Worker heterogeneity, selection, and employment dynamics in a pandemic: Appendix*

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This appendix provides details on the basic model, the competitive equilibrium, the social planner's allocation, efficiency, the model of temporary layoffs, alternative policy rules and alternative paramterizations.

1 The basic model

The model builds on that of Ravenna and Walsh (2012), and further details on the model, as well as an analysis of the role selection plays in the dynamic response to productivity shocks can be found there. In this section, details on labor flows, the household's decision problem, and the derivation of match surpluses are provided to supplement those provided in the main text.

1.1 Labor flows

The endogenous fraction of existing matches filled by low-productivity workers is ξ_t. Those that survived the exogenous separation hazard ρ^x_t receive a new productivity shock and are retained if and only if $a^l_{i,t} > \bar{a}^l_t$. This occurs with probability $1 - F(\bar{a}^l_t) = 1 - \rho^n_t$. All high-productivity workers surviving the exogenous separation hazard are retained. Thus, actual employment in period t is equal to

$$\begin{aligned} N_t &= (1-\rho^x_t)\left[\left(1-\xi_{t-1}\right)+\xi_{t-1}(1-\rho^n_t)\right]N_{t-1}+H_t \\ &= (1-\rho^x_t)\left(1-\xi_{t-1}\rho^n_t\right)N_{t-1}+H_t \end{aligned}$$

where H_t equals new hires. ξ_t evolves according to

$$\xi_t = (1-\rho^n_t)\left[\frac{(1-\rho^x_t)\xi_{t-1}N_{t-1}+\gamma_t k^w_t S_t}{N_t}\right], \tag{1}$$

where S_t is the number of job seekers, k^w_t is the fraction who are interviewed, and γ_t is the share of low-productivity workers among S_t. Thus, $\gamma_t k^w_t S_t$ type l workers are interviewed and the fraction $1 - \rho^n_t$ have productivity realizations that exceed $\bar{a}^l_t$ and so are hired.

Job seekers at t who are of quality l equal the total number of low-efficiency workers minus the number of matches of quality l that survive the exogenous separation hazard:

$$\gamma_t = \frac{L^l - (1-\rho_t^x)\xi_{t-1}N_{t-1}}{S_t}. \tag{2}$$

The efficiency-weighted average productivity of both employed workers and the pool of job seekers will change over time because γ_t is endogenous and persistent, even though $a_{i,t}^l$ is *i.i.d.*. During recessions, the outflow from employment rises and the inflow into employment falls, resulting in an increase in the average productivity among those still employed and a fall in the average efficiency level of those who are unemployed.

1.2 Households and retail firms

1.2.1 Households

The representative household maximizes

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left\{ D_t \frac{\mathcal{C}_{t+i}^{1-\sigma}}{1-\sigma} - \left[v(h_{t+i}^h)(1-\xi_{t+i})N_{t+i} + \xi_{t+i}N_{t+i} \int_{\bar{a}_t}^1 v(h_{i,t+i}^l) f(a) da \right] \right\}, \tag{3}$$

where $\sigma > 0$ is the coefficient of relative risk aversion, D_t is an aggregate preference shock, $\mathcal{C}_t$ is the sum of a market-purchased composite consumption good C_t and home-produced consumption by unemployed workers $C_t^u = (1-N_t)w^u$. In (3), the term

$$v(h_{t+i}^h)(1-\xi_{t+i})N_{t+i} + \xi_{t+i}N_{t+i} \int_{\bar{a}_t}^1 v(h_{i,t+i}^l) f(a^l) da^l$$

is the disutility to the household of having N_t members working, where hours worked depends on type and the idiosyncratic productivity shocks. We assume $v(h_{t+i}) = \ell h_{t+i}^{1+\chi}/(1+\chi)$.

Market consumption C_t is a Dixit-Stiglitz composite good consisting of the differentiated products produced by retail firms and is defined as

$$C_t = \left[\int_0^1 c_{k,t}^{\frac{\theta-1}{\theta}} dk \right]^{\frac{\theta}{\theta-1}} \qquad \theta > 0.$$

Given prices $p_{k,t}$ for the final goods, this preference specification implies the household's demand for good k is

$$c_{k,t} = \left(\frac{p_{k,t}}{P_t} \right)^{-\theta} C_t, \tag{4}$$

where the aggregate retail price index P_t is defined as

$$P_t = \left[\int_0^1 p_{k,t}^{1-\theta} dj \right]^{\frac{1}{1-\theta}}.$$

If i_t is the nominal rate of interest, the representative household's first order conditions imply the following must hold in equilibrium:

$$\lambda_t = \beta(1+i_t)\mathrm{E}_t\left(\frac{P_t}{P_{t+1}}\right)\lambda_{t+1}, \tag{5}$$

where $\lambda_t = D_t\mathcal{C}_t^{-\sigma}$.

1.2.2 Retail firms

There are a continuum of retail firms, indexed by j, who purchase the wholesale good and convert it into differentiated final goods that are sold to households and wholesale firms. Retail firms maximize profits subject to a CRS technology for converting wholesale goods into final goods, the demand functions (4), and a restriction on the frequency with which they can adjust their price. Each period a firm can adjust its price with probability $1-\omega$. For retail firms, real marginal cost is the price of the wholesale good relative to the price of final output, P_t^w/P_t.

A retail firm k that can adjust its price in period t chooses $p_t(k)$ to maximize

$$\sum_{s=0}^{\infty}(\omega\beta)^s\,\mathrm{E}_t\left[\left(\frac{\lambda_{t+s}}{\lambda_t}\right)\left(\frac{p_t(k)-P_{t+s}^w}{P_{t+s}}\right)Y_{t+s}(k)\right],$$

subject to

$$Y_{t+s}(k) = Y_{t+s}^d(k) = \left(\frac{p_t(k)}{P_{t+s}}\right)^{-\theta}Y_{t+s}^d, \tag{6}$$

where Y_t^d is aggregate demand for the basket of final goods. The first order condition for those firms adjusting their price in period t is

$$p_t(k)\mathrm{E}_t\sum_{s=0}^{\infty}(\omega\beta)^s\left(\frac{\lambda_{t+s}}{\lambda_t}\right)\left(\frac{p_t(k)}{P_{t+s}}\right)^{1-\theta}Y_{t+s} = \left(\frac{\theta}{\theta-1}\right)\mathrm{E}_t\sum_{s=0}^{\infty}(\omega\beta)^s\left(\frac{\lambda_{t+s}}{\lambda_t}\right)\left(\frac{1}{\mu_{t+s}}\right)\left(\frac{p_t(k)}{P_{t+s}}\right)^{1-\theta}Y_{t+s}.$$

All adjusting firms choose the same reset price p_t^*, and the aggregate price level is then

$$P_t^{1-\theta} = (1-\omega)\left(p_t^*\right)^{1-\theta} + \omega P_{t-1}^{1-\theta}.$$

When linearized around a zero-inflation steady state, these conditions yield a new Keynesian Phillips curve in which the retail price markup

$$\mu_t \equiv \frac{P_t}{P_t^w}$$

is the driving force for inflation; a rise in the markup implies a fall in real marginal costs for retail firms. As in a standard Phillips curve, the elasticity of inflation with respect to real marginal costs will be $\delta \equiv (1-\omega)(1-\beta\omega)/\omega$.

