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Exchange rate risk premiums

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A state space model which allows for the covariation of risk premiums and unexpected rates of depreciation is used to study exchange rate risk premiums. We find that exchange rate risk premiums have a high degree of persistence and the covariance of risk premiums and unexpected rates of depreciation is negative. Regressions of the estimated risk premium on its determinants implied by the equilibrium model of Lucas (1982) show limited support for the model. (JEL F31).

The presence of risk premiums in foreign exchange markets has significant implications for models of exchange rate determination, effectiveness of sterilized intervention, and individuals' investment decisions. Various approaches have been advanced to model and investigate foreign exchange rate risk premiums. Earlier efforts on the search for risk premiums in foreign exchange markets show little support for the risk premium hypothesis and are reviewed in, for example, Boothe and Longworth (1986), Frankel (1988), Froot and Thaler (1990), Hodrick (1987), Levich (1985), and Meese (1989). On the other hand, recent studies using advanced techniques to examine more complex models of risk premiums find mixed results. These studies include Baillie and Bollerslev (1990), Black and Salemi (1988), Bomhoff and Koedijk (1988), Giovannini and Jorion (1989), Hodrick (1989), Kaminsky and Peruga (1990), Levine (1989), and Obstfeld (1990). However, models of risk premiums based on, say, the asset pricing framework of Lucas (1982) are usually rejected by the data.

Most econometric models of risk premiums involve financial data and market fundamentals and are based on regression equations. It is noted that, in some cases, the choice of regressors is motivated by the availability of data (Hansen and Hodrick, 1983, p. 120). In some cases, the reduced-form equation is derived from strong assumptions on the source of uncertainty, the form of the utility function, and the stochastic process that generates the exogenous variables.[1]

Wolff (1987) suggests a state space model and the Kalman filtering technique to study exchange rate risk premiums. The advantage of this signal extraction approach is that we can empirically characterize the temporal behavior of risk premiums using only data on spot and forward exchange rates. We can avoid

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the limitations of specifying (the functional form of) the underlying economic determinants of risk premiums and strong assumptions (mentioned earlier) associated with the regression-based approach. On the other hand, the state space model yields little information on the relation of exchange rate risk premiums and other economic variables. Hence, we see the signal extraction approach is a complement to, rather than a substitute for, the traditional regression-based approach.

The standard state space model used in Wolff (1987) assumes that the risk premium and the unexpected rate of depreciation at time t are not correlated. However, it is obvious that both the risk premium and unexpected change in the spot rate at time t depend on the information which comes to the market from time $t-1$ to t. The information that contributes to the unexpected change in the spot rate may also affect the risk premium. For example, using the model discussed in Hodrick and Srivastava (1986), it can be shown that both the risk premium and unexpected spot rate change are functions of the conditional means and conditional variances of the domestic and foreign money supplies. This means shocks to monies can induce comovements in risk premiums and unexpected changes in the spot rate.

In this paper a state space model that allows for correlations in risk premiums and unexpected rates of depreciation is used to study exchange rate risk premiums. This generalized state space model provides a flexible structure to capture the possible linkage between risk premiums and unexpected rates of depreciation. Moreover, we use the estimated risk premium to study the relationship between risk premiums and macro variables implied by the intertemporal asset pricing model of Lucas (1982).

The remainder of this paper is organized as follows. Section I presents the state space model that allows for the correlation of risk premiums and unexpected rates of depreciation. Section II reports the estimation results, compares the forecasting performance of the state space model with that of the random walk hypothesis, and presents results of regressing the estimated risk premium on macro variables. Section III summarizes the paper.

I. The state space model

The state space model used to study the temporal behavior of risk premiums can be written as follows:

$$\langle 1 \rangle \qquad D_t = P_t + v_{t+1},$$

$$\langle 2 \rangle \qquad P_t = \phi P_{t-1} + a_t,$$

$$\langle 3 \rangle \qquad \begin{pmatrix} a_t \\ v_t \end{pmatrix} \sim iidN\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q^2 & C \\ C & R^2 \end{pmatrix}\right],$$

