Nash Bargaining with the Option to Wait*

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September 1996

Abstract

This paper derives the Generalized Nash Bargaining Solution for two-player games where each player has the option of postponing bargaining till a future period. This option endogenously determines the threat points of the initial game. The outcome is compared with the case where there is no such option, and with the usual Rubinstein bargaining game where one offer per period is made.

JEL Classification: C78

*This research was supported by grants from the UCSC Division of Social Sciences and Academic Senate Committee on Research. I am thankful to Daniel Friedman, Kenneth Kletzer and Donald Wittman for helpful discussions and suggestions. I alone am responsible for errors and omissions.
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Introduction

The Generalized Nash Bargaining Solution (GNBS) is an appealing cooperative solution concept for two-person bargaining games, because it approximates the equilibrium of a well-defined noncooperative bargaining procedure (Rubinstein, 1982, Binmore, Rubinstein and Wolinsky, 1986). This approximation result holds where utility is transferable, so a fixed surplus is being divided. The noncooperative game assumes that there is no surplus if there is no agreement, and this is the assumption for the corresponding cooperative game. More specifically, if utilities are scaled so that utility is zero for each player if no agreement is reached in the noncooperative game, this is also the exogenously given disagreement point for the corresponding cooperative game. In the noncooperative game, players make (possibly rapid) offers and counter offers, until an offer or counter offer is accepted. In the cooperative game, if the players do not agree on a split, they get nothing.

In this paper, we consider a situation where a cooperative bargaining game takes place within one period, but both players realize that if agreement is not reached in the current period, they will be able to resume in the next period. Either player can choose to postpone negotiations by one period. Hence, the disagreement or threat point for the first period's bargaining is not zero utility for each, but the payoffs they would receive from postponing bargaining and agreeing next period. These have a lower present value because both players discount the future. If next period is identical, stationarity allows us to solve for the endogenous threat points of the bargaining game. We derive this solution in this paper. We also show that the payoff of the more patient person is higher when this postponement opportunity exists, compared to the case where it does not.

Note that since there is no uncertainty or incomplete information, there is no delay in agreement. The option of waiting is not exercised in equilibrium. The option to wait is similar to the usual case of an outside option. Binmore, Shaked and Sutton (1989), have provided theoretical and empirical reasons for the limited relevance of outside options for the Nash Bargaining Solution: that they matter only as constraints on the solution, unless the breakdown in communications that leads to the outside option is beyond the players' control. In making our argument, therefore, we assume that players can commit to waiting in the specified manner, because of external constraints. This is plausible, for example, if bargaining takes place only in the day, and must be broken off at night. It is also worth noting that the situation we consider is somewhat different from that in Binmore, Shaked and Sutton, since the value of the 'outside' option here is endogenous. Another way to think of the difference from the usual case, is that we have a Rubinstein game where there are short intervals between offers during the 'day', and no offers at 'night', when the wait is exogenously enforced.

Model and Results

There are two players, indexed by subscript i, i = 1, 2. Their utilities are u_i , and their utilities in the case of no agreement are zero. The disagreement payoffs in the first period are d_i . The total surplus in the case of agreement is denoted by H. The bargaining powers of 1 and 2 are, respectively, α and $(1 - \alpha)$.

In this case, it is easy to show that the GNBS is given by:

(1) $u_1 = \alpha(H - d_2) + (1 - \alpha)d_1$,

(2) $u_2 = \alpha d_2 + (1 - \alpha)(H - d_1)$.

This is, of course standard. If there is no other option available to either player, then the disagreement payoffs, the d_i's, are zero, and the payoffs above reduce to:

(3) $u_1^0 = \alpha H$,

(4) $u_2^0 = (1 - \alpha)H$.

Now suppose, instead, that either player can postpone the bargaining to the next period, when the situation at the beginning of period 2 will be identical to the situation at the beginning of period 1. However, both players discount the future, with per period discount factors given by δ_i . Note that we are assuming that commitment to such postponement is possible. This may be because of natural limits, and physical barriers. For example, people have to break off negotiations to sleep, which creates separate bargaining periods, and they can physically absent themselves for a day, to avoid bargaining that day. Since the situation is stationary, it must be the case that

(5) $d_1 = \delta_1 u_1$,

(6) $d_2 = \delta_2 u_2$.

The four equations (1), (2), (5), (6) may now be solved to yield the following result.

Proposition 1

In the multiperiod bargaining situation described above, where players have the option of postponing bargaining, the GNBS is given by:

$u_1^m = \alpha H (1 - \delta_2) / [1- (1 - \alpha) \delta_1 - \alpha\delta_2]$,

$u_2^m = (1 - \alpha) H (1 - \delta_1) / [1- (1 - \alpha) \delta_1 - \alpha\delta_2]$.

