Workers, Capitalists, Wages, Employment and Welfare*

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Abstract

I construct a sticky-price, sticky-wage model of workers and capitalists in which the functional distribution of income matters. Workers face idyosyncratic income risk and a limited ability to share this risk. The model yields a direct wage channel on aggregate demand; a decline in wages can reduce aggregate demand and employment, a channel absent in standard models. Wage inflation generates a cross-sectional dispersion of consumption across workers that is welfare reducing. Using empirically plausible shock processes in a calibrated version of the model, welfare is shown to increase as wages become more flexible under an optimal monetary policy. However, when the monetary authority is constrained by an effective lower bound on interest rates, greater wage flexibility unambigiously worses the subsequent recession.

J.E.L. Codes: E52, E32, E25

In neoclassical models of the labor market, wage rigidity translates into unemployment in the face of adverse shocks to labor demand or positive shocks to labor supply.[1] In search and matching models of unemployment in the Mortensen-Pissarides tradition, Shimer (2005) showed that unemployment fluctuations are small if wages are flexible, and some degree of wage rigidity is viewed as necessary if the observed volatility of unemployment is to be matched.[2]

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[1] This view of unemployment as arising from wage rigidity is reflected in the way major undergraduate textbooks introduce unemployment; e.g., Accemoglu, et al 2015, pp. 207-212 and Jones, 2014, p. 181, p. 419.

[2] See, for example, Hall (1980), Hall and Milgrom (2008). However, when a search and matching model is imbeded in a sticky-price new Keynesian model with flexible wages, Lago Alves (2012) shows that the Shimer puzzle disappears if trend inflation is positive, as it is in the data.

In a typical new Keynesian model, however, wages have no direct effect on employment. The dynamic evolution of employment and output are "decoupled from wages. In other words, *there is no direct impact of wage adjustment on labor demand and employment.*" (Galí (2013), p. 991, emphasis in original). New Keynesian models emphasize the effects of wages on marginal costs and inflation, not on aggregate demand, and wages affect employment and output only through the induced response of monetary policy to the inflation consequences of wage changes. Thus, the consequences of greater wage flexibility for employment fluctuations and welfare can be ambiguous, depending importantly on how monetary policy responds to inflation. Galí (2013) concluded that increases in wage flexibility can reduce welfare under an optimal monetary policy, and Galí and Monacelli (2016) showed the effects of increased wage flexibility in the open economy depend on the response of monetary policy to the exchange rate.

An alternative view, common in popular discussions, is that there is a direct *positive* link between labor income and employment. A fall in wages that reduces total labor income may result in a fall in aggregate spending which, in turn, worsens output and employment unless offset by monetary policy. Hence, recoveries could be hindered by weak aggregate demand due to a failure of real wages to rise. And policies that prevent wages and labor income from falling may help stabilize total output and employment. This view stands in contrast to the neoclassical view that high unemployment during a recession is a sign that real wages are too high, and that the failure of wages to fall hinders the return to full employment. Standard models are not useful in comparing these views as they exclude any role for a direct aggregate demand channel of labor income.[3]

In normal times, a demand channel for labor income would simply affect the monetary authority's policy setting consistent with any desired output gap. However, such a channel is likely to be particularly relevant if the monetary authority's policy rate has been driven to its effective lower bound. In this case, a fall in employment and the associated fall in labor income can further depress demand, output, and inflation. Greater wage flexibility in this circumstance would worsen economic conditions.

In this paper, I investigate the role of wage flexibility in a model with incomplete markets in which the functional distribution of income between labor and non-labor income matters. To do so in a simple manner, I introduce heterogeneity by distinguishing between capitalists and workers. Capitalists own the economy's production technology and receive income as a return to capital and as profits. Workers receive wage income and do not have access to financial markets.

[3] The "wage channel" in standard new Keynesian models usually refers to the effects of wages on inflation, not aggregate demand; see Christoffel and Kuester (2008).

For a given real interest rate and employment level, a temporary rise in real wages has two effects in the model: first, it raises the relative price of labor, and second, it shifts income to workers and away from capitalists. This second effect increases aggregate demand as workers' consumption rises fully with the wage increase and capitalists consumption falls by less than the decline in profits as they attempt to smooth their consumption intertemporally. These are partial equilibrium effects, and the general equilibrium effects will depend on other aspect of the model's structure, including the response of the monetary authority to the wage increase. The worker-capitalist set up is designed to isolate clearly a demand channel of wages, allowing a more complete analysis of the consequences of greater wage flexibility for business cycle fluctuations. It is worth emphasizing that this not a paper arguing wages are rigid, but instead it investigates whether a wage-demand channel can make wage flexibility undesirable.

Galí (2013) and Galí and Monacelli (2016) also address the effects of wage flexibility in closed and open economy new Keynesian model, respectively. They focus on the how the effects of wage flexibility depend on the conduct of monetary policy. However, they employ a representative agent framework in which there is no direct effect of wages on aggregate demand, nor do they consider the consequence of the effective lower bound on interest rates.

Danthine and Donaldson (2002) and Lansing (2015) develop models of workers and capitalists to study asset pricing in a real business cycle framework with flexible prices and wages. Both papers assume only capitalists hold financial and/or real assets, while workers simply consume their wage income. This parallels the structure I adopt. However, these earlier papers assumed workers are homogeneous, so workers faced aggregate business cycle but not idiosyncratic risk. In addition, they assumed the labor supply of workers is inelastic; this implies workers are unable to smooth consumption risk through adjustments of their labor supply.[4] In contrast, I introduce sticky prices and wages, elastic labor supply, and heterogeneity across workers who face uninsurable idiosyncratic labor income risk as well as business cycle risk.

Another branch of the literature has introduced rule-of-thumb households to analyzed the effects of monetary and fiscal policy. For example, Bilbiie (2008) showed how heterogeneity in access to asset markets can reverse the sign of the impact of interest rates on aggregate demand. Mankiw (2000), Galí, López-Salido, and Vallés (2004), Galí, López-Salido, and Vallés (2007), Amato and Laubach (2003), Colciago (2011), and Furlanetto and Seneca (2012) all developed models with rule-of-thumb households to study fiscal policy. These models are new Keynesian

[4] In Danthine and Donaldson (2002), firms offer wage contracts that partially insure workers against aggregate consumption risk. Domeij and Flod??n (2006) emphasize the role labor supply adjustment can play in smoothing consumption in the face of shocks.

in spirit in assuming the presence of nominal rigidities, but they differ from the model developed here in their treatment of the labor market. Many rule-of-thumb models assume that both rule-of-thumb households and optimizing households are identical when it comes to the labor market. For example, in Galí, López-Salido, and Vallés (2007), all households face the same wage and supply the same amount of labor. Colciago (2011) assumed differentiated labor types, but each household supplies all labor types. In this case, labor income is identical across households, and all rule-of-thumb households have the same consumption. Thus, despite lacking access to financial markets, rule-of-thumb households are able to pool labor income and ensure against any idiosyncratic consumption risk. Importantly, labor income is also independent of whether the household is a rule-of-thumb household or an optimizing household.[5] I assume workers supply differentiated labor services that are imperfect substitutes in production, but rather than assume each worker/household supplies all types of labor services, I assume each worker is the monopoly supplier of a single labor type. Thus, heterogeneity with respect to hours and labor income will arise if staggered wage adjustment leads to a dispersion of relative wages, and workers will face idiosyncratic labor income risk. I allow workers to insure partially against this risk; the standard practice in new Keynesian models of assuming rule-of-thumb households completely insure against idiosyncratic consumption risk and the alternative of complete autarky are special cases of the present model. When such insurance is incomplete, the model generates a cross-sectional distribution of labor income and consumption among workers, as well as consumption differences between workers and capitalists; both forms of consumption heterogeneity have implications for welfare.[6]

A long literature has investigated the role of consumption heterogeneity in incomplete market environments.[7] Werning (2015) provides a recent general treatment and discusses how the presence of incomplete markets affects the aggregate relationship between consumption and real interest rates. Kaplan, Moll, and Violante (2016) study the general equilibrium implications of heterogeneous households facing financial transaction costs that give rise to assets of

[5] Furlanetto (2011) discusses alternative labor market specifications in a model with rule-of-thumb households. Natvik (2012) discusses the implications for determinacy and the effects of fiscal policy of the standard assumption that rule-of-thumb and optimizing households supply the same labor hours.

[6] In a recent paper, Broer, Krusell, Hansen, and Oberg (2016) also explore the implications of a model based on workers and capitalists. In contrast to the present model, they assume capitalists do not have access to financial markets, while workers do. They emphasize the importance of wage rigidity for the ability of heterogeneous-agent models to generate an effect of monetary policy on output.

[7] See Walsh (1985) for an example of a model with borrowing constraints and idiosyncratic income risk in which the fraction of constrained households is endogenously determined and for references to the earlier literature dealing with this issue.

varying liquidity. Their model is consistent with the presence of wealthy households whose consumption is still constrained by the lack of liquid assets, the so-called wealthy hand-to-mouth consumers identified by Kaplan, Violante, and Weidner (2014).[8] Challe, Matheron, Ragot, and Rubio_Ramirez (2014) focus on the interaction between precautionary savings in the face of incomplete markets and labor market frictions, interactions that lead to both aggregate demand and aggregate supply effects arising from precautionary saving. An adverse shock leads households to increase saving, amplifying the impact of the shock by reducing aggregate demand. At the same time, an aversion to disinvest in bad times helps smooth macro volatility. They find in an estimated version of their model that, relative to a representative agent version of the model, monetary policy shocks have large effects on output but the output effects of other shocks are similar in their baseline and representative agent models. In contrast to these models, in which the distribution of wealth across households is endogenous, I treat the capital stock as fixed and assume the owners of this stock (the capitalists) perfectly share consumption risk. Ignoring capital accumulation allows analytical results to be derived that can be compared directly to those in standard new Keynesian models which also ignore capital accumulation. In particular, the basic sticky-price, sticky-wage model of Erceg, Henderson, and Levin (2000) is a special case of the model developed here. In addition, my focus is on the role of wage rigidity, something not analyzed in any of the above cited papers. By allowing for cross-sectional variation in consumption between workers and capitalists, as well as among workers unable to fully insure against idiosyncratic labor income risk, the framework also highlights the potential welfare and policy implications of cross-sectional consumption heterogeneity.

The cyclical implications of the model will clearly depend on the assumptions made about monetary policy. I consider a policy implemented via a simple instrument rule, but I also evaluate the role of wage rigidity when monetary policy is designed to maximize a weighted sum of the utility of workers and capitalists. Welfare depends on price inflation, wage inflation, and output gap volatility, as in standard sticky price, sticky wage models. However, welfare also depends on the cross-sectional dispersion of consumption between workers and capitalists and on the cross-sectional dispersion of consumption among workers.[9] Thus, consumption inequality directly affects optimal policy. When shocks are calibrated to be consistent with the estimates of Smets and Wouters (2007), the welfare costs of inflation volatility and the dispersion of consumption between workers and capitalists are found to constitute the largest source of welfare loss under

[8] See also Bayer, Lütticke, Do, and Tjaden (2015) and Luetticke (2015).

[9] Coibion, Gorodnichenko, Kueng, and Silvia (2012) focus on the distributional consequences of macroeconomic fluctuations and monetary policy. See also Gornemann, Kuester, and Nakajima (2013).

the optimal time-consistent monetary policy. Importantly, welfare loss is increasing in wage rigidity except for extreme values of nominal wage rigidity.

As is well known, optimal monetary policy should neutralizes the impact of demand shocks on the output gap, inflation and wage inflation. When it can do so, the details of the demand side of the economy lose relevance. In particular, whether there is a direct wage channel on aggregate demand is irrelevant. However, at the effective lower bound (ELB) for nominal interest rates, monetary policy cannot offset the effects of falling wages on demand. Previous studies of the role of wage flexibility have not investigated the consequences at the ELB. I find that increased wage flexibility is extremely costly at the ELB. The decline in employment leads to a fall in real wages. This decline in labor income further reduces the fall in output and employment, exacerbating the consequences of the ELB. This result is the aggregate demand side parallel to the argument of Coibion, Gorodnichenko, Kueng, and Silvia (2012) that downward wage rigidity lowers the probability of falling into a deflationary trap at the ELB. Their argument focuses on the effects of wages on firms' marginal costs while I focus on demand side channels.

The rest of the paper is organized as follows. The details of the model are described in section 1. The connections between wages, aggregate demand, and employment are evaluated in section 2. Section 3 discusses the calibration of the model and compares the impulse responses to various aggregate shocks in the worker-capitalist model and the benchmark sticky wages and prices model of Erceg, Henderson, and Levin (2000). The implications of wage flexibility for welfare under optimal discretionary monetary policy when shocks are calibrated to be consistent with the estimates of Smets and Wouters (2007) are studied in section 4. The consequences of the effective lower bound on the nominal interest rate are discussed in section 5. Conclusions are summarized in a final section.