1.3 Match surplus in the competitive equilibrium

We first discuss the derivation of the surplus generated by a producing match of type h and then indicate the modifications needed when considering a type l worker.

1.3.1 Type h workers

A type h worker in a match produces $\phi^h h_t^h$, where ϕ^h is the worker's fixed hourly productivity and h_t^h is hours worked. For such a worker who produces in period t, the surplus is the value of output expressed in terms of retail goods prices $\phi^h h_t^h/\mu_t$ plus the continuation value q_t^h value of the match net of the utility cost of hours $v(h_t^h)/\lambda_t$ and the worker's outside opportunity $w_t^{u,h}$:

$$s_t^h = \left(\frac{\phi^h h_t^h}{\mu_t}\right) + q_t^h - \frac{v(h_t^h)}{\lambda_t} - w_t^{u,h}, \tag{7}$$

where h_t^h is chosen to maximize

$$\left(\frac{\phi^h h_t^h}{\mu_t}\right) - \frac{v(h_t^h)}{\lambda_t}.$$

Equation (7) highlights that the surplus s_t^h is the value in excess of the outside opportunity to the workers and firm. For the firm, the job posting conditions implies the value of a vacancy is zero in equilibrium and so it is the worker's outside opportunity of searching $w_t^{u,h}$ that is relevant. The continuation value q_t^h equals the probability the match survives the exogenous separation hazard times the expected future value of a match plus the probability the match does not survive times the expected future outside opportunity cost:

$$\begin{aligned} q_t^h &= \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \left[(1-\rho_{t+1}^x)\left(s_{t+1}^h + w_{t+1}^{u,h}\right) + \rho_{t+1}^x w_{t+1}^{u,h}\right] \\ &= \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \left[(1-\rho_{t+1}^x) s_{t+1}^h + w_{t+1}^{u,h}\right]. \end{aligned} \tag{8}$$

The worker's outside opportunity is

$$w_t^{u,h} = w^u + \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \left[(1-\rho_{t+1}^x) k_{t+1}^w \eta s_{t+1}^h + w_{t+1}^{u,h}\right], \tag{9}$$

where η is the worker's share of the surplus. Subtracting (9) from (8) yields

$$q_t^h - w_t^{u,h} = -w^u + \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) (1-\rho_{t+1}^x)\left(1-\eta k_{t+1}^w\right) s_{t+1}^h.$$

Using this in (7),

$$s_t^h = \left(\frac{\phi^h h_t^h}{\mu_t}\right) - \frac{v(h_t^h)}{\lambda_t} - w^u + \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) (1-\rho_{t+1}^x)\left(1-\eta k_{t+1}^w\right) s_{t+1}^h. \tag{10}$$

It may be useful to provide an alternative derivation of s_t^h, one that serves to make the timing assumptions of the model more transparent and highlights the separate match valuations of the worker and the firm.

N_t^h workers of type h are employed and producing in period t and $1 - N_t$ are unmatched. The value of being employed is denoted by e_t^h (for employed), and the valuation equation takes the form

$$e_t^h = w_t^h(h_t^h) - \frac{v(h_t^h)}{\lambda_t} + \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \left[\left(1 - \rho_{t+1}^x\right) e_{t+1}^h + \rho_{t+1}^x k_{t+1}^w e_{t+1}^h + \rho_{t+1}^x \left(1 - k_{t+1}^w\right) u_{t+1}^h\right],$$

where $w_t^h(h_t^h)$ is the wage of a type h worker as a function of hours worked, while $v(h_t^h)/\lambda_t$ is the disutility of supplying h_t^h hours, expressed in terms of retail goods. With probability $1 - \rho_t^x$ the worker survives the exogenous separation process and produces in period $t + 1$. The term $\rho_{t+1}^x k_{t+1}^h e_{t+1}^h$ reflects the assumption that workers exogenously separated can search. With probability ρ_{t+1}^x, a worker separates, enters the labor market and, with probability k_{t+1}^w obtains an interview and is hired (as no type h workers are screened out). With probability $1 - k_{t+1}^w$, the exogenously separated workers do not make a match.

Denote by u_t^h the value of being unemployed. The valuation equation for u_t^h, is

$$u_t^h = w^u + \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \left[k_{t+1}^w e_{t+1}^h + \left(1 - k_{t+1}^w\right) u_{t+1}^h\right],$$

where w^h is any unemployment benefit or value of home production. The unemployed worker finds a match with probability k_{t+1}^w and receives e_{t+1}^h, and with probability $1 - k_{t+1}^w$ the worker remains unmatched.

The surplus to a type h worker of being in a match is

$$\begin{aligned} e_t^h - u_t^h &= \left[w_t^h(h_t^h) - w^h - \frac{v(h_t^h)}{\lambda_t} + \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \left(1 - \rho_{t+1}^x + \rho_{t+1}^x k_{t+1}^w\right) e_{t+1}^h + \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \rho_{t+1}^x \left(1 - k_{t+1}^w\right) u_{t+1}^h \right. \\ &\quad - \left[\beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) k_{t+1}^w e_{t+1}^h + \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \left(1 - k_{t+1}^w\right) u_{t+1}^h\right] \\ &= w_t^h(h_t^h) - \frac{v(h_t^h)}{\lambda_t} - w^h + \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \left(1 - \rho_{t+1}^x\right) \left(1 - k_{t+1}^w\right) \left(e_{t+1}^h - u_{t+1}^h\right). \end{aligned}$$

From the job-posting condition, the value of a vacancy is driven to zero, so the value J_t^h to a firm of a filled job with a type h worker is

$$J_t^h = \left(\frac{\phi^h h_t^h}{\mu_t}\right) - w_t^h(h_t^h) + \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \left(1 - \rho_{t+1}^x\right) J_{t+1}^h.$$