Proof Straightforward calculations. ■

The above expressions are derived for arbitrary bargaining powers (α, 1 - α). However, if we follow the line of reasoning mentioned in the introduction, which relates the cooprative and noncooperative approaches, the bargaining powers are themselves determined by the discount rates. In fact, in this case, we have[1]:

$$\alpha = \ln \delta_2 / (\ln \delta_1 + \ln \delta_2).$$

Substituting this value of α into the expressions of Proposition 1 then yields a solution to the game where in each bargaining period the two players bargain noncooperatively, with rapid offers and counter offers, but if no agreement is reached in the period, bargaining is resumed at the next meeting, with a fixed time period between meetings.

The solution derived in Proposition 1 may be compared to the GNBS where postponement to the next period is not possible. We have the following result.

Proposition 2

The player with the higher (lower) discount factor is better (worse) off if the possibility of postponement of bargaining to another period exists**.**

Proof We have $u_1^0 / u_1^m = [1- (1 - \alpha) \delta_1 - \alpha\delta_2] / (1 - \delta_2)$

$$= 1 - (1 - \alpha) (\delta_1 - \delta_2) / (1 - \delta_2) .$$

Hence $u_1^0 < u_1^m$ if and only if $\delta_1 > \delta_2$. The result for the other player follows immediately. ■

This result is not surprising, in that the more patient player benefits from the possibility of postponement. We can also compare this GNBS with the option to postpone to the other polar case, where each player makes one offer per period, i.e., the standard Rubinstein game. The equilibrium of this game is given by:

$$u_1^{r1} = H (1 - \delta_2) / (1- \delta_1 \delta_2) ,$$

$$u_2^{r1} = H \delta_2 (1 - \delta_1) / (1- \delta_1 \delta_2) ,$$

if player 1 makes the first offer, and by:

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[1] The logarithms come from the fact that $\delta^t = \exp [(\ln \delta)t]$. Hence the continuous rate of discount is $-(\ln \delta)$.

$u_1^{r2} = H\ \delta_1 (1 - \delta_2) / (1- \delta_1 \delta_2)$,

$u_2^{r2} = H\ (1 - \delta_1) / (1- \delta_1 \delta_2)$,

if player 2 makes the first offer. We have our next result.

Proposition 3

If player 1 makes the first offer when only one offer is made per period, she is better off than under the GNBS with the option of postponing bargaining to the next period if and only if $\ln \delta_1 < \delta_2 \ln \delta_2$.

Proof In the case where player 1 makes the first offer, we have

$u_1^{r1} / u_1^{m} = [1- (1 - \alpha) \delta_1 - \alpha\delta_2] / \alpha(1- \delta_1 \delta_2)$.

$$
\begin{aligned}
\text{Hence, } u_1^{r1} > u_1^{m} &\Leftrightarrow [1- (1 - \alpha) \delta_1 - \alpha\delta_2] > \alpha(1- \delta_1 \delta_2) \\
&\Leftrightarrow (1 - \alpha) (1 - \delta_1) > \alpha\delta_2 - \alpha\delta_1 \delta_2 \\
&\Leftrightarrow (1 - \alpha) / \alpha > \delta_2 \\
&\Leftrightarrow \ln \delta_1 / \ln \delta_2 > \delta_2 \\
&\Leftrightarrow \ln \delta_1 < \delta_2 \ln \delta_2 \quad \blacksquare
\end{aligned}
$$

The interesting feature of this proposition is that it places an upper bound on the discount factor of the player moving first in the period-by-period alternating offer game, relative to the discount factor of the other player. This upper bound is in fact, $\delta_2^{\delta_2}$. If player 1's discount factor exceeds this bound, she will prefer the GNBS with the option of waiting. Essentially, in the latter case, the ability to bargain rapidly within the bargaining period (approximated in the GNBS) outweighs the value of making a take-it-or-leave it offer in the first period. Note that, since $\delta_2 < 1$, $\delta_2^{\delta_2} > \delta_2$. Therefore the bound still permits player 1 to have a higher discount factor than player 2.

Finally, we can consider the case where the second player makes the first offer in the standard Rubenstein game. Again, we focus on player 1's outcomes.

Proposition 4

If player 2 makes the first offer when only one offer is made per period, player 1 is better off than under the GNBS with the option of postponing bargaining to the next period if and only if $\delta_1 \ln \delta_1 < \ln \delta_2$.

Proof The inequality follows from Proposition 3 and the symmetry of the players. ■

In this case, we again have an upper bound on the discount factor of player 1.

However, this upper bound is more stringent now, since player 1 loses by not being able to make the first offer in the standard Rubinstein game. For player 1 to prefer the one-offer-per-period game, her discount factor must be lower than a bound which is strictly less than the other player's discount factor.

Conclusion

This paper contributes to a program which seeks to connect cooperative and noncooperative bargaining, and to provide institutionally realistic models of bargaining. The latter is captured here by the idea that there are bargaining periods where rapid offers and counter offers can be made ('day'), punctuated by intervals where there is no bargaining ('night'). This situation is compared to the case where there is only one such bargaining period, and to the case where only one offer per period can be made. Outcomes are compared based on the relative patience of the bargainers.

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