1 A model of workers and capitalists

Two types of households populate the economy: capitalists and workers. There is a measure $0 < W < 1$ of workers and $1 - W$ of capitalists. Workers receive income solely from wages and do not have access to financial markets. I assume workers are able to partially insure against idiosyncratic consumption risk through non-financial market means. For example, the worker may belong to a family of workers with intra-family transfers allowing an individual worker's consumption to differ from the worker's own wage income as in Ortigueira and Siassi (2013). Capitalists own the economy's productive technology, have access to financial markets to share risk, and receive income from profits and from ownership of capital. Capitalists do not supply labor.[10] Capitalists operate firms that hire labor and rent capital to produce differentiated consumption goods. For simplicity, I treat the aggregate capital stock as fixed. In addition, there is a zero-profit employment agency that hires the differentiated labor serves of workers and rents a labor aggregate to firms who use it to produce output.

1.1 Capitalists

Capitalists are members of a representative family that maximizes

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \nu_{t+i} U\left(C_{t+i}^c\right) = \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \nu_{t+i}^i \frac{\left[C_{t+i}^c\right]^{1-\sigma^c}}{1-\sigma^c}, \tag{1}$$

where ν_t is a stochastic preference shock common to capitalists and workers and

$$C_t^c = \left[\int_0^1 \left(c_{j,t}^c\right)^{\frac{\theta_t^p - 1}{\theta_t^p}} dj\right]^{\frac{\theta_t^p}{\theta_t^p - 1}} \tag{2}$$

is a Dixit-Stiglitz aggregate of the consumption by capitalists of individual final goods $c_{j,t}^c$. The demand parameter θ_t^p is treated as stochastic with mean θ^p. Consistent with these consumption preferences (which will be shared by workers), the aggregate price level is given by

$$P_t \equiv \left[\int_0^1 P_{j,t}^{1-\theta_t^p} dj\right]^{\frac{1}{1-\theta_t^p}},$$

where $P_{j,t}$ is the price of consumption good j. The maximization of (1) is subject to a budget constraint given by

$$\Pi_t + \left(\frac{1+i_{t-1}}{1+\pi_t}\right) b_{t-1} + T_t^c - C_t^c - b_t \geq 0, \tag{3}$$

where Π_t is the capitalist's per capita real profit income plus income from the ownership of the fixed stock of capital, b are real bond holdings that pay a nominal interest rate i, $\pi_t = (P_t/P_{t-1}) - 1$ is the inflation rate, and T^c is a per-capita lump-sum fiscal transfer (tax if negative).

The first order condition for the optimal intertemporal allocation of consumption by capi-

[10] The income from capital ownership could also be interpreted as income from human capital and therefore as a form of wage income. The key distinction is that capitalists supply their human capital inelastically and the return to human capital is flexible. Lansing (2015) develops a real model of workers and capitalists and shows how fluctuations in the distribution of income between wages and profits can account for asset pricing puzzles.

talists is

$$\left(C_t^c\right)^{-\sigma^c} = \beta \mathrm{E}_t \left(\frac{1+i_t}{1+\pi_{t+1}}\right)\left(\frac{\nu_{t+1}}{\nu_t}\right)\left(C_{t+1}^c\right)^{-\sigma^c}.$$

Letting $\hat{x}$ denote the log deviation of a variable X_t around a zero-inflation steady state, and defining $\rho_t \equiv \beta^{-1} - 1 - (\mathrm{E}_t\hat{\nu}_{t+1} - \hat{\nu}_t)$, the Euler condition for capitalists can be expressed as[11]

$$\hat{c}_t^c = \mathrm{E}_t\hat{c}_{t+1}^c - \left(\frac{1}{\sigma^c}\right)(i_t - \mathrm{E}_t\pi_{t+1} - \rho_t). \tag{4}$$

Capitalists own the (fixed) aggregate stock of capital and the economy's production technology, and they operate a continuum of firms producing differentiated consumption goods. Firm j produces good $c_{j,t}$, expressed in per capita terms, using the technology

$$c_{j,t} = WZ_t k_{j,t}^a h_{j,t}^{1-a},\ 0 < a \le 1, \tag{5}$$

where k_j and h_j are capital per worker and hours per worker at firm j, respectively, and Z_t is a common stochastic productivity factor. All firms hire labor services from an employment agency at a common real wage ω_t and rent capital at a common rental rate r_t. In addition, firms face a payroll tax rate τ_t^p so that their gross-of-tax real labor cost is $(1+\tau_t^p)\,\omega_t$.

The firm faces a demand curve given by

$$c_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\theta_t^p} C_t, \tag{6}$$

where C_t is aggregate per capita consumption of both capitalists and workers.[12] The price elasticity of demand, θ_t^p, is common across all firms and may vary stochastically. When prices are flexible, profit maximization leads to the standard condition that $\mu_t^p mc_t = 1$, where $\mu_t^p = \theta_t^p/(\theta_t^p - 1)$ is the markup and $mc_t = (1+\tau_t^p)\,\omega_t h_{j,t}/\left[(1-a)c_{j,t}\right]$ is real marginal cost.

Firms are assumed to have sticky prices, modelled by the standard Calvo adjustment process, where $1-\phi^p$ of the firms are randomly chosen to adjust each period. The Calvo assumption leads to the well-known solution for the optimal price P_t^* chosen by those firms that can adjust

[11] Bonds are in zero net supply and are only held by capitalists, so in equilibrium the per capita budget constraint for capitalists is given by

$$C_t^c \equiv \Pi_t + T_t^c,$$

and the aggregate consumption of capitalists is $(1-W)\,C_t^c$. In equilibrium, capitalists consume their current income, but this is an equilibrium outcome and not a constraint on capitalists' individual choices.

[12] This demand curve is consistent with the definition of C_t^c given by (2) and the assumption (below) that workers' utility depends on a consumption aggregate defined similarly to C_t^c.

their price at time t, while the aggregate price level is given by

$$P_t^{1-\theta_t} = \left(1-\phi^P\right)\left(P_t^*\right)^{1-\theta_t} + \phi^p P_{t-1}^{1-\theta_t}.$$

As is also well-known, the Calvo model, when linearized around a zero steady-state inflation rate, leads to the standard new Keynesian Phillips curve given by

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa_p \left(\widehat{mc}_t + \hat{\mu}_t^p\right), \tag{7}$$

where $\hat{\mu}_t^p$ is the log-deviation of the average markup around its steady-state value given by $\mu^p \equiv \theta^p / \left(\theta^p - 1\right)$, and

$$\widehat{mc}_t = \hat{\omega}_t + \tau_t^p + a\hat{h}_t - \hat{z}_t \tag{8}$$

is real marginal cost,[13] $\hat{h}_t$ is aggregate hours, and the reduced-form parameter κ_p is defined as

$$\kappa_p \equiv \frac{\left(1-\phi^p\right)\left(1-\beta\phi^p\right)}{\phi^p}\left(\frac{1-a}{1-a+a\theta^p}\right).$$

1.2 Workers

Each worker is the monopoly supplier of a differentiated labor service, and real labor income can differ across workers. To provide for partial consumption insurance, I assume a fraction χ, $0 \leq \chi \leq 1$, of each individual worker's idiosyncratic income risk is borne directly by the worker. The remaining $1-\chi$ fraction can be shared among other workers.[14] As in Edmond and Weill (2012), I treat χ as exogenous. When $\chi = 0$, the model reduces to a standard RoT model in which all workers have the same consumption; when $\chi = 1$, the individual worker bears all the idiosyncratic income risk.

Specifically, assume all workers belong to an extended family. Let $C_t^w(s)$ denote consumption of worker s, and $h_t(s)$ hours worked by s. The preferences of the family are given by

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \int_0^W U^w\left(C_{t+i}^w(s), h_{t+i}(s)\right) ds, \tag{9}$$

[13] This derivation assumes the tax rate is small so that $\log(1+\tau_t^p)/(1+\tau^p) \approx 1 + \tau_t^p - \tau^p = 1 + \hat{\tau}_t^p$.

[14] An alternative to the modeling structure employed here is to assume individuals face a transactions cost if they wish to divorce consumption from income; see Schulhofer-Wohl (2011) and, for this approach in a new Keynesian model, see Lee (2012) and Lee (2014).

with

$$U^w\left(C^w_{t+i}(s), h_{t+i}(s)\right) = \nu_{t+i}\left[\frac{\left(C^w_{t+i}(s)\right)^{1-\sigma^w}}{1-\sigma^w} - \frac{\gamma h_{t+i}(s)^{1+\eta}}{1+\eta}\right]. \tag{10}$$

As with capitalists, the consumption aggregate that enters workers' utility function is defined as

$$C^w_t(s) = \left[\int_0^1 c_{j,t}(s)^{\frac{\theta^p_t - 1}{\theta^p_t}} dj\right]^{\frac{\theta^p_t}{\theta^p_t - 1}}.$$

The family utility is maximized subject to a budget constraint for each worker s of the form

$$C^w_t(s) = (1-\tau^w_t)\left[\chi\omega_t(s)h_t(s) + (1-\chi)\left(\frac{1}{W}\right)\int_0^W \omega_t(j)h_t(j)dj\right] + T^w_t. \tag{11}$$

where $\omega_t(s)$ is the wage of worker s, τ^w is a wage tax, and T^w_t is a per-capita transfer.[15] With no risk sharing ($\chi = 1$), the consumption of worker s is simply the worker's after tax wage income plus any transfer. If workers are able to completely insure against idiosyncratic wage risks, $\chi = 0$ and $C^w_t(s)$ is independent of s. Log linearizing (11) around the steady state, one obtains

$$\hat{c}^w_t(s) = \chi\psi\left[\hat{\omega}_t(s) - \hat{\tau}^w_t + \hat{h}_t(s)\right] + (1-\chi)\,\psi\left(\hat{\omega}_t - \hat{\tau}^w_t + \hat{h}_t\right) + (1-\psi)\,\hat{t}^w_t,$$

where $\psi \equiv (1-\tau^w)\,\omega h / C^w$. Aggregating (11) over all workers and defining $C^w_t \equiv (1/W)\int_0^W C^w_t(s)ds$, the linearized aggregate budget constraint for the worker-family is

$$\hat{c}^w_t = \psi\left(\hat{\omega}_t - \hat{\tau}^w_t + \hat{h}_t\right) + (1-\psi)\,\hat{t}^w_t. \tag{12}$$

Thus,

$$\hat{c}^w_t(s) - \hat{c}^w_t = \chi\psi\left[\left(\hat{\omega}_t(s) - \hat{\omega}_t\right) + \left(\hat{h}_t(s) - \hat{h}_t\right)\right], \tag{13}$$

and, unless $\chi = 0$, consumption differences across workers reflect any cross-sectional variations in wages and hours.

Worker s supplies labor hours $h_t(s)$ to a labor employment agency at wage $\omega_t(s)$. The employment agency takes wages as given and hires individual labor types to form a labor-hours

[15] I assume the wage tax and the transfer are common to all workers and not a function of the worker's individual type.

aggregate defined as

$$h_t = \left[\left(\frac{1}{W} \right) \int_0^W h_t(s)^{\frac{\theta_t^w - 1}{\theta_t^w}} ds \right]^{\frac{\theta_t^w}{\theta_t^w - 1}}. \tag{14}$$

The demand for type s labor by the employment agency as a function of relative wages is

$$h_t(s) = \left(\frac{\omega_t(s)}{\omega_t} \right)^{-\theta_t^w} h_t, \tag{15}$$

and the aggregate average wage index ω_t is

$$\omega_t = \left[\left(\frac{1}{W} \right) \int_0^W \omega_t(s)^{1-\theta_t^w} ds \right]^{\frac{1}{1-\theta_t^w}}. \tag{16}$$

By linearizing the demand for labor type s given by (15), the consumption of worker s relative to aggregate workers' consumption (13) can be written as

$$\hat{c}_t^w(s) - \hat{c}_t^w = \chi\psi \left(1 - \theta^w\right) \left(\hat{\omega}_t^*(s) - \hat{\omega}_t\right).$$

Now consider the family's problem of maximizing (9) when wages are flexible by choosing consumption, hours, and wages for each s subject to (11) and (15). In this case, the decision problem of the family can be written as a sequence of static problems:

$$\begin{aligned}
\mathcal{L} \;=\; & \max_{\forall s, C_t^w(s), h_t(s), \omega_t(s)} \int_0^W U^w\left(C_t^w(s), h_t(s)\right) ds \\
& + \left(1 - \tau_t^w\right) \int_0^W \lambda_t(s) \left[\chi \omega_t(s) h_t(s) + (1-\chi) \left(\frac{1}{W} \right) \int_0^W \omega_t^w(j) h_t(j) dj \right] ds \\
& - \int_0^W \lambda_t(s) \left[C_t^w(s) - T_t^w \right] ds + \int_0^W \zeta_t(s) \left[\left(\frac{\omega_t(s)}{\omega_t} \right)^{-\theta_t^w} h_t - h_t(s) \right] ds,
\end{aligned}$$

where $\lambda_t(s)$ and $\zeta_t(s)$ are the Lagrangian multipliers on the budget and labor demand constraints facing worker s. The term

$$\left(1 - \tau_t^w\right) (1-\chi) \int_0^W \lambda_t(s) \left[\left(\frac{1}{W} \right) \int_0^W \omega_t^w(j) h_t(j) dj \right] ds$$

can be written as

$$\left(1 - \tau^w\right) (1-\chi) \Lambda_t \int_0^W \omega_t^w(s) h_t(s) ds,$$

where

$$\Lambda_t \equiv \left(\frac{1}{W}\right)\int_0^W \lambda_t(j)dj$$

is the average Lagrangian multiplier across all family members.[16] The family's decision problem can now be expressed as

$$\begin{aligned}\mathcal{L} \;=\; & \max_{\forall s, C_t^w(s), h_t(s), \omega_t(s)} \int_0^W U^w\left(C_t^w(s), h_t(s)\right) ds - \int_0^W \lambda_t(s)\left[C_t^w(s) - T_t^w\right] ds \\ & + \int_0^W \bar{\lambda}_t(s)\left(1-\tau^w\right)\omega_t(s) h_t(s) ds + \int_0^W \zeta_t(s)\left[\left(\frac{\omega_t(s)}{\omega_t}\right)^{-\theta_t^w} h_t - h_t(s)\right] ds,\end{aligned}$$

where

$$\bar{\lambda}_t(s) \equiv \chi\lambda_t(s) + (1-\chi)\Lambda_t \tag{17}$$

is a weighted average of the marginal value of income to worker s ($\lambda_t(s)$) and to the entire family (Λ_t) with weights given by the degree to which the family provides consumption risk insurance to the individual worker.