where

$$\langle 4 \rangle \qquad D_t \equiv F_t - S_{t+1},$$

$$\langle 5 \rangle \qquad P_t \equiv F_t - E_t S_{t+1},$$

$$\langle 6 \rangle \qquad v_{t+1} \equiv E_t S_{t+1} - S_{t+1}.$$

F_t and S_t are the natural logarithms of the one period ahead forward rate and the spot exchange rate at time t respectively. E_t is the conditional expectations operator based on information available at time t. D_t is the forecast error from using F_t to predict S_{t+1}. P_t is the unobservable risk premium and v_t is the unexpected change in the spot rate.[2] Under the efficient market hypothesis $\{v_t\}$ is an uncorrelated, zero mean sequence. Here P_t is written as an AR(1) process with an innovation term a_t for exposition purposes. The vector version, which is used when P_t follows a different ARMA process, is given in Appendix A. Note that both a_t and v_t are affected by information that reaches the market from time $t-1$ to t. The possible comovement of a_t and v_t is modeled explicitly by incorporating a non-zero covariance term, C, in equation $\langle 3 \rangle$. Standard state space models like the one used in Wolff (1987) typically have $C \equiv 0$.

The Kalman filter algorithm for the state space model (equations $\langle 1 \rangle$, $\langle 2 \rangle$, and $\langle 3 \rangle$) is given by:

$$\langle 7 \rangle \qquad E_t P_{t+1} = \phi E_t P_t + C H_t^{-1}(D_t - E_{t-1} P_t),$$

$$\langle 8 \rangle \qquad V_t(P_{t+1}) = \phi^2 V_t(P_t) + Q^2 - C^2 H_t^{-1} - 2\phi C K_t,$$

$$\langle 9 \rangle \qquad K_t = V_{t-1}(P_t) H_t^{-1},$$

$$\langle 10 \rangle \qquad E_t P_t = E_{t-1} P_t + K_t(D_t - E_{t-1} P_t),$$

$$\langle 11 \rangle \qquad V_t(P_t) = V_{t-1}(P_t)[1 - K_t],$$

where $H_t \equiv V_{t-1}(P_t) + R^2$, $V_{t-1}(P_t) \equiv E(E_{t-1}P_t - P_t)^2$, and $V_t(P_t) \equiv E(E_t P_t - P_t)^2$; $t = 1, 2, \ldots, T$. The derivation is given in Appendix B. Without the normality assumption, the Kalman filter is the best linear unbiased filter instead of the best unbiased filter (Anderson and Moore, 1979). Baillie and Bollerslev (1990) show that monthly exchange rate data, which are used in this study, exhibit minimal ARCH effects and are reasonably approximated by the normal distribution.

The standard Kalman filter algorithm is obtained by setting $C = 0$ in equations $\langle 7 \rangle$ and $\langle 8 \rangle$. When C is non-zero, 'observing' v_t (via D_t) provides useful information about the realization of a_t. The modified Kalman filter exploits this fact and processes information more efficiently in predicting the risk premium, P_t, and its variance. As pointed out by a referee, the incorrect constraint $C = 0$ can be interpreted as a misspecification of equation $\langle 2 \rangle$.

The starting values (P_0 and $V(P_0)$) required to initialize the filter are set equal to the unconditional mean and variance of the risk premium. Moreover, we set $C = 0$ while updating P_1 with equations $\langle 7 \rangle$ and $\langle 8 \rangle$.

Define $r_t \equiv D_t - E_{t-1}P_t$ to be the one period ahead prediction error. As $r_t \sim iidN(0, H_t)$, the log likelihood function of the model can be written as

$$\langle 12 \rangle \qquad L = -\frac{T}{2} \ln 2\pi - \frac{1}{2} \sum_{t=1}^{T} \ln H_t - \frac{1}{2} \sum_{t=1}^{T} r_t^2 / H_t.$$

A time series model for the risk premium is required to calculate the maximum likelihood estimates of ϕ, R^2, Q^2, and C. Unfortunately, *a priori* information on the ARMA representation for the risk premium is usually not available. The following strategy is adopted to determine the time series model for the risk premium. First, sample autocorrelation and partial autocorrelation functions of $\{D_t\}$ are used to infer the possible time series representation for the risk premium.[3]

Then, for each possible representation for the risk premium, we maximize equation ⟨12⟩ with respect to the corresponding parameters. Finally, model selection is based on the Akaike Information Criterion (AIC) and the Schwartz Bayesian Criterion (SBC).