The joint surplus of a match is therefore

$$\begin{aligned} s_t^h &= J_t^h + e_t^h - w_t^u = \left(\frac{\phi^h h_t^h}{\mu_t}\right) - \frac{v(h_t^h)}{\lambda_t} - w^h + \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \left(1 - \rho_{t+1}^x\right) J_{t+1}^h \\ &\quad + \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \left(1 - \rho_{t+1}^x\right) \left(1 - k_{t+1}^w\right) \left(e_{t+1}^h - u_{t+1}^h\right), \end{aligned}$$

where, again, h_t^h is chosen to maximize output net of the disutility of supplying hours. With Nash bargaining, the worker's share is equal $e_{t+1}^h - u_{t+1}^h = \eta s_{t+1}^h$, and the firm's share is $J_{t+1}^h = (1-\eta)\, s_{t+1}^h$. Hence,

$$\begin{aligned} s_t^h &= \left(\frac{\phi^h h_t^h}{\mu_t}\right) - \frac{v(h_t^h)}{\lambda_t} - w^h + \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \left(1 - \rho_{t+1}^x\right) (1-\eta)\, s_{t+1}^h \\ &\quad + \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \left(1 - \rho_{t+1}^x\right) \left(1 - k_{t+1}^w\right) \eta s_{t+1}^h, \end{aligned}$$

or

$$s_t^h = \left(\frac{\phi^h h_t^h}{\mu_t}\right) - \frac{v(h_t^h)}{\lambda_t} - w^h + \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \left(1 - \rho_{t+1}^x\right) \left(1 - \eta k_{t+1}^w\right) s_{t+1}^h,$$

which is the same as (10).

1.3.2 Type l workers

For type l workers, the endogenous separation hazard and the stochastic productivity shock must be taken into account. Because workers differ, it is now also necessary to start with the joint surplus of a match with worker i of type l with current productivity $a_{i,t}^l \phi^l$:

$$s_{i,t}^l = \left(\frac{a_{i,t}^l \phi^l h_{i,t}^l}{\mu_t}\right) - \frac{v(h_{i,t}^l)}{\lambda_t} + q_t^l - w_{i,t}^{u,l},$$

where h_t^l will vary with $a_{i,t}^l$ and is chosen to maximize

$$\left(\frac{a_{i,t}^l \phi^l h_{i,t}^l}{\mu_t}\right) - \frac{v(h_{i,t}^l)}{\lambda_t}.$$

In place of (8) and (9), one now has

$$\begin{aligned} q_t^l &= \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \left[\left(1 - \rho_{t+1}^x\right) \left(1 - \rho_{t+1}^n\right) \left(s_{t+1}^i + w_{t+1}^{u,l}\right) + \left[\rho_{t+1}^x + \left(1 - \rho_{t+1}^x\right) \rho_{t+1}^n\right] w_{t+1}^{u,l}\right] \\ &= \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \left[\left(1 - \rho_{t+1}^x\right) \left(1 - \rho_{t+1}^n\right) s_{t+1}^l + w_{t+1}^{u,l}\right]. \end{aligned}$$

where $\rho_{t+1}^{n,l} = F(\bar{a}_t^l)$ is the probability the worker's productivity realization falls below the threshold level $\bar{a}_t^l$, where $F(.)$ is the cumulative distribution of the idiosyncratic productivity shock. Notice that the expectation incorporates the probability of surviving the exogenous separation but then receiving a low productivity draw and experiencing endogenous separation. This occurs with probability $\left(1-\rho_{t+1}^x\right)\rho_{t+1}^n$. Because idiosyncratic productivity shocks are i.i.d., the expected future value of s_{t+1}^l is independent of i. Similarly, the worker's outside opportunity is

$$w_t^{u,l} = w^u + \beta \mathrm{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left[\left(1-\rho_{t+1}^x\right)\left(1-\rho_{t+1}^n\right)k_{t+1}^w \eta s_{t+1}^l + w_{t+1}^{u,l}\right],$$

where the assumption of Nash bargaining as been used. Combining these equations for $s_{i,t}^l$, q_t^l and $w_t^{u,l}$ yields

$$s_{i,t}^l = \left(\frac{a_{i,t}^l \phi^l h_{i,t}^l}{\mu_t}\right) - \frac{v(h_{i,t}^l)}{\lambda_t} - w^u + \beta \mathrm{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(1-\rho_{t+1}^x\right)\left(1-\rho_{t+1}^n\right)\left(1-\eta k_{t+1}^w\right)s_{t+1}^l. \quad (11)$$

Equation (11) can also be derived by using the valuations $e_{i,t}^l$, u_{t+1}^l and $J_{i,t+1}^h$ as previously done for a type h worker.

1.3.3 Optimal hours

Since hours will be chosen to maximize the surplus of a match, $v'(h_t^h)/\lambda_t = \phi^h/\mu_t$ for a type h worker, which, using the functional form for the disutility of leisure implies

$$\frac{v'(h_t^h)}{\lambda_t} = \frac{\ell\left(h_t^h\right)^\chi}{\lambda_t} = \frac{\phi^h}{\mu_t} \Rightarrow h_t = \left(\frac{\lambda_t \phi^h}{\ell \mu_t}\right)^{\frac{1}{\chi}}. \quad (12)$$

Hence,

$$\frac{v(h_t)}{\lambda_t} = \frac{1}{\lambda_t}\frac{\ell\left(h_t^h\right)^{1+\chi}}{1+\chi} = \frac{1}{\lambda_t}\frac{\ell\left(h_t^h\right)^\chi}{1+\chi}h_t^h = \frac{1}{1+\chi}\frac{\phi^h h_t^h}{\mu_t},$$

and

$$\frac{\phi^h h_t^h}{\mu_t} - \frac{v(h_t^h)}{\lambda_t} = \frac{\phi^h h_t^h}{\mu_t} - \frac{1}{1+\chi}\frac{\phi^h h_t^h}{\mu_t} = \left(\frac{\chi}{1+\chi}\right)\frac{\phi^h h_t^h}{\mu_t},$$

or,

$$\frac{\phi^h h_t^h}{\mu_t} - \frac{v(h_t^h)}{\lambda_t} = \left(\frac{\chi}{1+\chi}\right)\frac{\phi^h h_t^h}{\mu_t} = \left(\frac{\chi}{1+\chi}\right)\left(\frac{\lambda_t}{\ell}\right)^{\frac{1}{\chi}}\left(\frac{\phi^h}{\mu_t}\right)^{\frac{1+\chi}{\chi}} \quad (13)$$

For type l workers,

$$\frac{a_{i,t}^l \phi^l h_{i,t}^l}{\mu_t} - \frac{v(h_{i,t}^l)}{\lambda_t} = \left(\frac{\chi}{1+\chi}\right)\left(\frac{\lambda_t}{\ell}\right)^{\frac{1}{\chi}}\left(\frac{a_{i,t}^l \phi^l}{\mu_t}\right)^{\frac{1+\chi}{\chi}}. \quad (14)$$

1.3.4 Cutoff productivity

The match with a type l worker ends, of an interview with a type l worker fails to result in a match if the joint surplus is non-positive. This occurs for a worker with idiosyncratic productivity $a_{i,t}^l$ if $s_{i,t}^l \leq 0$, or when $a_{i,t}^l \leq \bar{a}_t^l$, where

$$\bar{a}_t^l = \left(\frac{\mu_t}{\phi^l}\right) \left[\left(\frac{1+\chi}{\chi}\right) \left(\frac{\ell}{\lambda_t}\right)^{\frac{1}{\chi}} \left(w_t^{u,l} - q_t^l\right)\right]^{\frac{\chi}{1+\chi}}.$$

The RHS does not depend on the firms, implying all firms employ the same cutoff productivity level for hiring and retention decision.