The first-order condition for the choice of $C_t^w(s)$ is

$$v_t\left[C_t^w(s)\right]^{-\sigma^w} = \lambda_t(s),$$

while the first-order conditions for hours and wages are

$$v_t\gamma h_t(s)^\eta + \bar{\lambda}_t(s)\left(1-\tau_t^w\right)\omega_t(s) - \zeta_t(s) = 0,$$

$$\bar{\lambda}_t(s)\left(1-\tau_t^w\right)\omega_t(s) - \theta_t^w\zeta_t(s) = 0.$$

Combining these last two conditions yields

$$\frac{\gamma v_t h_t(s)^\eta}{\bar{\lambda}_t(s)} = \left(\frac{\theta_t^w - 1}{\theta_t^w}\right)\left(1-\tau_t^w\right)\omega_t(s) = \frac{\left(1-\tau_t^w\right)\omega_t(s)}{\mu_t^w}, \tag{18}$$

where $\mu_t^w \equiv \theta_t^w / \left(\theta_t^w - 1\right)$ is the wage markup.

The key difference relative to standard models is the appearance of $\bar{\lambda}_t(s)$ in the definition of the marginal rate of substitution in (18) rather than the marginal utility of consumption for worker s, $\lambda_t(s)$. If $\chi = 1$ and the individual worker bears all the idiosyncratic labor income

[16] See Edmond and Weill (2012).

risk, $\bar{\lambda}_t(s) = \lambda_t(s)$ and the standard expression is obtained. If $\chi = 0$ and risk is shared across all workers, then $\bar{\lambda}_t(s) = \Lambda_t$ and the average marginal utility of consumption across all workers is used to value the wage income from additional hours of work by family member s. Because $\bar{\lambda}_t(s) = \lambda_t(s) + (1-\chi)\left[\Lambda_t - \lambda_t(s)\right]$, $\bar{\lambda}_t(s) > \lambda_t(s)$ for workers with above average consumption. The optimal allocation of hours for the family calls for such workers to supply more hours, ceteris paribus $(\gamma v_t h_t(s)^\eta = (1-\tau_t^w)\,\omega(s)\bar{\lambda}(s)/\mu_t^w \geq (1-\tau_t^w)\,\omega(s)\lambda(s)/\mu_t^w)$.[17]

If each worker s chooses wages every period, the underlying symmetry of the model implies wages will be equal across worker types, as will consumption. In this case, $\bar{\lambda}_t(s) = \lambda_t(s)$ for all s and t. If instead wages are sticky and adjust in a staggered, overlapping pattern, then wage dispersion will result. As a consequence, hours worked, labor income, and, when $\chi > 0$, consumption will differ across workers.

To model sticky wages, suppose wage adjustment follows the standard Calvo process in which each period, a randomly chosen fraction ϕ^w of all workers adjust their wage. All nominal wages adjusted at time t will be reset to the same value w_t^*. Let $C_{t+i/t}^w$ and $h_{t+i/t}$ denote the consumption and hours at $t+i$ of a worker whose wage was last set at time t. The value of w_t^* will be chosen to maximize

$$\mathrm{E}_t \sum_{i=0}^{\infty} (\beta\phi^w)^i \left\{ \begin{array}{c} v_t \left[\frac{C_{t+i/t}^w(s)^{1-\sigma^w}}{1-\sigma^w} - \gamma \frac{h_{t+/t}(s)^{1+\eta}}{1+\eta} \right] \\ +\bar{\lambda}_{t+i/t}\,(1-\tau^w)\left(\frac{w_t^*}{P_{t+i}}\right) h_{t+i/t} - \lambda_{t+i/t}\left(C_{t+i/t}^w - T_{t+i}^w\right) \end{array} \right\}$$

subject to the demand for labor type s given by (15). The first order condition for the optimal choice of w_t^* takes the form

$$\frac{w_t^*}{P_t} = \frac{\mathrm{E}_t \sum_{i=0}^{\infty} (\beta\phi^w)^i\, \bar{\lambda}_{t+i/t}\theta_{t+i}^w MRS_{t+i/t} h_{t+i/t}}{\mathrm{E}_t \sum_{i=0}^{\infty} (\beta\phi^w)^i\, \bar{\lambda}_{t+i/t}\left(\theta_{t+i}^w - 1\right)\left(1-\tau_{t+i}^w\right)\left(\frac{P_t}{P_{t+i}}\right) h_{t+i/t}}, \tag{19}$$

where $MRS_{t+i/t} = \gamma v_{t+i}\left(h_{t+i/t}\right)^\eta / \bar{\lambda}_{t+i/t}$.[18] The evolution of the aggregate nominal wage among workers is

$$(w_t)^{1-\theta^w} = (1-\phi^w)\left(w_t^*\right)^{1-\theta^w} + \phi^w \left(w_{t-1}\right)^{1-\theta^w}. \tag{20}$$

[17] The use of family labor supply adjusting to provide partial consumption insurance is consistent with the evidence of Blundell, Pistaferri, and Saporta-Eksten (2012). Domeij and Flod??n (2006) discuss how the positive correlation between wages and consumption due to borrowing restrictions can bias estimates of the wage elasticity of labor supply.

[18] The preference shock ν_t does not affect $MRS_{\,t+i/t}$ as it also appears in $\bar{\lambda}_{t+i/t}$.

Linearizing (19) and (20) around a zero inflation steady state yields[19]

$$\pi_t^w = \beta \mathrm{E}_t \pi_{t+1}^w + \kappa_w \left(\widehat{mrs}_t + \hat{\mu}_t^w - \hat{\omega}_t + \hat{\tau}_t^w \right), \tag{21}$$

where

$$\widehat{mrs}_t = \eta \hat{h}_t + \sigma^w \hat{c}_t^w, \tag{22}$$

$$\kappa_w \equiv \frac{(1-\phi^w)(1-\beta\phi^w)}{\phi^w} \left(\frac{1}{1+\xi} \right),$$

and

$$\xi \equiv \eta \theta^w + \chi \psi \sigma^w (\theta^w - 1).$$

When the individual worker bears aggregate labor income risk but not idiosyncratic risk, $\chi = 0$ and $\xi = \eta\theta^w$, which is the result obtained in standard sticky-wage new Keynesian models. The term $\chi\sigma^w\psi(\theta^w - 1) \geq 0$ arises because the individual worker internalizes the direct impact of her own labor income on her own consumption (and therefore on her marginal utility of consumption) to the extent (measured by χ) that the worker bears the labor income risk. When $\chi = 1$, $\xi \equiv \eta\theta^w + \psi\sigma^w(\theta^w - 1) > \eta\theta^w$. It is only through the parameter ξ that χ affects the dynamic behavior of the economy, and ξ only appears in κ_w in the wage inflation equation.[20]

Finally, the aggregate real wage evolves according to

$$\omega_t = \omega_{t-1} + \pi_t^w - \pi_t. \tag{23}$$

1.3 Monetary and fiscal policy

To assess the basic dynamic properties of the model, monetary policy is represented by a simple Taylor instrument rule of the form

$$1 + i_t = \beta^{-1} (1 + \pi_t)^{\varphi_\pi} x_t^{\varphi_x}$$

where $X_t \equiv Y_t / Y^e$ is output relative to the efficient output, defined as equilibrium output with flexible prices and wages, no markups, and consumption equalized across all households.[21] Log

[19] See the online appendix for details.

[20] Lee (2014) also shows how financial frictions affect the slope of the new Keynesian Phillips curve.

[21] Optimal policy is considered in section 4.

linearizing the policy rule around the zero inflation steady state yields

$$i_t = \rho + \varphi_\pi \pi_t + \varphi_y \hat{x}_t. \tag{24}$$

Fiscal policy is Ricardian. Total employment is Wh_t, so the government's revenue from the payroll tax is $\tau_t^p \omega_t W h_t$, and Its revenue from the wage income tax is $\tau_t^w \int_0^W \omega_t(s) h_t(s) ds$. This revenue is used to finance transfers (taxes if negative) of T_t^w per worker and T_t^c per capitalist. Thus, the government's budget constraint is

$$\tau_t^p \omega_t W h_t + \tau_t^w \int_0^W \omega_t(s) h_t(s) ds = W T_t^w + (1 - W) T_t^c,$$

or

$$W T_t^w + (1 - W) T_t^c = (\tau_t^p + \tau_t^w) \omega_t W h_t. \tag{25}$$

The steady-state transfers will be used to ensure the economy fluctuates around an efficient steady state. Because of the presence of workers who simply consume their after-tax wage income plus transfers, stochastic variations in transfers to workers will affect aggregate spending. I assume the government adjusts these transfers according to tax rules of the form

$$W \left(T_t^w - T^w\right) = \delta W \left[(\tau_t^p + \tau_t^w)\omega_t h_t - (\tau^p + \tau^w)\, \omega h\right]$$

and

$$(1 - W)\left(T_t^c - T^c\right) = (1 - \delta)\, W \left[(\tau_t^p + \tau_t^w)\omega_t h_t - (\tau^p + \tau^w)\, \omega h\right].$$

Linearizing these rules yields

$$\left(\frac{T^w}{\omega h}\right) \hat{t}_t^w = \delta \left[(\tau^p + \tau^w)\left(\hat{\omega}_t + \hat{h}_t\right) + \hat{\tau}_t^p + \hat{\tau}_t^w\right] \tag{26}$$

and

$$\left(\frac{1 - W}{W}\right)\left(\frac{T^c}{\omega h}\right) \hat{t}_t^c = (1 - \delta) \left[(\tau^p + \tau^w)\left(\hat{\omega}_t + \hat{h}_t\right) + \tau^p \hat{\tau}_t^p + \tau^w \hat{\tau}_t^w\right]. \tag{27}$$

To ensure steady-state output is efficient, both τ^p and τ^w will be negative, as subsidies are required to offset the steady-state distortions arising from markups in the goods and labor markets. Hence, $\delta \in [0\ 1]$ is the share any variation in the costs of these subsidies that are collected through taxes on workers.

Aggregating (11), combining it with the expression for T^w, and linearizing the result yields

$$\hat{c}_t^w = Q\frac{\omega h}{C^w}(\hat{\omega}_t + \hat{h}_t) - \frac{\omega h}{C^w}\hat{\tau}_t^w + \delta\frac{\omega h}{C^w}\left(\hat{\tau}_t^w + \hat{\tau}_t^p\right)$$

where $Q \equiv 1 - \tau^w + \delta\left(\tau^p + \tau^w\right)$. Two special cases of are note. First, if $\delta = 0$, all tax adjustment falls on capitalists. In this case,

$$\hat{c}_t^w = (1 - \tau^w)\frac{\omega h}{C^w}(\hat{\omega}_t + \hat{h}_t) - \frac{\omega h}{C^w}\hat{\tau}_t^w.$$

The wage tax directly affects after tax wage income, but budget adjustments resulting from stochastic variation in government revenues do not, as these are offset by adjusting taxes on capitalists. Second, if $\delta = 1$, all tax adjustment falls on workers. In this case,

$$\hat{c}_t^w = (1 + \tau^p)\frac{\omega h}{C^w}(\hat{\omega}_t + \hat{h}_t) + \frac{\omega h}{C^w}\hat{\tau}_t^p.$$

The wage tax does not affect workers' consumption as the taxes paid are directly returned to workers in transfers.