II. Empirical results

II.A. State space model

The state space model (equations ⟨1⟩, ⟨2⟩, and ⟨3⟩) is used to study the risk premiums of three bilateral US dollar exchange rates: the British pound (BP), the Deutsche mark (DM), and the Japanese yen (JY). One month forward rates and the corresponding spot rates measured at the end of each month are used. The data were taken from the Chicago Mercantile Exchange and the International Monetary Market yearbooks. The state space model is estimated using data from July 1973 to December 1987. Observations in 1988 and 1989 are reserved for out-of-sample analysis. The $\{D_t\}$ series is scaled by a factor 100.

The sample autocorrelation and partial autocorrelation functions of the $\{D_t\}$ series are reported in Table 1. These sample estimates suggest that the risk premiums have low order ARMA representations.[4] Therefore, the following

TABLE 1. Autocorrelations and partial autocorrelations of the $\{D_t\}$ series.

	Lag	British pound	Deutsche mark	Japanese yen
AC:	1	0.115	−0.048	0.109
	2	0.163	0.199	0.071
	3	0.003	0.007	0.108
	4	0.058	0.031	0.100
	5	0.095	0.048	0.087
	6	0.076	−0.040	−0.076
	7	0.077	0.169	0.077
	8	0.079	−0.013	0.039
	9	0.079	0.080	0.071
	10	0.079	0.035	−0.061
PAC:	1	0.115	−0.048	0.109
	2	0.152	0.197	0.060
	3	−0.032	0.025	0.096
	4	0.038	−0.007	0.077
	5	0.093	0.044	0.060
	6	−0.083	−0.043	−0.112
	7	0.035	0.154	0.073
	8	0.056	0.016	0.014
	9	0.042	0.019	0.068
	10	0.017	0.038	−0.085

Notes: AC and PAC give the sample autocorrelations and partial autocorrelations, respectively. The asymptotic standard error of autocorrelations ranges from 0.076 to 0.082 and that of partial autocorrelations is 0.076.

ARMA representations: AR(i), MA(i), ARMA(i,j); $i,j = 1, 2$, for risk premiums are examined. The likelihood function (equation ⟨12⟩) is maximized with each of these time series model specifications for risk premiums. The Davidon–Fletcher–Powell algorithm is used to obtain the maximum likelihood estimates. R and Q rather than R^2 and Q^2 are estimated to ensure that variance estimates are non-negative. The AIC and SBC are computed for each specification. Both AIC and SBC suggest an AR(1) model for the risk premiums of BP and JY and an AR(2) representation for the risk premium of DM. Maximum likelihood estimates of these models are presented in Table 2.

A few remarks on the results are in order. First, the parameter estimates are all significant. In particular, the likelihood values decrease significantly when the covariance term C is set equal to zero. For the BP model the log likelihood decreases by 2.2, which corresponds to a p-value of 0.04. For the DM and JY models, the log likelihood values drop by 6.2 (p-value = 0.0004) and 1.5 (p-value = 0.08), respectively.

TABLE 2. Maximum likelihood estimates of the risk premium model.

	British pound	Deutsche mark	Japanese yen
ϕ_1	0.8260	0.3427	0.8323
	(5.11)	(2.46)	(8.35)
ϕ_2		0.4561	
		(5.42)	
R	2.6005	2.7123	2.9538
	(10.8)	(21.0)	(10.7)
Q	1.1248	1.9720	0.9951
	(2.00)	(11.7)	(2.78)
C	−1.9284	−3.3259	−1.0705
	(2.75)	(3.20)	(5.06)
$V(P)$	3.9813	8.1430	3.2223
ρ_{pv}	−0.3717	−0.4297	−0.2019
C = 0	2.20	6.17	1.46
$Q(10)$	5.17	5.53	6.72
$Q^2(10)$	3.68	6.30	11.7

Notes: Maximum likelihood estimates of the risk premium model (equations ⟨1⟩ to ⟨3⟩) that has the smallest AIC and SBC are reported together with the corresponding absolute asymptotic t-statistics in the parentheses. $V(P)$ is the variance of the risk premium computed from the parameter estimates. ρ_{pv} is the sample correlation of the risk premium (P_t) and the unexpected rate of depreciation (v_t). 'C = 0' reported the decrease in the log-likelihood value when $C = 0$. $Q(10)$ and $Q^2(10)$ are the Box–Pierce portmanteau statistics based on the first ten serial correlation coefficients of the levels and squares of standardized residuals.