1.4 Complete set of equilibrium conditions: Market equilibrium with permanent layoffs

We list the conditions for the market equilibrium of the model with permanent separation. The main text presents the equilibrium conditions to be changed to introduce temporary layoffs. The next section provides additional details on the derivation of the equilibrium with temporary layoffs.

1.
$$\gamma_t = \frac{S_t^l}{S_t},$$

2.
$$\xi_t = \frac{N_t^l}{N_t}.$$

3.
$$S_t^h = L^h - (1 - \rho_t^x) N_{t-1}^h$$

4.
$$S_t = S_t^h + S_t^l$$

5.
$$N_t = N_t^l + N_t^h$$

6.
$$S_t = 1 - (1 - \rho_t^x) N_{t-1}$$

7.
$$\xi_t = (1 - \rho_t^n) \left[\frac{\xi_{t-1}(1 - \rho_t^x) N_{t-1} + \gamma_t k_t^w S_t}{N_t}\right].$$

8.
$$H_t = (1 - \gamma_t \rho_t^n) k_t^w S_t.$$

9.
$$N_t = \left(1 - \xi_{t-1}\rho_t^n\right)(1 - \rho_t^x) N_{t-1} + H_t$$

10.
$$\rho_t = \rho_t^x + (1 - \rho_t^x)\xi_{t-1}\rho_t^n.$$

11.
$$U_t^h = 1 - \frac{N_t^h}{L^h}$$

12.
$$U_t^l = 1 - \frac{N_t^l}{L^l}$$

13.
$$U_t = 1 - N_t.$$

14.
$$\theta_t = \frac{V_t}{S_t}$$

15.
$$k_t^w = \psi\theta_t^{1-\alpha}$$

16.
$$k_t^f = \psi\theta_t^{-\alpha}$$

17. Assuming pdf for $a_{i,t}$ is $U[0,1]$:
$$\rho_t^n = F(\bar{a}_t) = \bar{a}_t,$$

18.
$$Q_t = Y_t f_t$$

19. Assuming $v(h^l) = \frac{\ell}{1+\chi} h_t^{l^{1+\chi}}$:
$$\begin{aligned} Q_t &= N_t^l E_t(a_{i,t} h_{i,t}^l | a_{i,t} > \overline{a}_t) + h_t^h N_t^h = \\ &= N_t^l \frac{\int_{\bar{a}_t}^1 a_{i,t} h_{i,t}^l dF(a_i)}{1 - F(\overline{a}_t)} + h_t^h N_t^h = \end{aligned}$$

20.
$$\mathcal{C}_t = C_t + (L^l - N_t^l)w^l + (L^h - N_t^h)w^h$$

21.
$$Y_t = C_t + \kappa V_t$$

22.
$$\lambda_t = \beta(1+i_t)\mathrm{E}_t\left(\frac{1}{\Pi_{t+1}}\right)\lambda_{t+1}$$

23.
$$\lambda_t \equiv D_t\left(\mathcal{C}_t\right)^{-\sigma}$$

24.
$$\left(\frac{\bar{a}_t}{\mu_t}\right)\lambda_t = v_h(\hat{h}_t^l) = \ell \widehat{h}_t^{l\chi}$$

25.
$$\bar{a}_t = \frac{\mu_t\left(w_t^{u,l} + \frac{v(\hat{h}_t^l)}{\lambda_t} - q_t^l\right)}{z_t \hat{h}_t^l}$$

26. Define $\mathcal{X}_t = E_t(s_{i,t}^l | a_{i,t} \geq \overline{a}_t)$

$$\begin{aligned}
\mathcal{X}_t \quad &= \quad \left(\frac{\lambda_t z_t}{\mu_t \ell}\right)^{1/\chi} \frac{z_t}{\mu_t} \frac{(1-\overline{a}_t^{1+1+1/\chi})}{(1-\overline{a}_t)} \frac{1}{1+1+1/\chi} \\
&\quad -\frac{1}{\lambda_t}\frac{\ell}{1+\chi}\left(\frac{\lambda_t z_t}{\mu_t \ell}\right)^{(1+\chi)/\chi} \frac{(1-\overline{a}_t^{1+(1+\chi)/\chi})}{(1-\overline{a}_t)} \frac{1}{1+(1+\chi)/\chi} - w_t^{u,l} + q_t^l
\end{aligned}$$

27.
$$\begin{aligned}
q_t^l \quad &= \quad \beta \mathrm{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left[(1-\rho_{t+1}^x)(1-\rho_{t+1}^n)E_t(s_{i,t+1}^l | a_{i,t} > \overline{a}_{i,t}) + w_{t+1}^{u,l}\right] \\
&= \quad \beta \mathrm{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left[(1-\rho_{t+1}^x)(1-\rho_{t+1}^n)\int_{\bar{a}_{t+1}}^1 s_{i,t+1}^l\left[\frac{f(a_i)}{1-\overline{a}_{t+1}}\right]da_i + w_{t+1}^{u,l}\right]
\end{aligned}$$

28.
$$\begin{aligned}
w_t^{u,l} \quad &= \quad w^l + \beta \mathrm{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left\{(1-\rho_{t+1}^x)k_{t+1}^w \eta (1-\rho_{t+1}^n)E_t(s_{i,t+1}^l | a_{i,t} > \overline{a}_{i,t}) + w_{t+1}^{u,l}\right\} \\
&\quad\quad w^l + \beta \mathrm{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left\{(1-\rho_{t+1}^x)k_{t+1}^w \eta (1-\rho_{t+1}^n)\int_{\bar{a}_{t+1}}^1 s_{i,t+1}^l\left[\frac{f(a_i)}{1-\overline{a}_{t+1}}\right]da_i + w_{t+1}^{u,l}\right\}
\end{aligned}$$

29.
$$\left(\frac{1}{\mu_t}\right)\lambda_t = v_h(h_t^h) = \ell h_t^{h\chi}$$

30.
$$s_t^h = \left(\frac{h_t^h}{\mu_t}\right) - \frac{v(h_t^h)}{\lambda_t} - w_t^{u,h} + q_t^h$$