1.4 Market clearing

Goods market clearing for each firm implies

$$c_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\theta_t^p}\left[WC_t^w + (1 - W)\,C_t^c\right].$$

Defining aggregate output as

$$Y_t \equiv \left[\int_0^1 c_{j,t}^{\frac{\theta_t^p = 1}{\theta_t^p}}\,dj\right]^{\frac{\theta_t^p}{\theta_t^p - 1}},$$

goods market clearing implies

$$Y_t = \left[WC_t^w + (1 - W)\,C_t^c\right] \equiv C_t. \tag{28}$$

Linearizing (28) around the steady state yields

$$\hat{y}_t = \hat{c}_t = W\left(\frac{C^w}{Y}\right)\hat{c}_t^w + (1 - W)\left(\frac{C^c}{Y}\right)\hat{c}_t^c. \tag{29}$$

From the production function of firm j and the demand curve it faces,

$$h_{j,t} = \left(\frac{c_{j,t}}{Z_t \bar{A}}\right)^{\frac{1}{1-a}} = \left(\frac{Y_t}{Z_t \bar{A}}\right)^{\frac{1}{1-a}} \left(\frac{P_{j,t}}{P_t}\right)^{-\frac{\theta_t^p}{1-a}},$$

where $\bar{A} \equiv W k^a$ is a constant. Therefore

$$h_t = \int_0^1 h_{j,t} dj = \left(\frac{Y_t}{Z_t \bar{A}}\right)^{\frac{1}{1-a}} \Delta_{p,t} \tag{30}$$

where

$$\Delta_{p,t} \equiv \int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\frac{\theta_t^p}{1-a}} dj \geq 1$$

reflects relative price dispersion across firms.

The labor supplied by household s is given by (15). Define h_t^s as aggregate average hours per worker, and using (15),

$$h_t^s \equiv \left(\frac{1}{W}\right) \int_0^W h_t(s) ds = \left[\left(\frac{1}{W}\right) \int_0^W \left(\frac{\omega_t(s)}{\omega_t}\right)^{-\theta_t^w} ds\right] h_t = \Delta_{w,t} h_t,$$

where

$$\Delta_{w,t} \equiv \left[\left(\frac{1}{W}\right) \int_0^W \left(\frac{\omega_t(s)}{\omega_t}\right)^{-\theta_t^w} ds\right] \geq 1$$

is a function of relative wage dispersion. Hence,

$$h_t = \Delta_{w,t}^{-1} \left(\frac{1}{W}\right) \int_0^W h_t(s) ds < h_t^s. \tag{31}$$

Using (31),

$$Y_t = \Delta_{p,t}^{a-1} \bar{A} Z_t h_t^{1-a} = \left(\Delta_{p,t} \Delta_{w,t}\right)^{a-1} \bar{A} Z_t \left(h_t^s\right)^{1-a}, \tag{32}$$

illustrating how both wage and price dispersion drive wedges between average labor hours supplied per worker and final output produced per worker. However, to first order,

$$\hat{y}_t = \hat{z}_t + (1-a) \hat{h}_t, \tag{33}$$

as the measures of relative price and wage dispersion are both of second order, given the assumption of a zero-inflation steady state.

When the welfare implications of the model are evaluated, it will be useful to linearize the model around the efficient steady state. This involves using the steady-state values of the payroll tax, the wage tax, and the lump-sum taxes to ensure $\mu^p(1+\tau^p)=1$, $\tau^w=1-\mu^w$, and $C^w=C^w=C^c$. The conditions on the tax rates τ^p and τ^w ensure the effects of the steady-state markups are offset. Note that fiscal policy is assumed to ensure steady-state consumption is equalized across workers and capitalists, but taxes and transfers are not adjusted to offset the cyclical inefficiencies arising from imperfect consumption risk sharing in this economy. The complete set of equations characterizing the log linearized equilibrium of the model are collected together in the appendix.

2 Wages and aggregate demand

The presence of workers whose consumption is linked to their labor income introduces a direct channel from wages and employment to aggregate demand. This aggregate demand channel of wages is distinct from the standard wage channel in new Keynesian models through which wages affect firms' marginal costs and inflation. This section derives the demand channel of wages in the linearized version of the model.

Aggregate demand depends on the consumption of both workers and capitalists. The latter follow a standard Euler condition that links optimal consumption today to expected future consumption. Workers' consumption is equal to labor income plus transfers. Combining the log linearized Euler condition for capitalists (4) with the log linearized budget constraint for workers (12), the fiscal rule (26), the goods clearing condition (29), and the aggregate production function (33) yields[22]

$$\begin{aligned}\hat{y}_t &= \mathrm{E}_t\hat{y}_{t+1} - \Phi\left(\frac{1-W}{\sigma^c}\right)\left(i_t - \mathrm{E}_t\pi_{t+1} - \bar{\rho}_t\right) \\ &\quad + \Phi W Q\left(1-a\right)\left(\hat{\omega}_t - \mathrm{E}_t\hat{\omega}_{t+1}\right), \end{aligned} \tag{34}$$

[22]Equation (34) makes use of the fact that $\omega h/Y = (1-a)/[\mu^p(1+\tau^p)]$. In the efficient steady state, this is equal to $1-a$.

where $\Phi \equiv 1/(1-WQ)$, $Q \equiv 1-\tau^w+\delta\left(\tau^w+\tau^p\right)$, and

$$\begin{aligned}\bar{\rho}_t &\equiv \rho_t-\sigma^c\frac{W}{1-W}Q\left(\hat{z}_t-\mathrm{E}_t\hat{z}_{t+1}\right)\\ &\quad -\sigma^c\delta(1-a)\frac{W}{1-W}\left[(1-\delta)\left(\hat{\tau}_t^w-\mathrm{E}_t\hat{\tau}_{t+1}^w\right)-\delta\left(\hat{\tau}_t^p-\mathrm{E}_t\hat{\tau}_{t+1}^p\right)\right].\end{aligned}$$

For $WQ<1$, Φ reflects a Keynesian multiplier that arises from the impact of labor income on aggregate demand. A rise in output increases labor income. This directly increases the consumption of workers, re-enforcing the initial rise in aggregate demand. The multiplier depends on the fraction W of households who do not have access to financial markets; an increase in the fraction of households (workers) whose consumption raises automatically one-for-one with increases in aggregate income increases the multiplier.

The multiplier also depends on the wage and payroll taxes, as well as the fiscal adjustment parameter δ, as these enter into the determination of Q. Under the baseline calibration, τ^w and τ^p are set to ensure the flexible price steady-state equilibrium is efficient. This requires that the steady-state values of both tax rates be negative, representing subsidies designed to offset the markups in the labor and product markets. Specifically, $\tau^p=(1/\mu^p)-1<0$ and $\tau^w=1-\mu^w<0$; these values imply $Q=(1-\delta)\mu^w+\delta\left(1/\mu^p\right)>0$. In this case, $WQ<1$ if and only if

$$\delta>\delta^*\equiv\frac{\mu^p\left(1-W\mu^w\right)}{W\left(1-\mu^p\mu^w\right)}.$$

For the baseline calibration discussed below, $\delta^*=-0.1246$ so $WQ<1$ for all $\delta\in[0\ 1]$.

The fiscal parameter δ determines how the revenues needed to subsidize labor supply and labor demand to eliminate steady-state distortions are financed through "lump-sum" taxes on workers and capitalists. While these taxes do operate as lump-sum taxes on capitalists (the first-order conditions for capitalists' consumption are not affected by these taxes), this is not true with respect to the taxes on workers, as these directly affect workers' consumption. With $\tau^w+\tau^p<0$, $\partial Q/\partial\delta<0$, implying the multiplier is decreasing in δ. As output rises, the government must generate more revenue to finance the subsidy payments. If δ is large, this revenue is raised primarily by taxing workers, thereby reducing their after-tax labor income and consumption. This acts to reduce the multiplier.

Two special cases are worth noting. If $\delta=0$, so that all budget adjustment falls on capitalists,

$$\tilde{\rho}_t\equiv\rho_t-\sigma^c\frac{W}{1-W}Q\left(\hat{z}_t-\mathrm{E}_t\hat{z}_{t+1}\right)-\sigma^c\left(1-\tau^w\right)(1-a)\frac{W}{1-W}\left(\hat{\tau}_t^w-\mathrm{E}_t\hat{\tau}_{t+1}^w\right),$$

and demand is independent of the payroll tax, while a temporary rise in the wage tax is contractionary as it reduces workers' after tax labor income. Equivalently, a temporary wage subsidy, by boosting labor income, is expansionary. If $\delta = 1$, so that budget adjustment falls on workers,

$$\tilde{\rho}_t \equiv \rho_t - \sigma^c \frac{W}{1-W} Q \left(\hat{z}_t - \mathrm{E}_t \hat{z}_{t+1}\right) + \sigma^c (1-a) \frac{W}{1-W} \left(\hat{\tau}_t^p - \mathrm{E}_t \hat{\tau}_{t+1}^p\right).$$

In this case, a temporary rise in the payroll tax is expansionary since the tax revenues are given to workers and so increase aggregate demand. Changes in the wage tax do not affect aggregate demand directly, as any change in after-tax labor income is offset by an adjustment in transfers to workers.

Employing a standard new Keynesian model with homogeneous households, Galí (2013) argues that, if the central bank were to maintain a fixed real interest rate, employment would be independent of wages. This is no longer true when the distribution of income between capitalists and workers matters. The aggregate economy-wide Euler condition (34) can be solved forward to obtain[23]

$$\hat{y}_t = \left(\frac{1}{1-WQ}\right) \left\{ WQ\left(1-a\right)\hat{\omega}_t - \left(\frac{1-W}{\sigma^c}\right) \mathrm{E}_t \sum_{i=0}^{\infty} \left(i_{t+i} - \mathrm{E}_t \pi_{t+1+i} - \bar{\rho}_{t+i}\right) \right\}. \tag{35}$$

Substituting (35) into the aggregate production function (33) links employment positively to the real wage for a given current and expected future path of the real interest rate (relative to $\bar{\rho}_t$). A rise in wages *increases* employment. The direct effect of an increase in wages on worker consumption boosts aggregate demand and, with sticky prices, total output.

Rewriting (34) as

$$\hat{y}_t = \mathrm{E}_t \hat{y}_{t+1} - \Phi \left(\frac{1-W}{\sigma^c}\right) \left(i_t - \mathrm{E}_t \pi_{t+1} - \tilde{\rho}_t\right),$$

where $\tilde{\rho}_t \equiv \bar{\rho}_t + \sigma^c \left(\frac{W}{1-W}\right) Q\left(1-a\right)\left(\hat{\omega}_t - \mathrm{E}_t \hat{\omega}_{t+1}\right)$, shows that the net effect of the demand channel of wages on output, operating through $\tilde{\rho}_t$, will depend on the response of monetary policy. Any direct effect on aggregate demand could be offset by an adjustment of the nominal interest rate. If, however, the monetary authority would like to reduce the nominal interest rate to boost aggregate demand but is constrained by the zero lower bound, a rise in wages would be expansionary through two channels. First, $\hat{\omega}_t$ has a direct effect on $\hat{y}_t$ as shown in (35). Second,

[23] This uses the fact that $\lim_{T\to\infty} \hat{\omega}_T = 0$, which follows from restricting attention to stationary equilibria.

the rise in $\hat{\omega}_t$ increases firms' marginal costs and increases inflation through the standard wage channel on costs. If this effect is persistence, it also raises expected future inflation, further increasing aggregate demand through the resulting fall in the real interest rate.

3 Calibration results

In this section, I investigate the properties of a calibrated version of the model. Baseline values for the model's parameters are adopted from the calibration employed by Galí (2013) for the Erceg, Henderson, and Levin (2000) model (EHLM), together with the policy coefficients from Taylor (1993). These values are reported in Table 1. The steady-state level of output is normalized to equal one, while γ in (10) is set to ensure workers devote one-third of their time to market work in the steady-state.

Three additional parameters must be calibrated: the share of workers in the population W, the fiscal rule parameter δ, and the degree of risk sharing among workers χ. Lansing (2015) sets the ratio of workers to capitalists at 9 (implying $W = 0.9$) because he focuses on asset prices, and wealth in the United States is heavily concentrated in the top decile. In a new Keynesian models with rule-of-thumb households, Galí, López-Salido, and Vallés (2007) set $W = 0.5$, citing Mankiw (2000), and this value is also employed by Colciago (2011) and Furlanetto and Seneca (2012). In those models, however, optimizing households and rule-of-thumb households both earn labor income so the critical distinction is between those able to access financial markets and those who cannot. Kaplan, Violante, and Weidner (2014) argues that many households who behave as if they were hand-to-mouth consumers actually have significant wealth that is illiquid and so not easily used to smooth consumption fluctuations. They suggest 25 to 40 percent of households are hand-to-mouth. In the present model, W represents the fraction of hand-to-mouth households and the share who receive only labor income. The former interpretation would argue for a value of W of 0.5, as in Galí, López-Salido, and Vallés (2007), or lower, following Kaplan, Violante, and Weidner (2014). The latter interpretation suggests a value closer to the 0.9 employed by Lansing (2015). Using household data from the Consumer Expenditure Survey, Saijo (2014) divides households into asset holders and non-asset holders and finds the fraction of non-asset holders to be 75%. For the baseline calibration, I set $W = 0.75$.[24]

I set $\delta = 0.5$ in the baseline case, implying variations in government subsidy costs are split equally across workers and capitalists.