Second, the covariance term, C, is negative for these three exchange rate series. That is, the innovation in risk premiums (a_t) is negatively related to the most recent unexpected rate of depreciation $(E_{t-1}S_t - S_t)$. A smaller risk premium is likely to be associated with an unexpected depreciation. This is complementary to Fama's (1984) finding that risk premiums and expected depreciation rates are negatively correlated.[5]

The third remark is related to the persistence of risk premiums. The autoregressive parameter estimates show that the premiums have a high degree of persistence. The augmented Dickey–Fuller test was used to test whether the estimated premium series contains a unit root. It turned out that, for all the premium series, the unit root hypothesis was rejected at the 1 per cent level of significance.[6] That is, the estimated premiums exhibit only short-run but not unit root persistence.[7]

Finally, the estimated risk premiums of the BP and JY series are less volatile than unexpected changes and explain, respectively, 37 per cent and 27 per cent of the total variation in the forward rate forecast error. For the DM series, the risk premium accounts for more than one half of the total variation in the forecast error.[8] However, this variation decomposition result is not related to the predictive power of the model. Indeed, the variance estimates show that the forecast error, D_t, is mainly driven by the random components a_t and v_t. Sample correlation coefficients of risk premiums and unexpected changes in spot rates (ρ_{pv}) reported in Table 2 indicate that the comovement of these two variables is of practical interest.

Box–Pierce portmanteau Q-statistics calculated from standardized residuals, $r_i/H_i^{0.5}$, and from squares of these standardized residuals, r_i^2/H_i, are reported in the lower panel of Table 2. Similar statistics were used by Wolff (1987) to check the adequacy of the estimated state space models. The Q-statistic computed from standardized residuals is used to detect serial correlations in the data not captured by the estimated model. The other statistic is used to test for serial correlations in the second moment (see McLeod and Li, 1983). All these sample Q-statistics are smaller than the conventional critical values; that is, the space models reasonably describe the dynamics of the data.

Table 3 compares the out-of-sample forecast of spot rates obtained from the estimated state space models with a random walk model. The forecasting period is 1988–89. One month ahead forecasts are obtained from the $\{E_{t-1}P_t\}$ and $\{F_t\}$ series, and are labelled as XAF. These forecasts are *ex ante* as they are generated recursively, using only information actually available. The forecast generated from the random model is labelled as RW. The mean squared prediction error (MSPE) and the mean absolute prediction error (MAPE) of these forecasts are presented. We use the procedure discussed in Granger and Newbold (1977, pp. 280–281) to test the hypothesis that the expected squared prediction errors of the RW and XAF are the same. Asymptotic t-statistics are given below the MSPEs of the XAF.

In general, the MSPEs and MAPEs of XAF do not improve upon those of RW. This result corroborates those of others using different techniques and different data sets, such as Meese and Rogoff (1983), who find structural exchange rate models do not outperform the random walk model, and Diebold and Nason (1990), who cannot improve exchange rate forecasts using non-parametric prediction techniques.

TABLE 3. Post sample forecast errors.

		British pound	Deutsche mark	Japanese yen
MPSE	XAF	16.33	24.25	14.89
		(−0.78)	(−1.87)	(−1.34)
	RW	15.38	14.62	13.82
MAPE	XAF	3.20	4.00	3.12
	RW	3.05	3.33	2.92

Notes: Numbers reported in the tables are mean squared prediction errors (MSPEs) and mean absolute prediction errors (MAPEs) of one month ahead *ex ante* forecasts, XAF, generated from $\{E_{t-1}P_t\}$ and forecasts from the random walk model, RW. XAF is derived from models reported in Table 2. The forecasting period is 1988–89. t-statistics in the parentheses are for the hypothesis that the reported MSPE of XAF equals that of RW. A significant positive (negative) statistic implies the RW has a larger (smaller) expected squared error.