31.
$$q_t^h = \beta \mathrm{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left[(1-\rho_{t+1}^x)s_{t+1}^h + w_{t+1}^{u,h}\right]$$

32.

$$w_t^{u,h} = w^h + \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \left\{(1-\rho_{t+1}^x) k_{t+1}^w \eta s_{t+1}^h + w_{t+1}^{u,h}\right\}$$

33.

$$\begin{aligned}
\kappa &= k_t^f (1-\eta)\left[\gamma_t \int_{\bar{a}_t}^1 s_{i,t}^l f(a_i) da_i + (1-\gamma_t) s_t^h\right] \\
&= k_t^f (1-\eta)\left[\gamma_t (1-\rho_t^n) \int_{\bar{a}_t}^1 s_{i,t}^l \left[\frac{f(a_i)}{1-\rho_t^n}\right] da_i + (1-\gamma_t) s_t^h\right]
\end{aligned}$$

34.

$$[(1+\pi_t)]^{1-\theta} = \theta_p + (1-\theta_p)\left[\frac{\tilde{G}_t}{\tilde{H}_t}(1+\pi_t)\right]^{1-\theta}.$$

35.

$$\tilde{G}_t = \mu \lambda_t \mu_t^{-1} Y_t + \theta_p \beta \tilde{G}_{t+1} (1+\pi_{t+1})^{\theta}$$

36.

$$\tilde{H}_t = \lambda_t Y_t + \theta_p \beta \tilde{H}_{t+1} (1+\pi_{t+1})^{\theta-1}$$

37.

$$\begin{aligned}
f_t &= (1-\theta_p)\left(\frac{\tilde{G}_t}{\tilde{H}_t}\right)^{-\theta} + \theta_p (1+\pi_t)^{\theta} f_{t-1}. \\
f_t &\equiv \int_0^1 \left[\frac{P_t(z)}{P_t}\right]^{-\theta} dz
\end{aligned}$$

38. monetary policy

$$\frac{\left(1+\overline{R}n_{t,t+1}\right)}{(1+Rn^{ss})} = \left(\frac{1+\pi_t}{1+\pi_{SS}}\right)^{\omega_\pi}.$$

39. Probability l-worker enters into productive match

$$\begin{aligned}
k_t^{w,l} &= k_t^w \Pr(s_{i,t}^l > 0) \\
&= k_t^w (1-\bar{a}_t)
\end{aligned}$$

40. Unconditional probability of entering into productive match

$$k_t^{job,w} = \gamma_t k_t^w (1-\bar{a}_t) + (1-\gamma_t) k_t^w$$

41. Unconditional probability of filling a vacancy

$$k_t^{job,f} = \gamma_t k_t^f (1-\bar{a}_t) + (1-\gamma_t) k_t^f$$

2 The social planner's problem

The appendix presents the social planner's problem for the basic model with idiosyncratic productivity shocks only to type l workers and then extends this problem to the case in which both worker types experience idiosyncratic productivity shocks.

The planner's problem is to maximize

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \delta_t^i \left[D_t \frac{\mathcal{C}_{t+i}^{1-\sigma}}{1-\sigma} - (1-\xi_{t+i}) N_{t+i} v(h_{t+i}^h) - \xi_{t+i} N_{t+i} \int_{\bar{a}_t^l}^1 v(h_{i,t+i}^l) f(a) da \right]$$

subject to

1. Labor market flows:
$$N_t = (1-\rho_t^x)\left(1-\xi_{t-1}\rho_t^n\right) N_{t-1} + H_t,$$
where
$$\xi_t = (1-\rho_t^n)\left[\frac{(1-\rho_t^x)\xi_{t-1}N_{t-1} + \gamma_t k_t^w S_t}{N_t}\right],$$
$$S_t = 1 - (1-\rho_t^x) N_{t-1},$$
$$I_t = \psi V_t^{1-a} S_t^a, \qquad 0 < \alpha < 1,\ \psi > 0,$$
$$H_t = (1-\gamma_t) k_t^w S_t + \gamma_t (1-\rho_t^n) k_t^w S_t = (1-\gamma_t \rho_t^n) k_t^w S_t,$$
$$\gamma_t \equiv \frac{S_t^l}{S_t} = \frac{L^l - (1-\rho_t^x)\xi_{t-1}N_{t-1}}{S_t}.$$

2. Production technology:
$$Y_t = \left\{ (1-\xi_t)\phi_t^h h_t^h + \xi_t \phi^l \left[\frac{\int_{\bar{a}_t^l}^1 a_{i,t}^l h_{i,t}^l dF_l(a_i)}{1-F_l(\bar{a}_t^l)} \right] \right\} N_t.$$

3. Goods clearing:
$$\phi^h h_t^h N_t^h + \phi^l \int_{\bar{a}_t^l}^1 a_{i,t}^l h_{i,t}^l di + w^u \left(L - N_t^h - N_t^l\right) - \kappa V_t - \mathcal{C}_t L \geq 0$$
$$\mathcal{C}_t = C_t + w^u \left(L - N_t^h - N_t^l\right)$$
$$Y_t = C_t + \kappa V_t,$$

In deriving the solution, let A_t^j be the set of employed type j workers and let $\#N_t^j$ be their number. In addition, let $\bar{a}_t^{sl}$ denote the threshold productivity level for type l workers in the social planners solution.

The social planner's problem is described by

$$W_t(N^h_{t-1}, N^l_{t-1}) = \max_{\mathcal{C}_t, C_t, V_t, I_t, \bar{a}^l_t, N^h_t, \# N^l_t, h^h_t, h^l_{i,t}} \left[\begin{array}{c} U(\mathcal{C}_t)L - \int_{i\in A^h_t} v\left(h^h_{i,t}\right) di \\ - \int_{i\in A^l_t} v\left(h^l_t\right) di + \beta \mathrm{E}_t W_{t+1}(N^h_t, N^l_t) \end{array} \right]$$

subject to

$$\phi^h \int_{i\in A^h_t} h^h_t di + \phi^l \int_{i\in A^l_t} a^l_{i,t} h^l_{i,t} di - \kappa V_t - C_t L \geq 0$$

$$C_t L + w^u \left(L - N^h_t - N^l_t\right) - \mathcal{C}_t L \geq 0$$

$$\left[(1-\rho^x_t) N^h_{t-1} + (1-\gamma_t) I_t\right] - N^h_t = 0$$

$$\left[1 - F(\bar{a}^{sl}_t)\right] \left[(1-\rho^x_t) N^l_{t-1} + \gamma_t I_t\right] - N^l_t = 0$$

$$\psi V_t^{1-\alpha} \left[1 - (1-\rho^x_t)\left(N^h_{t-1} + N^l_{t-1}\right)\right]^\alpha - I_t = 0$$

where

$$\gamma_t \equiv \frac{L^l - (1-\rho^x_t) N^l_{t-1}}{\left[1 - (1-\rho^x_t)\left(N^h_{t-1} + N^l_{t-1}\right)\right]} = \frac{S^l_t}{S^h_t + S^l_t}.$$