[24] While this value may seem high, by potentially overstating the demand channel of wages, a finding that greater wage flexibility increases welfare would only be strengthened if W were set to a lower value.

The final parameter to set is χ, the share of idiosyncratic labor income risk borne by the individual worker. As discussed previously, the dynamics of the linearized model are affected by χ only through its impact on the slope of the wage inflation equation (κ_w). For the benchmark calibration, $\kappa_w = 0.0032$ when $\chi = 1$ and $\kappa_w = 0.0036$ when $\chi = 0$. Thus, the degree to which workers are able to insure against idiosyncratic wage income risk has little effect on the aggregate dynamics of the model, though it does affect the cross-sectional distribution of consumption among workers and therefore welfare. For the benchmark case, I assume workers are unable to share idiosyncratic labor income risk and set $\chi = 1$.

3.1 Impulse responses

A useful way to begin an evaluation of the model is to compare the effects of a demand shock in the worker-capitalist model (WCM) with the results from EHLM. The preference shock, ν_t, increases capitalists' and workers' marginal utility of consumption without affecting workers' labor supply. It is equivalent, in EHLM, to the demand shock considered by Galí (2013). The shock affects ρ_t in the Euler equation (4) for capitalists, and is assumed to be persistent, with an AR(1) coefficient of 0.9. Such a shock is equivalent to a expansionary monetary policy shock with the same serial correlation properties, as both the policy shock and the preference shock affect equilibrium by affecting $i_t - \rho_t$.

The effects of the demand shock in the worker-capitalist model (WCM) are shown by the circles in Figure 1.[25] The diamonds shows the results in EHLM. The basic responses are similar in the two models. The chief differences are the weaker initial output response and the stronger, more persistent inflation response in WCM. These differences arise primarily because $i_t - \rho_t$ initially affects only the consumption of capitalists and not total consumption, so the rise in ρ_t has a smaller direct impact on demand than it does in EHLM. In addition, with the baseline parameter calibrations (specifically $\sigma^w = 1$), the real wage does not have a direct negative effect on wage inflation, unlike the case in EHLM. In EHLM, the rise in ω_t dampens wage inflation by reducing the gap between the marginal rate of substitution between leisure and consumption and the real wage. Thus limits the rise in firms' marginal costs, reducing the inflationary effects of the shock. In contrast, in WCM the wage has offsetting effects on the gap between workers' marginal rate of substitution between leisure and consumption and the real wage. For example, suppose $\psi = 1$ in (12) so that $\hat{c}_t^w = \hat{\omega}_t + \hat{h}_t$.[26] Then $\widehat{mrs}_t + \hat{\mu}_t^w - \hat{\omega}_t$, the driver for wage

[25] The four graphs in the upper two rows of figure 1 for EHLM replicate Galí's figure 11, p. 985.

[26] Under the baseline calibration, $\psi = 0.96$.

inflation, is equal to $\eta\hat{h}_t + \hat{c}_t^w + \hat{\mu}_t^w - \hat{\omega}_t = (1+\eta)\,\hat{h}_t + \hat{\mu}_t^w$ which is not affected directly by the real wage. With log preferences, the effect of a rise in ω_t on workers' consumption increases the marginal rate of substitution between leisure and consumption by the exact percentage amount by which the real wage rises, leaving the gap between the two, and therefore wage inflation, initially unaffected.[27]

As the lower panels of the figure show, consumption of workers and capitalists are affected differentially by the demand shocks.[28] While a positive demand shock initially increases the consumption of capitalists, real marginal costs for firms rise as the employment expansion increases real wages, leading eventually to a fall in profits and the consumption of capitalists.

Because a positive demand shock is equivalent to an (appropriately scaled) expansionary monetary policy shock, the results in figure 1 also illustrate the effects of a policy shock. The worker-capitalist model implies an exogenous interest rate cut increases the consumption of workers relative to capitalists. The impulse responses for worker and capitalist consumption reported in figure 1 are similar (with opposite sign) to the effects of contractionary monetary policy on low-net worth and high-net worth households reported by Coibion, Gorodnichenko, Kueng, and Silvia (2012). Their results imply a monetary expansion leads initially to a rise in consumption of both types of households followed by a sustained fall in the consumption of high-net worth households.[29]

The effects of a positive and persistent productivity shock are shown in figure 2. In EHLM, output rises but by less than efficient output does, implying the output gap falls. Hours decline, in line with the evidence in Galí and Rabanal (2004), as do price inflation and wage inflation. Price inflation falls more than wage inflation, so the real wage rises. In the worker-capitalist model, the drop in employment reduces aggregate demand through the impact of declining hours on workers' consumption. Because of sticky prices, the fall in demand amplifies the decline in employment and results in a large, negative effect on the output gap. The consumption of workers falls and follows the path of hours closely, as the decline in real wages is small. In contrast, consumption of capitalists rises significantly in the face of the positive productivity shock. Thus, a positive productivity shock boosts the consumption of the owners of capital and lowers the consumption of workers. This distributional effect of the productivity shock reduces

[27] As emphasized by Broer, Krusell, Hansen, and Oberg (2016), if prices are sticky but wages are flexible, a worker-capitalist model with log utility of consumption implies (in my notation) that $(1+\eta)\,\hat{h}_t + \hat{\mu}_t^w = 0$. Hence, employment is unaffected by monetary policy shocks.

[28] In the EHL model, consumption is equal across all households, so the consumption response is the same as the output response shown in the upper left panel of the figure.

[29] See their figure 9, p. 41.

aggregate demand as the share of workers in the population is large, and it leads to a sustained fall in both employment and output.

The impulse responses in figures 1 and 2 were based the assumption that each worker's consumption was linked to the worker's individual labor income. Results are similar when workers are able to pool all labor income, the standard case in models with rule-of-thumb households. As discussed earlier, the degree of risk sharing only affects the linearized model through the parameter κ_w in the wage inflation equation – see (22). The quantitative effects of χ on κ_w are small and so the dynamics are little affected by allowing for risk sharing among workers. Consequently, only results for the base case of no risk sharing are shown.

3.2 The role of transfers

In a standard representative agent model such as EHLM, fluctuations that have consequence for the government's budget are offset through adjustment in lump-sum taxes or transfers. In the face of heterogeneity and workers' hand-to-mouth behavior, any adjustment of T_t^w can have a direct effect on aggregate demand. An increase in the wage tax $\hat{\tau}_t^w$, for example, will have effects on labor supply decisions, but it will also matter for demand whether any associated increase in government revenues are rebated to workers or to capitalists. These effects can be seen most clearly by comparing the effects of the wage tax and the payroll tax on the economy's flexible wage/price equilibrium.

Consider first the standard model with flexible wages and prices. In this case, equilibrium employment and output can be found by noting that the specification of household preferences, the production function, and the labor market equilibrium condition imply

$$\eta\hat{h}_t + \sigma\hat{c}_t + \hat{\tau}_t^w + \hat{\mu}_t^w = \hat{\omega}_t = \hat{z}_t - a\hat{h}_t - \hat{\tau}_t^p + \hat{\mu}_t^p. \tag{36}$$

With goods market clearing and the production function implying $\hat{c}_t = \hat{y}_t = (1-a)\hat{h}_t + z_t$, output with flexible prices and wages equals

$$\hat{y}_t^{ehlf} = \frac{(1+\eta)\,\hat{z}_t - (1-a)\left(\hat{\tau}_t^w + \hat{\tau}_t^p + \hat{\mu}_t^w + \hat{\mu}_t^p\right)}{\eta + \sigma + a(1-\sigma)}. \tag{37}$$

In the worker-capitalist model, (36) is modified to take the form

$$\eta\hat{h}_t + \sigma^w\hat{c}_t^w + \hat{\tau}_t^w + \hat{\mu}_t^w = \hat{\omega}_t = \hat{z}_t - a\hat{h}_t - \hat{\tau}_t^p + \hat{\mu}_t^p, \tag{38}$$

reflecting the fact that only workers supply labor. While total consumption still equals output in equilibrium, $\hat{c}_t^w$ is equal to labor income plus transfers. Using (12) and (26) in (38) and using the aggregate production function, output with flexible prices and wages in the worker-capitalist model is

$$
\begin{aligned}
\hat{y}_t^{wcf} \quad = \quad & \frac{(1+\eta)\,\hat{z}_t - (1-a)\,(\tau_t^p + \mu_t^p + \hat{\tau}_t^w + \hat{\mu}_t^w)}{\eta + \tilde{\sigma}^w Q + a\,(1-\tilde{\sigma}^w Q)} \\
& + \frac{(1-a)\,\tilde{\sigma}^w Q\,(\tau_t^p + \hat{\mu}_t^p + \hat{\tau}_t^w) - \delta\,(1-a)\,\tilde{\sigma}^w Q\,(\hat{\tau}_t^w + \hat{\tau}_t^p)}{\eta + \tilde{\sigma}^w Q + a\,(1-\tilde{\sigma}^w Q)}. \qquad (39)
\end{aligned}
$$

In comparing (37) and (39), the first terms in each take a similar form and would be equal if $\sigma = \tilde{\sigma}^w Q$.[30] The additional terms in (39) reflect the direct impact of wages on workers' consumption, and these effects depend on the extend to which changes in government revenues affect transfers to workers. For example, if $\delta = 1$, the direct effects of tax induced wage chages on workers' labor income is offset by adjustments in transfers. In this case, the effects of taxes on flex-price and wage output will be similar in the two models. Equation (39) also highlights that it is only the preferences of workers that govern labor supply and theorefore $\hat{y}_t^{wcf}$.

Figure 3 presents the effects of a 1 percentage point cut in the wage tax for $\delta = 0$, 0.5, and 1. When $\delta = 0$ (the solid line), the fall in government revenues when the wage tax is cut reduces transfers to capitalists; the wage subsidy has income and substitution effects on workers and leads to a reduction in labor supply. In contrast, when $\delta = 1$, the fall in $\hat{\tau}_t^w$ reduces transfers to households, negating the direct income effect of the tax cut on after-tax wage income. The substitution effect leads to a rise in labor supply and a rise both in output with flexible prices and wages and in the output gap when prices and wages are sticky.

The sign of the net effect of a price markup shock on output in the worker-capitalist model is determined by $1-\tilde{\sigma}^w Q$. A rise in the price markup reduces labor demand and results in a fall in real wages. If $\tilde{\sigma}^w Q = 1$, income and substitution effects on labor supply are offsetting. Employment remains unchanged and $\hat{\mu}_t^p$ has no effect on $\hat{y}_t^{wcf}$. If $\tilde{\sigma}^w Q > 1$, the income effect dominates and labor supply rises, increasing output; if $\tilde{\sigma}^w Q < 1$, the substitution effect dominates, labor supply declines and so does output.

[30] For the baseline calibration ($\sigma = \sigma^w = 1$), $\tilde{\sigma} Q = 0.963$ for $\delta = 0$, 0.815 for $\delta = 0.5$, and 0.667 for $\delta = 1$.

3.3 The effects of wage flexibility

If output is constrained by aggregate demand and wages have a direct effect on spending, a fall in real wages could reduce labor demand and exacerbate unemployment rather than reduce it. In the face of a negative preference shock that reduces demand, sticky wages might help stabilize the economy by limiting the decline in wage income and workers' spending. The feedback or multiplier effect that arises when current labor income directly affects current spending could, in principle, be neutralized by monetary policy to the extent that the central bank is not constrained by the zero lower bound on its policy interest rate; no adjustment of wages or prices would be required. If monetary policy failed to fully insulate spending in the face of such a shock, then the potential feedback from a fall in labor demand to wages, labor incomes and spending would come into play.

Even if demand shocks are neutralized by monetary policy, the degree of wage flexibility will be relevant in the face of other shocks, such as productivity and markup shocks. To focus on the role of wage flexibility, therefore, I consider the responses to shocks holding the monetary rule fixed while varying the Calvo parameter ϕ^w.

Figure 4 shows the response to an aggregate demand shock for $\phi^w = 0.5$, 0.65, and 0.75 (the baseline value). Recalling that an increase in ϕ^w implies an increase in wage rigidity, output and employment are significantly more stable with more flexible wages. Thus, despite the fact the model builds in a labor-income spending multiplier, and the real wage rises more in the face of the positive demand shock when wages are more flexible, wage flexibility reduces the overall responses of employment and output. The lower left plot in the figure shows that the response of consumption spending by workers is largest when wages are the most rigid (i.e., when $\phi^w = 0.75$) as hours rise more with greater wage rigidity, and this accounts for the larger expansion of workers' consumption, output and employment. Conversely, a negative demand shock causes a larger decline in employment and workers' consumption when wages are relatively rigid. Of course, as wages becomes more rigid, the impact of the demand shock on firms' marginal costs and therefore on inflation is dampened.[31]

The effects of wage flexibility on the economy's response to a productivity shock are shown in figure 5. The output gap, hours, and worker consumption are more stable when wages are more flexible. The multiplier effect generated by the link between labor income and worker consumption leads to a fall in worker's consumption in the face of a positive productivity shock,

[31] Coibion (2012) show that downward nominal wage rigidity can be stabilizing at the zero lower bound on nominal interest rates by reducing the chances the economy experiences a destabilizing deflation. It does so by limiting the fall in real marginal costs that would otherwise occur. See section 5 below.

particularly when wages are quite sticky ($\phi^w = 0.75$). Recall that the output gap is defined as output relative to the efficient level of output, so greater wage rigidity moves the economy further from the efficient output path in response to productivity shocks.