II.B. The estimated premium and macro variables

In this subsection, we examine the relationship between the risk premium extracted from the state space model and its determinants implied by general equilibrium theory. Specifically, we consider the three different formulations for risk premiums discussed in Domowitz and Hakkio (1985), Hodrick and Srivastava (1986), and Hodrick (1989). These risk premium representations are all derived from the intertemporal asset pricing model of Lucas (1982), but are based on different assumptions on the economy and the exogenous variables. In solving the asset pricing model, these authors show that the risk premium is related to a number of macro variables.[9] For instance, Hodrick and Srivastava (1986, p. S10) show that

$$\langle 13 \rangle \qquad P_t = n_t - E_t n_{t+1} - m_t + E_t m_{t+1} + 0.5(\sigma^2_{mt+1} + \sigma^2_{nt+1}) + \log(1 - \exp(-\sigma^2_{mt+1})),$$

where n_t and m_t are the natural logarithms of the monies in the two-country model. $E_t x_{t+1}$ and σ^2_{xt+1} are the conditional mean and conditional variance of x_{t+1}, respectively. Based on different assumptions, Domowitz and Hakkio (1985, p. 51) derive

$$\langle 14 \rangle \qquad P_t = 0.5(\sigma^2_{nt+1} - \sigma^2_{mt+1}).$$

The expression for risk premiums given in Hodrick (1989, p. 444) includes output variables and can be written as

$$\langle 15 \rangle \qquad P_t = -\alpha_1\sigma^2_{yt+1} + \alpha_2\sigma^2_{zt+1} - \alpha_3\sigma^2_{mt+1} + \alpha_4\sigma^2_{nt+1} + \xi_t,$$

where σ^2_{yt+1} and σ^2_{zt+1} are the conditional variances of domestic and foreign outputs y and z, parameter α_is depend on the intertemporal substitution for goods, and ξ_t captures the effect of government spendings. Following Hodrick (1989), ξ_t is not included in the following empirical analysis.

In order to test these specifications for risk premiums, we use data on monthly money supplies, as measured by M1, and industrial production indexes obtained from the OECD Main Economic Indicators as proxies for the macro variables described in these models. The data were transformed by taking the first log difference. Each data series is assumed to follow the AR(q_1) − GARCH(q_2, q_3) process defined by

$$\langle 16 \rangle \qquad x_t = \beta_0 + \sum_{i=1}^{q_1} \beta_i x_{t-i} + \varepsilon_t, \qquad \varepsilon_{t|t-1} \sim N(0, h_t),$$

$$h_t = \omega_0 + \sum_{i=1}^{q_2} \omega_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q_3} \gamma_i h_{t-i},$$

where β_0 and β_is are the AR parameters and ω_0, ω_is, and γ_is are positive coefficients satisfying the constraint $\sum \omega_i + \sum \gamma_i < 1$ (see Engle, 1982; and Bollerslev, 1986). Parameters of the AR-GARCH models are estimated by the maximum likelihood method.[10] The adequacy of the estimated models is checked by Box–Pierce portmanteau stastistics computed from the first ten serial coefficients of the levels and squares of standardized residuals. To summarize, the highest order of AR polynomial used is twelfth. The conditional variance of British money supply is described by a GARCH(8, 0) process while that of others is modeled by a first-order GARCH process. The conditional means and variances from these estimated AR-GARCH processes are then used to test the relationship between risk premiums and macro variables.

Table 4 reports the results of regressing the estimated risk premium on (i) n_t, $E_t n_{t+1}$, m_t, $E_t m_{t+1}$, σ_{mt+1}^2, σ_{nt+1}^2, and ζ_{t+1}; (ii) σ_{mt+1}^2 and σ_{nt+1}^2; and (iii) σ_{mt+1}^2, σ_{nt+1}^2, σ_{yt+1}^2, and σ_{zt+1}^2, where $\zeta_{t+1} \equiv 1 - \exp(-\sigma_{mt+1}^2)$. Regressions (i), (ii), and (iii) correspond to equations $\langle 13 \rangle$, $\langle 14 \rangle$, and $\langle 15 \rangle$, respectively, and the USA is taken as the home country. The estimates are corrected for first-order serial correlation because of the significant Durbin–Watson statistics obtained in the preliminary analysis. Absolute asymptotic t-statistics are reported in parentheses.

The results in Table 4 offer only limited support for these risk premium models. Typically, there are a few significant estimates and the adjusted R^2, $\bar{R}^2$, is small. Specification (i) for the DM series has the highest, $\bar{R}^2$ of 0.13. However, the coefficient estimates are significantly different from the values implied by the models. In fact, estimates of specifications (i) and (ii) are all different from their theoretical values. The coefficient estimates of specification (iii) indicate that the intertemporal substitution for goods is different across countries. Overall, the macro variables account for some variation in risk premiums. However, the estimated models do not satisfy the restrictions imposed by the theory.