Define $\lambda_{1,t}$, $\lambda_{2,t}$ τ^h_t, $\tau^l_{i,t}$, and ζ_t as the Lagrangian multipliers on these five constraints. The first-order conditions take the form:

$$\mathcal{C}_t\colon\ U'(\mathcal{C}_t)L - \lambda_{2,t} L = 0$$

$$C_t\colon\ -\lambda_{1,t} L + \lambda_{2,t} L = 0 \Rightarrow \lambda_{1,t} = \lambda_{2,t} = U'(\mathcal{C}_t)$$

$$V_t\colon\ -\kappa \lambda_{1,t} + \psi (1-\alpha) \left(\frac{I_t}{V_t}\right) \zeta_t = 0$$

$$I_t\colon\ (1-\gamma_t) \tau^h_t + \gamma_t \left[1 - F(\bar{a}^{sl}_t)\right] \tau^l_t - \zeta_t = 0$$

$$h^h_t\colon\ -v'(h^h_t) + \lambda_{1,t} \phi^h = 0 \Rightarrow \frac{v'(h^h_t)}{U'(\mathcal{C}_t)} = \phi^h$$

$$h^l_{i,t}, \text{ for } a^l_{i,t} \geq \bar{a}^{sl}_t\colon\ -v'\left(h^l_{i,t}\right) + \lambda_{1,t} \phi^l a^l_{i,t} = 0 \Rightarrow \frac{v'(h^l_{i,t})}{U'(\mathcal{C}_t)} = a^l_{i,t} \phi^l$$

$$N^h_t\colon \text{ of type } h\colon - v(h^h_t) + \lambda_{1,t} \phi^h h^h_t - \lambda_{2,t} w^u - \tau^h_t + \beta \mathrm{E}_t \frac{\partial W_{t+1}(N^h_t, N^l_t)}{\partial N^h_t} = 0$$

$$N^l_t \text{ of type } l\colon\ - v(h^l_{i,t}) + \lambda_{1,t} a^l_{i,t} \phi^l h^l_{i,t} - \lambda_{2,t} w^u - \tau^l_{i,t} + \beta \mathrm{E}_t \frac{\partial W_{t+1}(N^h_t, N^l_t)}{\partial N^l_t} = 0$$

The first order conditions for V and I imply

$$\kappa\lambda_{1,t} = \left\{(1-\gamma_t)\left[1-F(\bar{a}_t^h)\right]\tau_t^h + \gamma_t\left[1-F(\bar{a}_t)\right]\tau_t^l\right\}\psi\,(1-\alpha)\left(\frac{I_t}{V_t}\right)$$

while the ones for N^h and N^l imply

$$\tau_t^h = U'(\mathcal{C}_t)\left[\phi^h h_t^h - \frac{v(h_t^h)}{U'(\mathcal{C}_t)} - w^u\right] + \beta \mathrm{E}_t \frac{\partial W_{t+1}(N_t^h, N_t^l)}{\partial N_t^h}$$

$$\tau_{i,t}^l = U'(\mathcal{C}_t)\left[a_{i,t}^l \phi^l h_{i,t}^l - \frac{v(h_{i,t}^l)}{U'(\mathcal{C}_t)} - w^u\right] + \beta \mathrm{E}_t \frac{\partial W_{t+1}(N_t^h, N_t^l)}{\partial N_t^l}$$

Integrating this last condition over i yields

$$\tau_t^l \equiv \int_{i\in A_t^l} \tau_{i,t}^l di = U'(\mathcal{C}_t)\left\{\phi^l \int_{i\in A_t^l}\left[a_{i,t}^l h_{i,t}^l - v(h_{i,t}^l)\right]di - w^u\right\} + \beta \mathrm{E}_t \frac{\partial W_{t+1}(N_t^h, N_t^l)}{\partial N_t^l}$$

To evaluate terms of the form $\beta \mathrm{E}_t\left[\partial W_{t+1}/\partial N_t^j\right]$, use the envelope conditions,

$$\begin{aligned}\frac{\partial W_t(N_{t-1}^h, N_{t-1}^l)}{\partial N_{t-1}^h} &= \tau_t^h\left\{(1-\rho_t^x) - I_t\frac{\partial\gamma_t}{\partial N_{t-1}^h}\right\}\\ &\quad +\tau_t^l\left[1-F(\bar{a}_t^l)\right]\left(I_t\frac{\partial\gamma_t}{\partial N_{t-1}^h}\right) - \zeta_t\,(1-\rho_t^x)\,\alpha\left(\frac{I_t}{S_t}\right)\end{aligned}$$

and

$$\begin{aligned}\frac{\partial W_t(N_{t-1}^h, N_{t-1}^l)}{\partial N_{t-1}^l} &= \tau_t^h\left(-I_t\frac{\partial\gamma_t}{\partial N_{t-1}^l}\right)\\ &\quad +\tau_t^l\left[1-F(\bar{a}_t^l)\right]\left\{(1-\rho_t^x) + I_t\frac{\partial\gamma_t}{\partial N_{t-1}^l}\right\}\\ &\quad -\zeta_t\,(1-\rho_t^x)\,\alpha\left(\frac{I_t}{S_t}\right),\end{aligned}$$

together with

$$\frac{\partial\gamma_t}{\partial N_{t-1}^h} = \frac{(1-\rho_t^x)\left[L^l-(1-\rho_t^x)N_{t-1}^l\right]}{S_t^2} = (1-\rho_t^x)\,\frac{\gamma_t}{S_t}$$

$$\frac{\partial\gamma_t}{\partial N_{t-1}^l} = \frac{-(1-\rho_t^x)\,S_t + (1-\rho_t^x)\left[L^l-(1-\rho_t^x)N_{t-1}^l\right]}{S_t^2} = -(1-\rho_t^x)\,\frac{1-\gamma_t}{S_t} \leq 0.$$

Noting that the first-order condition for I_t can be written as

$$\zeta_t = (1-\gamma_t)\,\tau_t^h + \gamma_t\left[1-F(\bar{a}_t^{sl})\right]\tau_t^l = \tau_t^h - \gamma_t x_t,$$

where $x_t \equiv (1-\rho_t^x)\left\{\tau_t^h - \left[1-F(\bar{a}_t^{sl})\right]\tau_t^l\right\}$, one then obtains, after noting that $k_t^w = I_t/S_t$, that[1]

$$\frac{\partial W_t(N_{t-1}^h, N_{t-1}^l)}{\partial N_{t-1}^h} = \tau_t^h\left(1-\alpha k_t^w\right) - (1-\alpha)\gamma_t k_t^w x_t.$$