In standard new Keynesian models, markup shocks in labor and product markets are inefficient shocks that pose trade-offs for monetary policy makers. The response to a wage markup shock is shown in figure 6. Greater wage flexibility amplifies the effects of the wage markup shock on output, and employment as the contractionary effects of the shock lower labor income, workers' consumption and aggregate demand. Not surprisingly, the responses of price and wage inflation become more volatile as the Calvo parameter falls. Greater wage rigidity affects the response to price markup shocks in a manner similar to its effects on productivity and demand shocks.

To summarize, increased wage flexibility serves to stabilize the output gap in the face of aggregate demand, productivity, and price markup shocks despite the presence of a direct aggregate demand channel for wages. However, greater wage flexibility leads to larger fluctuations in price and wage inflation in the response to these shocks. Thus, the benefits of greater wage flexibility is ambiguous from the perspective of a typical flexible-inflation targeting central bank that cares about stability of the output gap and inflation. In contrast, the effects of markup shocks on both the output gap and inflation are dampened by greater wage rigidity.

4 Welfare and wage rigidity

The previous sections have employed a simple Taylor rule to represent monetary policy. Because greater nominal wage flexibility increased the volatility of price inflation and wage inflation in the face of shocks and decreased the volatility of the output gap, any conclusions about the costs of wage rigidity will depend on the relative importance of the alternative shocks and on the relative costs attached to the volatility of output, inflation, and wage inflation. In this section, I employ a second order approximation to the welfare of workers and capitalists to assess the implications of wage rigidity.

Define social welfare V_t as the expected present discounted sum of the welfare of workers and capitalists, divided by total population. Thus,

$$V_t = \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left\{ \int_0^W \nu_{t+i} \left[\frac{\left(C^w_{t+i}(s)\right)^{1-\sigma^w}}{1-\sigma^w} - \frac{\gamma H_{t+i}(s)^{1+\eta}}{1+\eta} \right] ds + \int_W^1 v_{t+i} \frac{\left(C^c_{t+i}(h)\right)^{1-\sigma^c}}{1-\sigma^c} dh \right\}. \tag{40}$$

The appendix shows that the second order approximation of welfare around an efficient steady state with $\sigma^c = \sigma^w = \sigma$ is given by

$$V_t \approx \bar{V} - \frac{1}{2} U_c \bar{C} \left(\frac{\theta^p}{\kappa_p} \right) \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i L_{t+i} + tip,$$

where $\bar{V}$ is steady-state welfare, *tip* represents terms independent of policy, and the period loss function is

$$L_t = \pi_{t+i}^2 + \lambda_x x_{t+i}^2 + (\lambda_{1,\pi^w} + \lambda_{1,\pi^w}) \left(\pi_{t+i}^w \right)^2 + \lambda_c \left(\hat{c}_t^c - \hat{c}_t^w \right)^2 . \tag{41}$$

The weights in (41) are given by

$$\lambda_x \equiv \left(\frac{a + \eta + \sigma(1-a)}{1-a} \right) \left(\frac{\kappa_p}{\theta^p} \right)$$

$$\lambda_{1,\pi^w} \equiv (1-a) \left[\frac{(1 + \eta\theta^w)}{1 - \chi\psi\sigma + (\chi\psi\sigma + \eta)\,\theta^w} \right] \left(\frac{\kappa_p}{\theta^p} \right) \left(\frac{\theta^w}{\kappa_w} \right)$$

$$\lambda_{2,\pi^w} \equiv \chi^2 \left[\frac{W\psi^2 (1-\theta^w)^2}{1 - \chi\psi\sigma + (\chi\psi\sigma + \eta)\,\theta^w} \right] \left(\frac{\kappa_p}{\theta^p} \right) \left(\frac{\theta^w}{\kappa_w} \right)$$

$$\lambda_c \equiv \sigma (1 - W)\, W \left(\frac{\kappa_p}{\theta^p} \right).$$

The weights λ_x and λ_{1,π^w} are identical in form to those appearing on the output gap and wage inflation terms in the standard sticky-price, sticky-wage new Keynesian model. In the worker-capitalist model, an additional weight, λ_{2,π^w}, is given to wage inflation volatility.[32] This additional cost arises because relative wage dispersion generates an inefficient dispersion of consumption across workers. Further, the final term in the loss function reflects imperfect risk sharing across workers and capitalists. In an efficient equilibrium, consumption for the two groups would move together. When consumption is equalized across all households, $\lambda_{1,\pi^w} = (1-a)(\kappa_p/\theta^p)(\theta^w/\kappa_w)$, $\lambda_{2,\pi^w} = 0$, and $\hat{c}_t^c = \hat{c}_t^w$, yielding the standard quadratic ap-

[32] While λ_{1,π^w} is identical in form to the weight on wage inflation volatility in EHL, its value differs as κ_w has $1 + \eta\theta^w + \chi\psi\sigma(\theta^w - 1)$ in the denominator rather than $1 + \eta\theta^w$ as in EHL. The standard new Keynesian model assumes $\chi = 0$ (complete risk sharing), in which case

$$\lambda_{1,\pi^w} \equiv (1-a) \left(\frac{\kappa_p}{\theta^p} \right) \left(\frac{\theta^w}{\kappa_w} \right),$$

and, as consumption is equalized across workers, $\lambda_{2,\pi^w} = 0$.

proximation to welfare (see Erceg, Henderson, and Levin (2000) or Galí (2015)). When workers, who correspond to the rule-of-thumb households in this model, are able to share consumption risk, $\lambda_{2,\pi^w} = 0$ but $\hat{c}_t^c$ and $\hat{c}_t^w$ can still differ so that consumption inequality between workers and capitalists reduces overall welfare.

To evaluate L_t as the degree of wage rigidity varies, I let ϕ^w range from 0.25 to 0.95. By fixing the parameters in the policy rule while varying the degree of wage rigidity, the exercise serves to isolate the effects of wage rigidity without compounding the effects of any endogenous response of policy to changes in the economy's structure. I then repeat this exercise under an optimal, time-consistent monetary policy. Results are reported individually for demand shocks and productivity shocks to illustrate the crucial role the assumption about policy plays in an assessment of the welfare costs of wage rigidity. I then use the results from Smets and Wouters (2007) to obtain an empirically reasonable calibration of the variance and serial correlation properties of the underlying shocks. Based on these values, the welfare effects of wage rigidity can be assessed when the model is subject to a full set of disturbances.

When monetary policy is given by the Taylor rule and only demand (preference) shocks are present, the five components of L_t due to inflation, the standard wage inflation cost, idiosyncratic labor income risk, output gap volatility, and asymmetric consumption volatility are shown in figure 7. When wages are fairly flexible, the bulk of the loss arises from inflation variability. This falls significantly as wages become more rigid. However the cost of output gap volatility increases as wages become more rigid and becomes the major source of welfare loss as wages become very rigid. However, the overall loss declines dramatically as wages become more rigid once $\phi^w > 0.65$. This, the greater stability of π and π^w in response to demand shocks as ϕ^w increases, seen in figure 4, dominates in welfare terms over the greater output gap instability that occurs when wages are stickier.

A similar decomposition for the case of productivity shocks is shown in figure 8. As with demand shocks, the bulk of the welfare loss arises from inflation variability when wages are quite flexible, though the loss arising from imperfect consumption risk sharing between workers and capitalists is larger then in response to demand shocks. Similar to the case of demand shocks, the cost of output gap volatility comes to dominate the total welfare loss as wages become very rigid. Again, the greater stability of π and π^w in response to productivity shocks as ϕ^w increases, seen in figure 5, dominates in welfare terms over the greater output gap instability that occurs when wages are stickier.

The results in figures 7 and 8 were conditional on a fixed rule describing monetary policy. Using the loss function L_t, one can evaluate loss as a function of wage rigidity under an optimal monetary policy that will itself depend on the degree of wage rigidity. I focus on the case of optimal discretionary policy.

Outcomes under an optimal, time-consistent monetary policy are quite different than those under a fixed Taylor rule. This is clearest for the case of demand shocks. An optimal monetary policy insulates prices, wages, and output completely in the face of demand shocks. Thus, the welfare loss is zero, regardless of the degree of nominal wage rigidity.[33]

Now consider the situation when the only disturbance arises from productivity shocks. With sticky prices and sticky wages, optimal policy will be unable to neutralize the welfare effects generated by such shocks. Under the Taylor rule, figure 8 showed that loss declined as wages became very rigid. Contrast that finding with the results under an optimal monetary policy, shown in figure 9. Loss is monotonically *increasing* in wage rigidity in the face of productivity shocks. The costs arising from wage inflation and output gap volatility fall as wages become more rigid, but this is more than offset by rising costs of price inflation and the costs of asymmetric consumption movements. In fact, the losses arising from imperfect consumption risk sharing between workers and capitalists is the largest source of welfare loss under optimal policy, regardless of the degree of wage rigidity.

The results for demand and productivity shocks illustrate how critical the assumption about monetary policy is for any analysis of the effects of wage rigidity. If demand and productivity shocks were the only two disturbances, and policy was optimal, if would be straightforward to conclude that welfare would improved if wages were more flexible. However, this result is not true for other shocks. Welfare actually falls under optimal policy as wage rigidity increases when only price markup shocks are present, a shown in figure 10. A similar result holds when wages become very rigid if wage markup shocks are the only disturbance (not shown). Thus, the welfare effects of greater wage rigidity differ depending on the source of disturbance, and any overall conclusion about the welfare effects of wage flexibility will depend on the relative importance of the various shocks.

To assess fully the connection between wage rigidity and welfare, it is important to allow for the full set of shocks and to calibrate their volatilities in a manner that reflects their relative empirical importance. To do so, I map the shocks in the estimated model of Smets and Wouters (2007) into the shocks in the worker-capitalist model. This is straightforward, with the exception of the demand (preference) shock in the theoretical model, as the Smets-Wouter model incorporates risk premium and spending shocks, implying multiple sources of what in the

[33] The analysis ignores the constraint posed by an effective lower bound on nominal interest rates which would limit the ability of monetary policy to neutralize the effects of demand shocks. See section 5.

basic model is captured by the preference shock. Therefore, I take the estimated standard errors of shocks for the productivity, price markup, wage markup, and monetary policy shocks from Smets and Wouter and then set the standard deviation of the model's preference shock ρ_t to match the standard deviation of HP-filtered U.S. real GDP for the 1996:1-2004:4 sample period used by Smets and Wouter. Table 2 summarizes the values used when simulating the model in the presence of all five shocks.

Results for welfare as a function of wage rigidity under the Taylor rule are shown in figure 11, while the results under optimal time-consistent policy are shown in figure 12. These two figures illustrate that the costs of wage rigidity differ significantly depending on the assumption made about monetary policy. First, notice that vertical scales in the two figures are vastly different; at the benchmark value of wage rigidity ($\phi^w = 0.75$), welfare costs are over 4 percent of steady-state consumption under the Taylor rule, while they are less than 0.1 percent under the optimal policy. Second, welfare costs fall markedly as wages become very rigid under the Taylor rule, while welfare costs are increasing with ϕ^w until ϕ^w rises above 0.85. Third, the composition of the welfare cost also differs under the alternative policies. Under the Taylor rule, the standard welfare costs associated with price and wage stickiness dominate. Output gap volatility becomes important only for extremely high degrees of wage rigidity, and the costs from imperfect risk sharing are relatively small as a share of total costs. Contrast this with the situation under optimal policy in figure 12. Two components of welfare costs tend to dominate the total loss; these are the costs associated with price stickiness and the costs arising from dispersion in consumption between workers and capitalists. The costs of output gap volatility and idiosyncratic labor income risk are small. The results under optimal policy suggest variations in the functional distribution of income between labor and non-labor income can, to the extent they reflect imperfect risk sharing as they do in the present model, interact with wage rigidity to be a significant component of the costs of economic fluctuations.

5 The ELB

Under an optimal monetary policy and in the absence of any concern about an effective lower bound (ELB) for the nominal interst rate, the monetary authority will always adjust the policy interest rate to control aggregate demand. Thus, the exact details of the monetary transmission mechanism, including any direct demand channel of wages, will be irrelevant for the behavior of the output gap.[34] However, this will no longer be true if monetary policy is constrained by the presence of an effective lower bound (ELB) on the nominal policy rate. In this case, the negative impact on demand of a fall in labor income cannot be offset by a reduction in the policy rate. If, in response to a contractionary shock that pushes the nominal interest rate to zero, labor income falls more when wages are flexible the demand effect of labor income will worsen the output effects of the shock.