III. Summary

This paper introduces the state space model which allows for covariation of risk premiums and unexpected rates of depreciation to examine risk premiums in foreign exchange markets. This more general state space model enables us to examine the comovement of two unobservable variables, the risk premium and the unexpected depreciation. Estimation results reveal that unexpected depreciation is inversely related to the risk premium.

The risk premium estimated from the state space model exhibits short-run persistence. Consistent with studies reported in the literature, we do not obtain

TABLE 4. Extracted risk premiums and macro variables.†

	British pound			Deutsche mark			Japanese yen		
	(i)	(ii)	(iii)	(i)	(ii)	(iii)	(i)	(ii)	(iii)
μ	−4.455	0.125	0.094	1.413	−0.111	−0.295	−1.299	0.009	0.187
	(1.68)	(0.68)	(0.45)	(0.60)	(0.95)	(1.10)	(0.49)	(0.05)	(0.89)
m_t	−0.016			−0.027			−0.004		
	(1.98)			(2.54)			(0.45)		
$E_t m_{t+1}$	−0.011			−0.007			−0.002		
	(2.26)			(0.72)			(0.34)		
σ^2_{mt+1}	3.623	−4.357	−4.353	1.567	4.257	4.683	−1.151	−4.115	−3.832
	(0.76)	(1.87)	(1.85)	(0.32)	(1.98)	(2.16)	(0.24)	(1.68)	(1.58)
n_t	0.001			−0.032			0.009		
	(0.63)			(3.70)			(1.53)		
$E_t n_{t+1}$	0.002			0.016			0.006		
	(1.35)			(2.67)			(1.87)		
σ^2_{nt+1}	0.013	0.009	0.003	−0.650	−0.266	−0.206	0.352	0.261	0.296
	(0.18)	(0.12)	(0.05)	(0.86)	(0.33)	(0.26)	(0.67)	(0.52)	(0.59)
ζ_{t+1}	−0.000			0.000			−0.000		
	(1.78)			(0.41)			(0.46)		
σ^2_{yt+1}			0.008			−0.110			−0.050
			(0.05)			(0.41)			(0.43)
σ^2_{zt+1}			0.020			0.232			−0.454
	0.856		(0.94)			(1.18)			(2.26)
ρ	(19.5)	0.830	0.833	−0.082	0.054	0.049	0.833	0.829	0.833
		(18.1)	(18.2)	(1.01)	(0.68)	(0.61)	(18.9)	(18.9)	(19.1)
$\bar{R}^2$	0.040	0.010	0.002	0.132	0.013	0.010	0.003	0.044	0.027

Notes: m and n are the domestic and foreign money supplies, as measured by M1. y and z are the domestic and foreign industrial production indexes. The USA is the domestic country. First log differences of these variables are used. $E_t x_{t+1}$ and σ^2_{xt+1} are the conditional mean and variance of x_{t+1} and $\zeta_{t+1} \equiv 1 - \exp(-\sigma^2_{mt+1})$. The risk premium extracted from the state space model is regressed on (i) n_t, $E_t n_{t+1}$, m_t, $E_t m_{t+1}$, σ^2_{mt+1}, σ^2_{nt+1}, and ζ_{t+1}, (ii) σ^2_{mt+1} and σ^2_{nt+1}, and (iii) σ^2_{mt+1}, σ^2_{nt+1}, σ^2_{yt+1}, and σ^2_{zt+1}. μ is a constant. Regressions are all adjusted for first-order serial correlation. The estimated serial coefficients are reported in the 'ρ' row. Absolute asymptotic t-statistics are reported in the parentheses. The adjusted R^2 is given in the row labeled '$\bar{R}$'.

forecasts that are better than those from the random walk model. When we use data on estimated risk premiums, money supplies, and industrial production indexes to examine the risk premium models derived from the intertemporal asset pricing model of Lucas (1982), we find that the data reject restrictions imposed by the theory though some parameter estimates are significant.