Similarly,[2]

$$\frac{\partial W_t(N_{t-1}^h, N_{t-1}^l)}{\partial N_{t-1}^l} = \tau_t^l\left[1-F(\bar{a}_t^l)\right]\left(1-\alpha k_t^w\right) + (1-\alpha)(1-\gamma_t)k_t^w x_t$$

Therefore

$$\mathrm{E}_t\frac{\partial W_{t+1}(N_t^h, N_t^l)}{\partial N_t^h} = \mathrm{E}_t\left\{\begin{array}{c}\left(1-\rho_{t+1}^x\right)\tau_{t+1}^h\left(1-\alpha k_{t+1}^w\right) \\ -(1-\alpha)\gamma_{t+1}k_{t+1}^w x_{t+1}\end{array}\right\}$$

[1] This uses the following steps:

$$\begin{aligned}
\frac{\partial W_t(N_{t-1}^h, N_{t-1}^l)}{\partial N_{t-1}^h} &= \mu_t^h(1-\rho^x)\left\{1-\gamma_t k_t^w\right\} \\
&\quad +\mu_t^l(1-\rho^x)\left[1-F(\bar{a}_t^l)\right](\gamma_t k_t^w) - \zeta_t(1-\rho^x)\alpha k_t^w \\
&= (1-\rho^x)\mu_t^h - (1-\rho^x)\gamma_t k_t^w\left\{\mu_t^h - \mu_t^l\left[1-F(\bar{a}_t^{sl})\right]\right\} \\
&\quad -\zeta_t(1-\rho^x)\alpha k_t^w \\
&= (1-\rho^x)\mu_t^h - \gamma_t k_t^w x_t - \left[\mu_t^h(1-\rho^x) - \gamma_t x_t\right]\alpha k_t^w \\
&= \mu_t^h(1-\rho^x)(1-\alpha k_t^w) - (1-\alpha)\gamma_t k_t^w x_t.
\end{aligned}$$

[2] The steps are

$$\begin{aligned}
\frac{\partial W_t(N_{t-1}^h, N_{t-1}^l)}{\partial N_{t-1}^l} &= \mu_t^h(1-\rho^x)(1-\gamma_t)\left(\frac{I_t}{S_t}\right) \\
&\quad +\mu_t^l\left[1-F(\bar{a}_t^l)\right]\left\{(1-\rho^x) - (1-\rho^x)(1-\gamma_t)\frac{I_t}{S_t}\right\} \\
&\quad -\zeta_t(1-\rho^x)\alpha\left(\frac{I_t}{S_t}\right) \\
&= \mu_t^h(1-\rho^x)(1-\gamma_t)k_t^w + \mu_t^l(1-\rho^x)\left[1-F(\bar{a}_t^{sl})\right]\left[1-(1-\gamma_t)k_t^w\right] \\
&\quad -\left[(1-\gamma_t)\mu_t^h + \gamma_t\left[1-F(\bar{a}_t^{sl})\right]\mu_t^l\right](1-\rho^x)\alpha k_t^w \\
&= \mu_t^h(1-\rho^x)(1-\gamma_t)k_t^w(1-\alpha) \\
&\quad +\mu_t^l(1-\rho^x)\left[1-F(\bar{a}_t^{sl})\right]\left[1-(1-\gamma_t)k_t^w - \gamma_t\alpha k_t^w\right] \\
&= \mu_t^h(1-\rho^x)(1-\gamma_t)k_t^w(1-\alpha) \\
&\quad +\mu_t^l(1-\rho^x)\left[1-F(\bar{a}_t^{sl})\right]\left[1-\alpha k_t^w - (1-\gamma_t)k_t^w + \alpha k_t^w - \gamma_t\alpha k_t^w\right] \\
&= \mu_t^l(1-\rho^x)\left[1-F(\bar{a}_t^{sl})\right]\left[(1-\alpha k_t^w) - (1-\gamma_t)(1-\alpha)k_t^w\right] \\
&\quad +\mu_t^h(1-\rho^x)(1-\gamma_t)k_t^w(1-\alpha) \\
&= \mu_t^l(1-\rho^x)\left[1-F(\bar{a}_t^{sl})\right](1-\alpha k_t^w) + (1-\alpha)(1-\gamma_t)k_t^w x_t.
\end{aligned}$$

and

$$\mathrm{E}_t \frac{\partial W_{t+1}(N_t^h, N_t^l)}{\partial N_t^l} = \mathrm{E}_t \left\{ \begin{array}{c} \left(1-\rho_{t+1}^x\right) \tau_{t+1}^l \left[1 - F(\bar{a}_{t+1}^l)\right] \left(1 - \alpha k_{t+1}^w\right) \\ + \left(1-\alpha\right) \left(1-\gamma_{t+1}\right) k_{t+1}^w x_{t+1} \end{array} \right\}$$

Using these results in the expressions for τ_t^h and $\tau_{i,t}^l$ yields

$$\begin{aligned} \tau_{i,t}^h &= U'(\mathcal{C}_t) \left[\phi^h a_{i,t}^h h_t^h - \frac{v(h_t^h)}{U'(\mathcal{C}_t)} - w^{h,u}\right] \\ &\quad + \beta \left(1-\rho_{t+1}^x\right) \mathrm{E}_t \left[1 - F(\bar{a}_{t+1}^h)\right] \left(1 - \alpha k_{t+1}^w\right) \tau_{t+1}^h \\ &\quad - \beta \left(1-\alpha\right) \mathrm{E}_t \gamma_{t+1} k_{t+1}^w x_{t+1} \end{aligned}$$

$$\begin{aligned} \tau_{i,t}^l &= U'(\mathcal{C}_t) \left[a_{i,t}^l \phi^l h_{i,t}^l - \frac{v(h_{i,t}^l)}{U'(\mathcal{C}_t)} - w^{l,u}\right] + \beta \left(1-\rho_{t+1}^x\right) \mathrm{E}_t \left(1 - \alpha k_{t+1}^w\right) \left[1 - F(\bar{a}_{t+1})\right] \tau_{t+1}^h \\ &\quad + \beta \left(1-\alpha\right) \mathrm{E}_t \left(1-\gamma_{t+1}\right) k_{t+1}^w x_{t+1}. \end{aligned}$$

