessarily deal with finite-sample data. If the finite-sample critical values are different from the asymptotic ones, using the latter will bias the KPSS tests toward rejecting the null of stationarity either too often or too infrequently. The second issue concerns the choice of the lag truncation parameter, which has to be determined to compute the heteroskedasticity and autocorrelation consistent variance estimator. Under appropriate conditions, the asymptotic critical values for the KPSS statistics should not depend on the lag truncation parameter. In finite samples, nonetheless, the choice of lag may affect the appropriate critical values.

Based on response surface analysis, this study illustrates the effects of the sample size and the lag truncation specification on the finite-sample critical values of the two KPSS statistics. To account for both effects directly, the finite-sample critical values for a wide range of sample sizes and lag truncation parameters are summarized in a few response surface equations.

The two KPSS statistics are described in the following section. Section

III discusses the Monte Carlo design and the results of response surface estimation. Concluding remarks are given in the last section.

II. The KPSS Test

A time series $\{Y_t\}_{t=1,\dots,T}$ is considered to have three components: a deterministic term, a random walk, and a stationary error. The stationary null hypothesis is specified as the variance of errors in the random walk component being equal to zero. The time series is difference-stationary under the alternative. Two possible null hypotheses are allowed for: level- and trend-stationary. Under the trend-stationary null, the deterministic term has a time trend component.

The KPSS statistics are conducted based on the least squares residuals from regressing Y on the deterministic term. For the level-stationary null, we first obtain the residual $e_t = Y_t - \bar{Y}$, where $\bar{Y}$ is the mean. The KPSS statistic, $\hat{\eta}_\mu$, is given by

$$\hat{\eta}_\mu = T^{-2} \sum_{t=1}^{T} S_t^2 \ / \ s^2(\ell), \tag{1}$$

where the S_t is the partial sum process of $\{e_t\}$ defined by

$$S_t = \sum_{i=1}^{t} e_i \tag{2}$$

and $s^2(\ell)$ is the serial correlation and heteroskedasticity consistent variance estimator given by (see Newey and West, 1987)

$$s^2(\ell) = T^{-1}\sum_{t=1}^{T} e_t^2 + 2T^{-1}\sum_{s=1}^{\ell} \omega(s,\ell) \sum_{t=s+1}^{T} e_t \, e_{t-s}. \tag{3}$$

Following Kwiakowski *et al.* (1992), the optimal weighting function $\omega(s,\ell) = 1 - s/(1+\ell)$ is used, which guarantees the nonnegativity of $s^2(\ell)$. The null of level-stationarity is rejected in favor of the unit-root alternative when $\hat{\eta}_\mu$ is larger than the appropriate critical value.

A similar procedure is employed to construct the KPSS statistic for the trend-stationary null hypothesis, with the exception that the residual e_t is obtained from regressing Y_t on a constant and a time trend. The resulting statistic is labelled $\hat{\eta}_{\tau}$.

The lag truncation parameter, ℓ, in equation (3) is a choice parameter. A necessary condition for the consistency of $s^2(\ell)$ under the null is $\ell \longrightarrow \infty$ as $T \longrightarrow \infty$. After considering a few ℓz-rules, which set $\ell = \text{INT}[z(T/100)^{1/4}]$, Kwiakowski *et al.* (1992) suggest the ℓ8-rule provides a good compromise between size and power.

III. Finite-Sample Critical Values

The response surface methodology is applied to estimate the finite-sample critical values of the two KPSS statistics. In general, response surface analysis examines the response of a variable, called the response variable, to systematic changes in a set of control variables. The interactions between the response and control variables are explored via Monte Carlo experiments. The information from the resulting experimental data is summarized using computationally simple response surface equations. Early econometric applications of the response surface technique include Hendry (1979) and Hendry and Harrison (1974). Hendry (1984) provides an excellent review on the use of response surface analysis in econometrics. Some recent applications are Cheung and Lai (1993), Ericsson (1991), and MacKinnon (1991).

In this study, the response variable is the finite-sample critical value and the control variables are the sample size T and lag truncation parameter ℓ. A Monte Carlo experiment, covering 371 combinations of T and ℓ with T = {30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100, 120, 150, 200, 250, 300, 350, 400, 450, 500} and ℓ = {0, 1, ...,

ℓ12), is used to generate data for estimating the response surface equations. In each given (T,ℓ)-experiment, the 10%, 5%, and 1% finite-sample critical values are all calculated directly as quantiles of the empirical distribution based on 20,000 replications.

The data generating process is

$$Y_t = \varepsilon_t, \tag{4}$$

where ε_t is an IIDN(0,1) error term. Sample series of T + 50 observations are first generated and then only the last T observations are used in the simulation. The routines RAN3 and GASDEV of Press *et al.* (1992) are used to generate the pseudo-random normal variates.

A part of the response surface analysis involves the choice of the functional form for the response surface equation. Although the critical values are expected to be dependent on T and ℓ in finite samples, they should approach their theoretical asymptotic levels for any given ℓ as T increases to infinity. This is a useful guideline for specifying a response surface equation. The response surface estimation results reported in Table 1 are based on the following regression equation:

$$CR_{T,\ell} = \alpha + \sum_{i=1}^{r} \beta_i (1/T)^i + \sum_{i=1}^{s} \gamma_i (\ell/T)^i + \varepsilon_{T,\ell}. \tag{5}$$

$CR_{T,\ell}$ is the critical value estimate for a sample size T and lag truncation parameter ℓ from the Monte Carlo simulation, and $\varepsilon_{T,\ell}$ is the error term. The terms involving 1/T and ℓ/T capture the effects of the sample size and the lag truncation parameter, respectively. Since these two terms go to zero as T approaches infinity, the intercept α can be interpreted as an estimator of the asymptotic critical value.