Appendix A

Suppose $P_t = \phi_1 P_{t-1} + \cdots + \phi_p P_{t-p} + a_t + \theta_1 a_{t-1} + \cdots + \theta_q a_{t-q}$. We follow Harvey (1981) to construct the state space model.

$$Y_t \equiv \begin{pmatrix} y_{t,1} \\ \vdots \\ \vdots \\ \vdots \\ y_{t,m} \end{pmatrix} = \left(\begin{array}{c:c} \phi_1 & \\ \vdots & I_{m-1} \\ \phi_{m-1} & \\ \hdashline \phi_m & 0' \end{array}\right) \begin{pmatrix} y_{t-1,1} \\ \vdots \\ \vdots \\ \vdots \\ y_{t-1,m} \end{pmatrix} + \begin{pmatrix} 1 \\ \theta_1 \\ \vdots \\ \vdots \\ \theta_{m-1} \end{pmatrix} a_t,$$

$$\equiv \Phi Y_{t-1} + \Theta a_t.$$

where $m = \max(p, q+1)$, $\phi_i = 0$ for $i > p$ and $\theta_i = 0$ for $i > q$. Then equations ⟨1⟩ and ⟨2⟩ can be re-written as

$$\langle A1 \rangle \qquad D_t = Z Y_t + v_{t+1},$$

$$\langle A2 \rangle \qquad Y_t = \Phi Y_{t-1} + \Theta a_t,$$

where $Z = (1\ 0\ \ldots)$, a $1 \times m$ vector.

The Kalman filter for this vector state space model is given in Appendix B.

Appendix B

The Kalman filter algorithm for state space models with $C \equiv 0$ can be found in a standard textbook such as Harvey (1981). In this appendix we follow Kalman (1963) to derive the algorithm (equations ⟨7⟩ to ⟨11⟩) given in the text. Consider the state space model

$$\langle B1 \rangle \qquad z_t = H x_t + v_t,$$

$$\langle B2 \rangle \qquad x_t = \phi x_{t-1} + \Gamma w_{t-1},$$

$$\langle B3 \rangle \qquad \operatorname{cov}\begin{pmatrix} w_t \\ v_t \end{pmatrix} = \begin{pmatrix} Q^2 & C \\ C & R^2 \end{pmatrix},$$

where z_t is $M \times 1$ vector of observable variables,
x_t is $N \times 1$ vector of state variables,
H is $M \times N$ parameter matrix,
ϕ is $N \times N$ parameter matrix,
v_t is $M \times 1$ vector of zero mean Gaussian noises,
Γ is $N \times R$ parameter matrix,
w_t is $R \times 1$ vector of zero mean Gaussian noises, and
Q^2, R^2, and C are the variance and covariance matrices of w_t and v_t.

The updating equations are given by

$$\langle B4 \rangle \qquad E_t x_t = E_{t-1} x_t + K_t (z_t - H E_{t-1} x_t),$$

$$\langle B5 \rangle \qquad V_t(x_t) = V_{t-1}(x_t) - K_t H V_{t-1}(x_t),$$

$$\langle B6 \rangle \qquad K_t = V_{t-1}(x_t) H^T [H V_{t-1}(x_t) H^T + R^2]^{-1}.$$

$C \neq 0$ implies that z_t contains useful information about the random variable w_t and, hence, helps to predict x_{t-1}. We can break down $E_t x_{t-1}$ into two parts. One depends on information available up to time $t-1$ and the other depends on information obtained between time $t-1$ and t.

$$\langle B7\rangle \qquad E_t x_{t+1} = E_{t-1}x_{t+1} + E(x_{t+1}|\tilde{z}_t)$$
$$= \phi E_{t-1}x_t + E(x_{t+1}|\tilde{z}_t),$$

where

$$\tilde{z}_t = z_t - HE_{t-1}x_t = H(x_t - E_{t-1}x_t) + v_t.$$

Since x_{t+1} and $\tilde{z}_t$ are jointly Gaussian,

$$E(x_{t+1}|\tilde{z}_t) = Ex_{t+1} + \operatorname{cov}(x_{t+1}, \tilde{z}_t)[\operatorname{cov}(\tilde{z}_t)]^{-1}(\tilde{z}_t - E\tilde{z}_t).$$

Note that

$$\operatorname{cov}(x_{t+1}, \tilde{z}_t) = \operatorname{cov}(\phi x_t + \Gamma w_t, H(x_t - E_{t-1}x_t) + v_t)$$
$$= \phi V_{t-1}(x_t)H^T + \Gamma C,$$
$$\operatorname{cov}(\tilde{z}_t) = HV_{t-1}(x_t)H^T + R^2.$$