For concreteness, the ELB is taken to be zero and the policy rule (24) is replaced by

$$i_t = \max\left(0, \rho + \varphi_\pi \pi_t + \varphi_y \hat{x}_t\right).$$

All shocks except the demand shock are set to zero, while the demand shock taks a negative realization that makes the ELB a binding constraint.[35] The responses of the output gap, price inflation, and wage inflation are shown in Figure 13 for the standard EHLM that does not include a demand channel for wages (denoted by circles) and for the worker-capitalist model (without circles). Solid lines correspond to the benchmark calibration for wage rigidity ($\phi^w = 0.75$) while dashed lines correspond to greater wage flexibility ($\phi^w = 0.5$). The upper panel of the figure illustrates the behavior of the output gap. In EHLM, the degree of wage flexibility has little effect on the contractionary effects due to the ELB (the solid and dashed lines with circles are indistinguishable). This is not the case when labor income directly affects aggregate demand. The drop in the the output gap is larger in WCM, and it is largest when wages are more flexible. The lower panel shows the significiant fall in wage inflation when $\phi^w = 0.5$. The adverse effects of wage flexibility reduce demand through two channels. The fall in real labor income directly reduces demand, and the fall in firms' marginal costs reduces inflation (see middle panel) which, since the decline is persistent, increases the real interest rate when the nominal rate cannot fall and reduces the consumption of capitalists.

The behavior of workers' and capitalists' consumption is shown in figure 14. Greater wage flexibility leads to a sharper and more persistent decline in the consumption or workers (top panel, dashed line). While greater wage flexibility initially leads to a drop in capitalists' consumption as the real interest rate rises, the fall in wages boosts profit income, leading to a boom in the consumption of capitalists. Thus, the ELB constraint on monetary policy leads to significiant redistributional effects if wages are relatively flexible.

[34] This is not the case if policy is suboptimal, as with simple instrument rules such as (24).

[35] The negative shock drives the nominal rate to zero for three periods and then decays at the rate 0.4. The model was solved using the toolkit of Guerrieri and Iacoviello (2015).

6 Conclusions

In this paper, I have developed a simply variant of a rule-of-thumb model that distinguishes between workers earning labor income and unable to participate in financial markets and capitalists who own the economy's productive technology and can engage in financial markets to share consumption risk. The model is very stylized, as are standard representative agent models, but the structure of the model serves to highlight the role the distribution of income between labor and capital may play for the way aggregate shocks affect output, employment, and inflation. The presence of workers who consume wage income introduces a direct aggregate demand channel for wages and employment that leads to a Keynesian multiplier effect. This channel captures the possibility that more flexible wages, by reducing labor income during a downturn, could further dampen aggregate demand and amplify the initial downturn.

While most models with nominal rigidities that include rule-of-thumb households have focused on government spending shocks, I consider a broader range of shocks and show in a calibrated version of the model how the economy's response to shocks depends on the presence of a labor income demand channel. A preference shock that increases aggregate demand raises labor income and boosts the consumption of workers relative to capitalists; the consumption of capitalists actually falls. An expansionary monetary policy has the opposite effect on workers' and capitalists' consumption.

When workers supply differentiated labor types, sticky wages generate relative wage dispersion and a cross-sectional variance of consumption among workers. This represents an additional welfare cost of wage inflation that is absent in representative agent models. It is also absent in traditional rule-of-thumb models which assume consumption is equalized across rule-of-thumb households. Welfare costs also arise from imperfect risk sharing between workers and capitalists. When workers and capitalists have identical preferences over consumption, a quadratic approximation to welfare was obtained that is directly comparable to quadratic loss functions in other new Keynesian models. The sources of welfare costs, and their dependence on the extend to which wages are rigid, are affected by the way monetary policy is conducted; results under a Taylor rule differ significantly from those under the optimal time-consistent policy. Under optimal monetary policy, the volatility of consumption differences between workers and capitalists constitutes the major welfare cost of wage rigidity.

If any constraints on monetary policy due to an effective lower bound on the nominal interest rate are ignored, the demand channel of labor income can always be offset by an appropriate adjustment of monetary policy. At the ELB, conventional interest policy is not able to offset the multiplier effect of a fall in labor income, and this channel becomes more important in amplifying the effects of shocks. It was shown that the labor income demand channel amplifies the negative effects of the ELB constraint significantly. And increased wage flexibility worsens the contractionary effects of a negative shock that pushes the economy to the ELB.

Even away from the ELB though, the dichotomy between workers and capitalists is still relevant in affecting the behavior of wage inflation, real wages, price inflation, and the welfare effects of fluctuations.

7 Appendix

Details of the model derivation are available in an online appendix. The log-linearized model consists of the following equations, which are jointly solved for $\hat{c}^c$, $\hat{c}^w$, $\hat{y}$, $\hat{h}$, i, $\widehat{mc}_t$, $\widehat{mrs}_t$, $\hat{\omega}$, π, π^w, $\hat{t}^w$ and $\hat{t}^c$:[36]

$$\hat{y}_t = W\left(\frac{C^w}{Y}\right)\hat{c}_t^w + (1-W)\left(\frac{C^c}{Y}\right)\hat{c}_t^c, \tag{42}$$

$$\hat{c}_t^c = \mathrm{E}_t\hat{c}_{t+1}^c - \left(\frac{1}{\sigma^c}\right)(i_t - \mathrm{E}_t\pi_{t+1} - \rho_t), \tag{43}$$

$$\hat{c}_t^w = \psi\left(\hat{\omega}_t - \hat{\tau}_t^w + \hat{h}_t\right) + (1-\psi)\,\hat{t}_t^w, \tag{44}$$

$$\hat{y}_t = \hat{z}_t + (1-a)\,\hat{h}_t, \tag{45}$$

$$\pi_t = \beta\mathrm{E}_t\pi_{t+1} + \kappa_p\left(\widehat{mc}_t + \hat{\mu}_t^p\right), \tag{46}$$

$$\widehat{mc}_t = \hat{\omega}_t + \hat{\tau}_t^p + a\hat{h}_t - \hat{z}_t, \tag{47}$$

$$\pi_t^w = \beta\mathrm{E}_t\pi_{t+1}^w + \kappa_w\left(\widehat{mrs}_t + \hat{\mu}_t^w - \hat{\omega}_t + \hat{\tau}_t^w\right), \tag{48}$$

[36] The equations that are linearized, in the order listed below, are (29), (4), (12), (33) (7), (8), (21), (22), (23), (24), (26) and (27).

$$\widehat{mrs}_t = \eta \hat{h}_t + \sigma^w \hat{c}_t^w, \tag{49}$$

$$\omega_t = \omega_{t-1} + \pi_t^w - \pi_t, \tag{50}$$

$$i_t = \rho + \phi_\pi \pi_t + \phi_x \hat{x}_t, \tag{51}$$

$$\left(\frac{T^w}{\omega H}\right) \hat{t}_t^w = \delta \left[(\tau^p + \tau^w) \left(\hat{\omega}_t + \hat{h}_t \right) + \hat{\tau}_t^w + \hat{\tau}_t^p \right], \tag{52}$$

$$\left(\frac{1-W}{W}\right) \left(\frac{T^c}{\omega H}\right) \hat{t}_t^c = (1-\delta) \left[(\tau^p + \tau^w) \left(\hat{\omega}_t + \hat{h}_t \right) + \hat{\tau}_t^w + \hat{\tau}_t^p \right], \tag{53}$$

where

$$\kappa_p \equiv \frac{(1-\phi^p)(1-\beta\phi^p)}{\phi^p} \left(\frac{1-a}{1-a+a\theta^p} \right),$$

$$\kappa_w \equiv \frac{(1-\phi^w)\, 1-\beta\phi^w}{\phi^w} \left(\frac{1}{1+\xi} \right),$$

$$\xi \equiv (\chi\psi\sigma^w + \eta)\, \theta^w - \chi\psi\sigma^w,$$

$$\psi = (1-\tau^w) \frac{\omega H}{C^w}.$$

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Table 1

	Parameter	Value
β	Discount factor	0.99
σ^c	1/EIS capitalists	1
σ^w	1/EIS workers	1
η	1/wage elasticity	5
θ^p	Goods demand elasticity	9
θ^w	Labor demand elasticity	4.52
ϕ^p	Calvo price adjustment	0.75
ϕ^w	Calvo wage adjustment	0.75
φ_π	Policy response, inflation	1.5
φ_x	Policy response, output	0.125
W	Pop. share of workers	0.75
δ	Fiscal rule	0.5

Table 2

Disturbance	Innovation standard deviation	AR(1) Coefficient
Productivity	0.72	0.95
Demand	0.20	0.90
Price markup	0.14	0.89
Wage markup	0.24	0.96
Policy	0.24	0.15

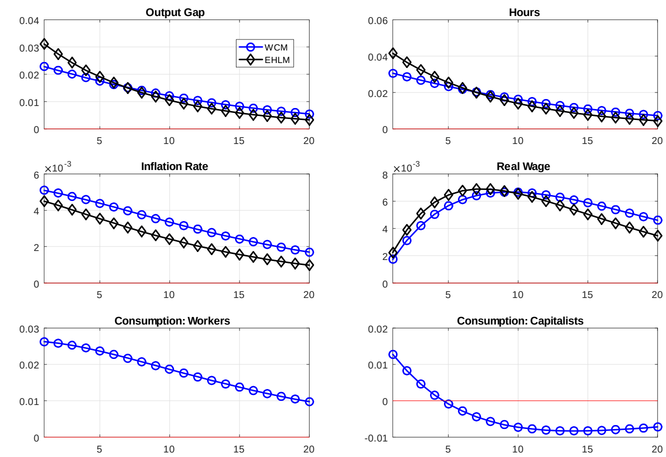

Figure 1: Response to a shock to $i_t - \rho_t$ due to a positive demand shock (preference shock) or a negative policy shock under the baseline calibration. Worker-capitalist model: circles. EHLmodel: diamonds.

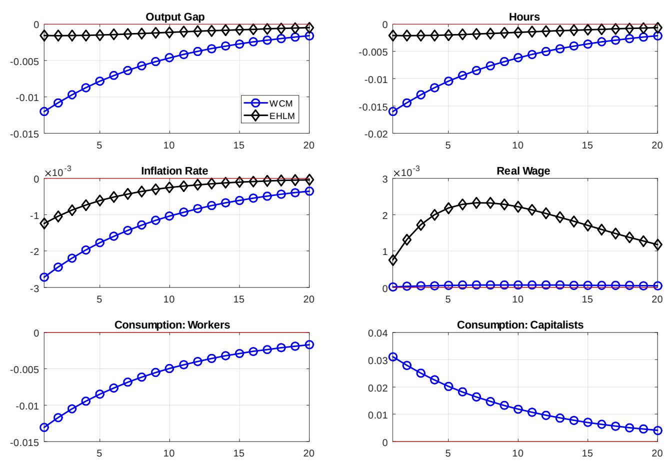

Figure 2: Response to a positive productivity shock under the baseline calibration. Worker-capitalist model: circles. EHLmodel: diamonds.

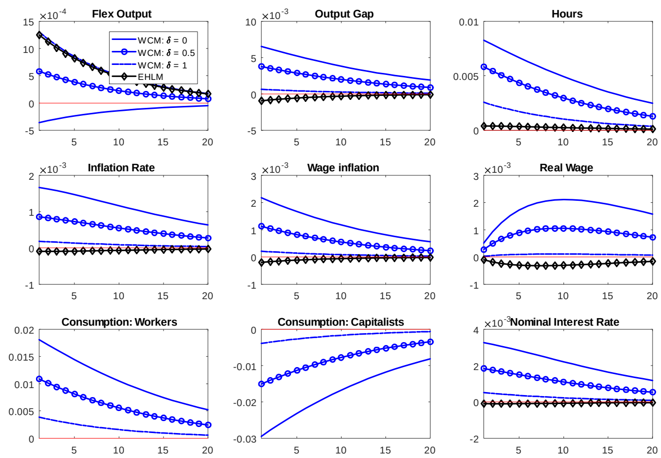

Figure 3: Response to a cut in the wage tax: solid line, $\delta = 0$; dashed line, $\delta = 0.5$, dot-dashed line, $\delta = 1$; black dashed line, EHL model.