Functional forms other than (5) were also explored. The additional specifications considered include (1) more general polynomial equations that incorporate $(1/T)^{i+1/2}$ and $(\ell/T)^{i+1/2}$, (2) the replacement of $T-\ell$ with T, and (3) the use of $CR_{T,\ell}/CR_{\infty}$ as the left-hand-side variable, where CR_{∞} is the asymptotic critical value from Kwiakowski *et al.* (1992). However, all these different specifications fail to yield better explanatory power than equation (5).

Six response surface equations are presented in Table 1 for the two KPSS statistics. The polynomial orders r and s are determined by their explanatory power. r = 2 and s = 1 are selected for the KPSS $\hat{\eta}_{\mu}$ statistic, and r = 2 and s = 3 for the KPSS $\hat{\eta}_{\tau}$ statistic. Both the 1/T and ℓ/T variables are significant, as evidenced by the heteroskedasticity consistent standard errors reported below the coefficient estimates. The $\bar{R}^2$ measure indicates that these specifications describe the data pretty well. The $\bar{R}^2$'s are mostly larger than 0.90. The only exception is the fitted response surface equation for the 10% critical values of the KPSS $\hat{\eta}_{\mu}$ statistic. For this case, the simulation experiment is repeated with 30,000 replications for re-estimating the corresponding response surface regression. The resulting explanatory power is very similar to that reported in Table 1. The low $\bar{R}^2$ result may indicate that the 10% critical values of the KPSS $\hat{\eta}_{\mu}$ statistic display not much variation with respect to T and ℓ.

On the other hand, the other measures of data fit consistently show that the six response surface equations capture the data information very well. The standard error of the regression ($\hat{\sigma}$) is in the narrow range of .0007 to .0155. The maximum and the mean of absolute estimated errors ($\mathrm{Max}|\hat{\varepsilon}_{T,\ell}|$ and $\overline{|\hat{\varepsilon}_{T,\ell}|}$) are also very small. We observe that the $\mathrm{Max}|\hat{\varepsilon}_{T,\ell}|$ is usually associated only with a small T and large ℓ.

The α-estimates are fairly close to the asymptotic critical values given in Kwiakowski *et al*. (1992). The 10%, 5%, and 1% critical values derived from the asymptotic distributions are, respectively, .347, .463, and .739. for the level-stationary null hypothesis and .119, .146, and .216 for the trend-stationary null. The corresponding values from the response surface regressions are .348, .462, and .738 for the former case and .119, .148, and .218 for the latter case. The close match in estimates further illustrates the ability of the response surface equations to model the critical values of the two KPSS statistics.

IV. Summary

The behavior of the finite-sample critical values of two KPSS statistics has been examined, with a special focus on their potential dependence on not only the simple size but also the lag truncation parameter in these tests. The latter type of dependence has generally been ignored in previous work. In this study, response surface analysis is used to investigate such possible dependence. The results show that both the sample size and the lag truncation parameter can systematically affect the finite-sample critical values for the KPSS test. The response surface equations are also provided, which can be used to compute approximate finite-sample critical values that correct for the sample size and lag truncation parameter effects.

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Table 1. Response Surface Equation*

	KPSS $\hat{\eta}_\mu$ Test			KPSS $\hat{\eta}_\tau$ Test		
	10%	5%	1%	10%	5%	1%
α	.348	.426	.738	.119	.148	.218
	(.001)	(.001)	(.003)	(.000)	(.000)	(.000)
β_1	-.281	-.361	-2.200	-.040	-.199	-.730
	(.103)	(.132)	(.487)	(.017)	(.025)	(.053)
β_2	10.527	4.947	41.001	3.826	6.033	14.861
	(2.736)	(3.608)	(14.111)	(.523)	(.726)	(1.559)
γ_1	-.048	-.362	-1.493	-.021	-.132	-.532
	(.005)	(.008)	(.035)	(.003)	(.007)	(.015)
γ_2				.267	.246	.642
				(.037)	(.075)	(.158)
γ_3				1.220	1.977	4.228
				(.100)	(.229)	(.474)
$\bar{R}^2$	.248	.932	.951	.989	.949	.988
$\hat{\sigma}$	.0035	.0045	.0156	.0007	.0010	.0021
$\text{Max}\lvert\hat{\varepsilon}_{T,\ell}\rvert$	.0089	.0100	.0429	.0020	.0032	.0072
$\overline{\lvert\hat{\varepsilon}_{T,\ell}\rvert}$	.0026	.0036	.0120	.0006	.0008	.0016

* The response surface equation is given by equation (5). The KPSS $\hat{\eta}_\mu$ test has the null hypothesis of level-stationarity, while the KPSS $\hat{\eta}_\tau$ test has the trend-stationary null. Both tests have difference stationarity as the alternative hypothesis. $\hat{\sigma}$ represents the standard error of the regression. $\text{Max}\lvert\hat{\varepsilon}_{T,\ell}\rvert$ and $\overline{\lvert\hat{\varepsilon}_{T,\ell}\rvert}$ correspond to the maximum and the mean of the absolute estimated errors. Heteroskedasticity consistent standard errors are given in parentheses.