Therefore (c.f equation ($\mathrm{II_d}$) in Kalman, 1963, p. 301),

$$\langle B8\rangle \qquad E_t x_{t+1} = \phi E_{t-1}x_t + [\phi V_{t-1}(x_t)H^T + \Gamma C][HV_{t-1}(x_t)H^T + R^2]^{-1}\tilde{z}_t$$
$$\equiv \phi E_{t-1}x_t + K^*\tilde{z}_t.$$

Substituting equation ⟨B4⟩ into equation ⟨B8⟩ and simplifying, we have

$$\langle B9\rangle \qquad E_t x_{t+1} = \phi E_t x_t + \Gamma C[HV_{t-1}(x_t)H^T + R^2]^{-1}(z_t - HE_{t-1}x_t).$$

Now, let us consider the predicting equation of the variance of $E_t x_{t+1}$. Note that

$$x_{t+1} - E_t x_{t+1} = \phi(x_t - E_{t-1}x_t) + \Gamma w_t - E(x_{t+1}|\tilde{z}_t).$$

Thus (c.f. ($\mathrm{III_d}$) in Kalman, 1963, p. 303),

$$\langle B10\rangle \qquad V_t(x_{t+1}) = \phi V_{t-1}(x_t)\phi^T + \Gamma Q^2\Gamma^T + K^*\operatorname{cov}(\tilde{z}_t)K^{*T}$$
$$- \operatorname{cov}(\phi(x_t - E_{t-1}x_t), K^*\tilde{z}_t) - \operatorname{cov}(K^*\tilde{z}_t, \phi(x_t - E_{t-1}x_t))$$
$$- \operatorname{cov}(\Gamma w_t, K^*\tilde{z}_t) - \operatorname{cov}(K^*\tilde{z}_t, \Gamma w_t)$$
$$= \phi V_{t-1}(x_t)\phi^T + \Gamma Q^2\Gamma^T + K^*[\phi V_{t-1}(x_t)H^T + \Gamma C]^T.$$

Substituting equation ⟨B5⟩ into equation ⟨B10⟩ and simplifying, we have

$$\langle B11\rangle \qquad V_t(x_{t+1}) = \phi V_t(x_t)\phi^T + \Gamma Q^2\Gamma^T - \Gamma C[HV_{t-1}(x_t)H^T + R^2]^{-1}\Gamma^T C^T$$
$$- \phi K_t C^T\Gamma^T - \Gamma C K_t^T\phi^T.$$

Equations ⟨B4⟩, ⟨B5⟩, ⟨B6⟩, ⟨B9⟩, and ⟨B11⟩ constitute the Kalman filter algorithm for the state space model given by equations ⟨B1⟩ to ⟨B3⟩. It is straightforward to obtain equations ⟨7⟩ to ⟨11⟩ stated in the text from the filter derived here.

Notes

1. A detailed discussion on these regression-based techniques is found in, for example, Hodrick (1987, pp. 84–132).
2. The hypothesis of expectational errors is an alternative interpretation of D_t. See the review articles listed in the introduction section for a more complete discussion.
3. See Engel (1984). Note that D_t is the sum of the risk premium and a white noise term v_t.
4. This is in accord with the results reported in Fama (1984) and Wolff (1987).
5. The correlations between the risk premium and the expected and unexpected depreciation rates can be used to infer the exchange rate expectations mechanism. A more detailed discussion is contained in an earlier version of this paper.

6. The augmented Dickey–Fuller test was applied to both the $\{E_tP_t\}$ and $\{E_{t-1}P_t\}$ series with the lag parameter equal to 1 to 4. The augmented Dickey–Fuller 't-statistics' range from -3.79 (JY $\{E_{t-1}P_t\}$) to -15.20 (DM $\{E_tP_t\}$). The critical value for the 1 per cent test is -3.51.
7. Garbade (1977) illustrates that the likelihood ratio statistic for testing whether the state variable is a random walk or a constant does not have an exact χ^2 distribution. Since we find no evidence of random walk in the risk premium series, the Garbade test is not reported.
8. While the magnitude of risk premiums reported here seems large compared with other studies, it is smaller than that presented in Wolff (1987). Some studies, see Fama (1984) and Frankel and Chinn (1991), also find evidence of large variations in risk premiums.
9. However, no direct tests of these risk premium models were reported by these authors.
10. The consistency and normality of maximum likelihood estimates without the normality assumption are proved in Weiss (1986).

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