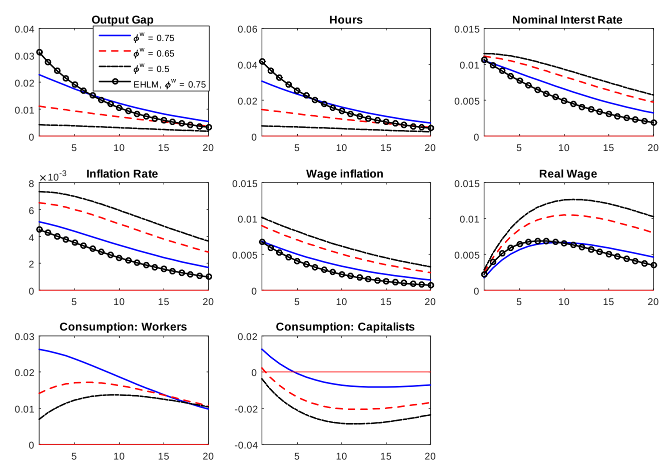

Figure 4: Wage rigidity and the response to a demand shock: dotted line ϕ^w= 0.5, dashed line ϕ^w= 0.65, solid line ϕ^w= 0.75, circles EHLM with ϕ^w= 0.75.

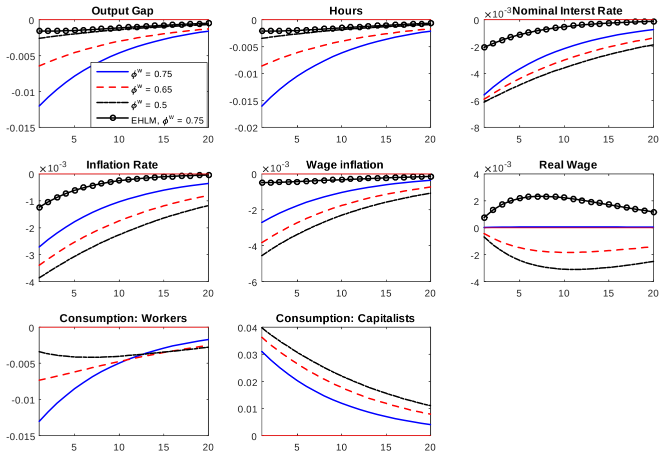

Figure 5: Wage rigidity and the response to a productivity shock: dotted line $\phi^w = 0.5$, dashed line $\phi^w = 0.65$, solid line $\phi^w = 0.75$, circles EHLM with $\phi^w = 0.75$.

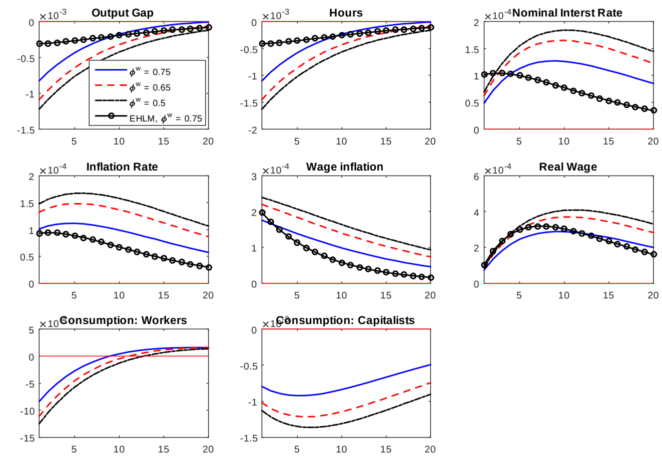

Figure 6: Wage rigidity and the response to a wage markup policy shock: dotted line $\phi^w = 0.5$, dashed line $\phi^w = 0.65$, solid line $\phi^w = 0.75$, circles EHLM with $\phi^w = 0.75$.

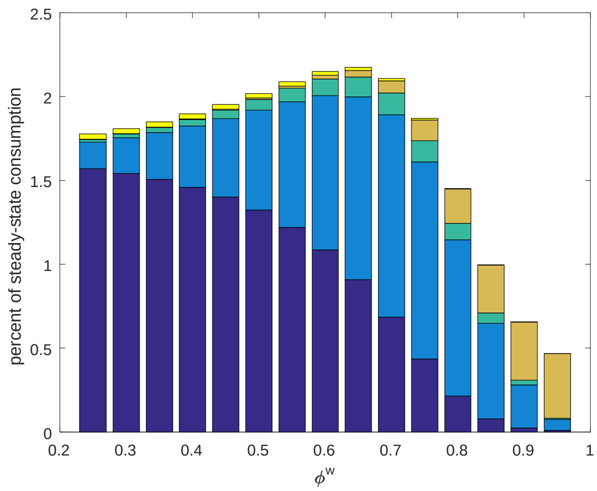

Figure 7: Welfare loss due to demand shocks as function of wage stickiness under the Taylor rule. Bottom to top: price inflation (dark blue), standard wage inflation (light blue), idiosyncratic labor income risk (green), output gap (brown), consumption inequality between workers and capitalists (yellow).

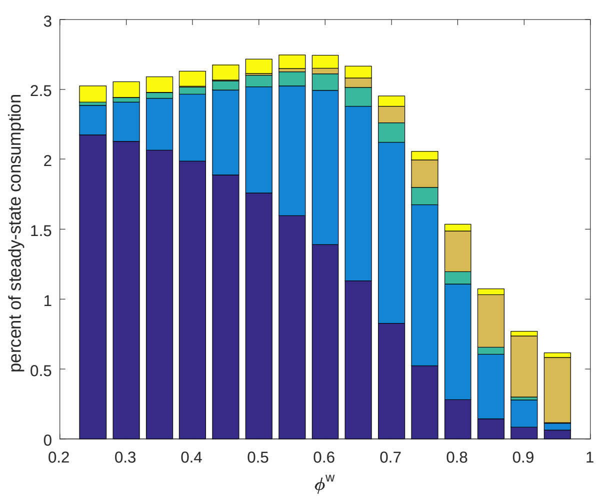

Figure 8: Welfare loss due to productivity shocks as function of wage stickiness under the Taylor rule. Bottom to top: price inflation (dark blue), standard wage inflation (light blue), idiosyncratic labor income risk (green), output gap (brown), consumption inequality between workers and capitalists (yellow).

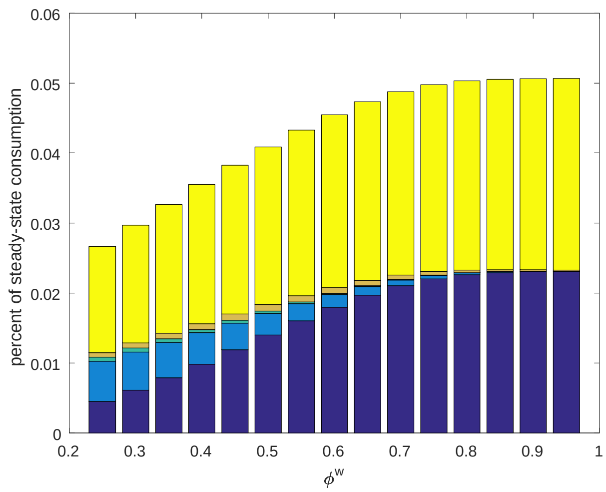

Figure 9: Welfare loss due to productivity shocks as function of wage stickiness under optimal discretion. Bottom to top: price inflation (dark blue), standard wage inflation (light blue), idiosyncratic labor income risk (green), output gap (brown), consumption inequality between workers and capitalists (yellow).

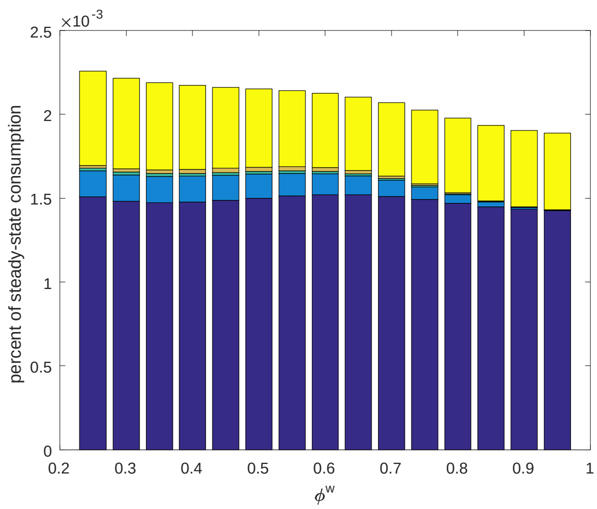

Figure 10: Welfare loss due to price markup shocks as function of wage stickiness under optimal discretion. Bottom to top: price inflation (dark blue), standard wage inflation (light blue), idiosyncratic labor income risk (green), output gap (brown), consumption inequality between workers and capitalists (yellow).

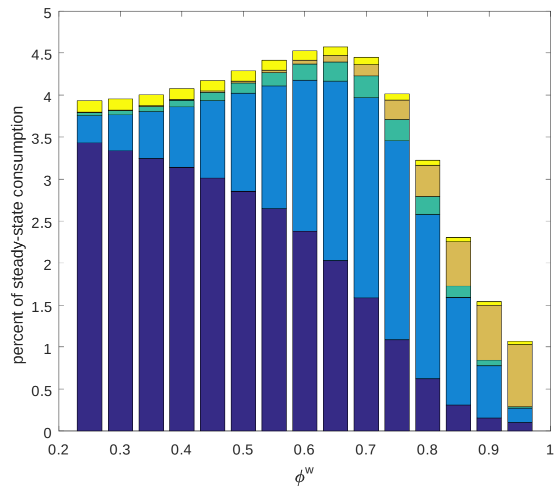

Figure 11: Loss under a Taylor rule, shock processes from Smets and Wouters (2007). Bottom to top: price inflation (dark blue), standard wage inflation (light blue), idiosyncratic labor income risk (green), output gap (brown), consumption inequality between workers and capitalists (yellow).

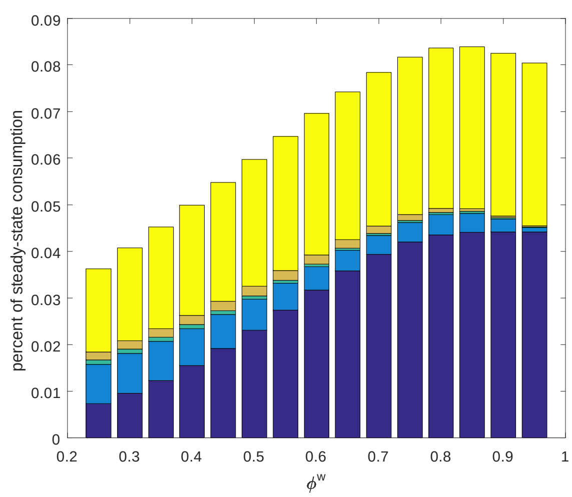

Figure 12: Loss under optimal time-consistent monetary policy, shock processes from Smets and Wouters (2007). Bottom to top: price inflation (dark blue), standard wage inflation (light blue), idiosyncratic labor income risk (green), output gap (brown), consumption inequality between workers and capitalists (yellow).

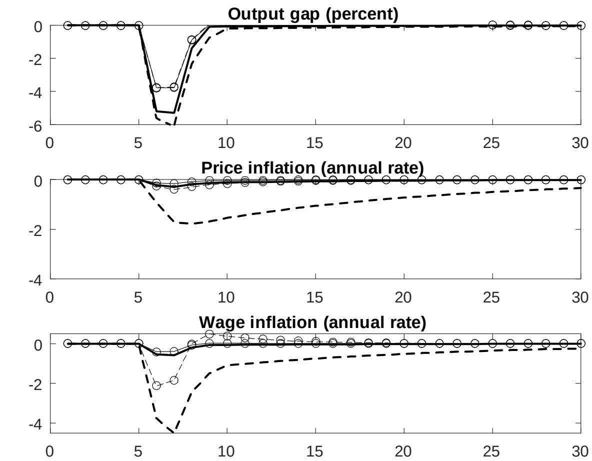

Figure 13: The effects at the ELB: Solid line: benchmark calibration ($\phi^w = 0.75$); dashed line: increase wage flexibility ($\phi^w = 0.5$); solid line with circles: EHL model ($\phi^w = 0.75$); dashed line with circles: EHL with increased wage flexibility ($\phi^w = 0.5$).

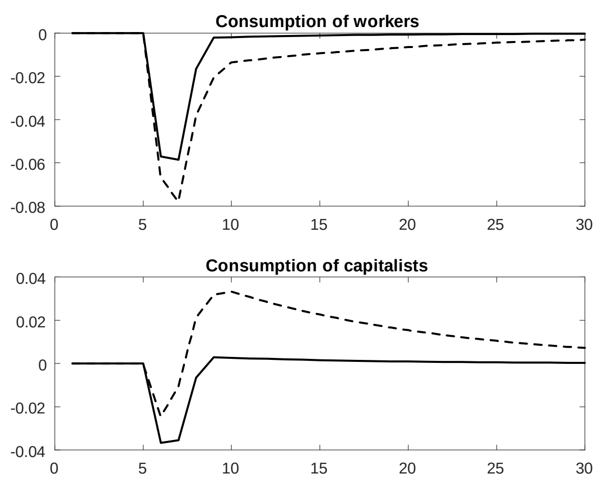

Figure 14: Consumption of workers and capitalists at the ELB: solid line: benchmark calibration ($\phi^w = 0.75$); dashed line: increase wage flexibility ($\phi^w = 0.5$).