Define $\bar{s}_t^j \equiv \tau_t^j / \lambda_{1,t} = \tau_t^j / U'\left(\mathcal{C}_t\right)$ and $X_t \equiv x_t / \lambda_{1,t}$. Then

$$\begin{aligned} \bar{s}_t^h &\equiv \frac{\tau_t^h}{\lambda_{1,t}} = \left[\phi^h a_{i,t}^h h_{i,t}^h - \frac{v(h_{i,t}^h)}{U'(\mathcal{C}_t)} - w^u\right] + \beta \left(1-\rho_{t+1}^x\right) \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \left(1 - \alpha k_{t+1}^w\right) \bar{s}_{t+1}^h \\ &\quad - \beta \left(1-\alpha\right) \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \gamma_{t+1} k_{t+1}^w X_{t+1}, \end{aligned} \tag{15}$$

and

$$\begin{aligned} \bar{s}_{i,t}^l &\equiv \frac{\tau_{i,t}^l}{\lambda_{1,t}} = \left[\phi^l a_{i,t}^l h_{i,t}^l - \frac{v(h_{i,t}^l)}{U'(\mathcal{C}_t)} - w^u\right] + \beta \left(1-\rho_{t+1}^x\right) \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \left(1 - \alpha k_{t+1}^w\right) \left[1 - F(\bar{a}_{t+1}^{sl})\right] \bar{s}_{t+1}^l \\ &\quad + \beta \left(1-\alpha\right) \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \left(1-\gamma_{t+1}\right) k_{t+1}^w X_{t+1}. \end{aligned} \tag{16}$$

2.1 Efficiency

To assess efficiency, it is necessary to compare equations (10) and (11) from the competitive market equilibrium with equations (15) and (16) from the social planner's allocation. We first, however, impose the conditions that would ensure the competitive market equilibrium is efficient in a new Keynesian model with homogeneous labor in a search and matching model. These conditions are shown in Ravenna and Walsh (2011) to require price stability, a subsidy to offset the steady-distortion due to imperfect competition, and that the Hosios condition holds. In this case, $\mu_t = 1$, $\eta = \alpha$, and (10) and (11) become

$$s_t^h = \left[\phi^h h_t^h - \frac{v(h_t^h)}{\lambda_t} - w^u\right] + \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \left(1-\rho_{t+1}^x\right) \left(1 - \alpha k_{t+1}^w\right) s_{t+1}^h. \tag{17}$$

$$s^l_{i,t} = \left[a^l_{i,t}\phi^l h^l_{i,t} - \frac{v(h^l_{i,t})}{\lambda_t} - w^u\right] + \beta \mathrm{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(1-\rho^x_{t+1}\right)\left(1-F(\bar{a}^l_{t+1})\right)\left(1-\alpha k^w_{t+1}\right)s^l_{t+1}. \tag{18}$$

Now consider the case in which workers are homogeneous and are all of type h. The surplus of a match in the social planner's allocation is obtained by setting $\gamma_t = 0$ (no searching workers are type l), in which case (15) becomes

$$\bar{s}^h_t = \left[\phi^h h^h_t - \frac{v(h^h_t)}{\lambda_t} - w^u\right] + \beta\left(1-\rho^x_{t+1}\right)\mathrm{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(1-\alpha k^w_{t+1}\right)\bar{s}^h_{t+1}. \tag{19}$$

Equations (17) and (19) imply $s^h_t = \bar{s}^h_t$, and the competitive equilibrium is efficient. This outcome corresponds to the homogeneous labor model of Ravenna and Walsh (2011). Similarly, if all workers are type l, then $\gamma_t = 1$ and (16) becomes

$$\bar{s}^l_{i,t} = \left[\phi^l a^l_{i,t} h^l_{i,t} - \frac{v(h^l_{i,t})}{\lambda_t} - w^u\right] + \beta\left(1-\rho^x_{t+1}\right)\mathrm{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(1-\alpha k^w_{t+1}\right)\left[1-F(\bar{a}^{sl}_{t+1})\right]\bar{s}^l_{t+1}, \tag{20}$$

which together with (18) implies $s^l_{i,t} = \bar{s}^l_{i,t}$, and the competitive equilibrium is efficient.

In the presence of labor heterogeneity, the social planner's valuation of matches given by (15) and (16) differ from (17) and (18) due to the addition terms in (15) and (16) that involve X_{t+1}. This term measures the difference in the expected future value of a type h worker over a type l worker. It captures the composition externality that arises in the presence of heterogeneous labor; employment and separation decisions by firms ignore the impact their decisions have on the composition of the pool of unemployed workers. The social planner takes this externality into account. That is, evaluated at the same allocation,

$$\begin{aligned} s^h_t - \bar{s}^h_t &= \beta\left(1-\rho^x_{t+1}\right)\mathrm{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(1-\alpha k^w_{t+1}\right)\left(s^h_{t+1}-\bar{s}^h_{t+1}\right) \\ &\quad +\beta\left(1-\alpha\right)\mathrm{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\gamma_{t+1}k^w_{t+1}X_{t+1} \end{aligned} \tag{21}$$

and

$$\begin{aligned} s^l_{i,t} - \bar{s}^l_{i,t} &= \beta\left(1-\rho^x_{t+1}\right)\mathrm{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(1-\alpha k^w_{t+1}\right)\left[1-F(\bar{a}^{sl}_{t+1})\right]\left(s^l_{t+1}-\bar{s}^l_{t+1}\right) \\ &\quad -\beta\left(1-\alpha\right)\mathrm{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(1-\gamma_{t+1}\right)k^w_{t+1}X_{t+1}, \end{aligned} \tag{22}$$

where

$$X_{t+1} \equiv \left(1-\alpha\right)\left(1-\rho^x_{t+1}\right)k^w_{t+1}\left[\bar{s}^h_{t+1} - \left(1-\rho^n_{t+1}\right)\bar{s}^l_{i,t+1}(a^l_{i,t+1})\right]. \tag{23}$$

Because $X_{t+i} \geq 0$, all else equal, type h workers are overvalued and type l workers are undervalued in the competitive equilibrium.

Both $s^l_{i,t}$ and $\bar{s}^l_{i,t}$ are increasing functions of the idiosyncratic productivity of worker i.

Since private matches involving type l workers end whenever $a^l_{i,t} < \bar{a}^l_t$, where $\bar{a}^l_t$ is defined such that $s^l_{i,t}(\bar{a}^l_t) = 0$, the fact that $\bar{s}^l_{i,t}(a_{i,t}) > s^l_{i,t}(a_{i,t})$ in the face of labor heterogeneity (i.e., $\mathrm{E}_t(\lambda_{t+1}/\lambda_t)\left(1-\gamma_{t+1}\right)X_{t+1} > 0$ in (22) when $0 < \gamma_{t+1} < 1$) implies

$$\bar{s}^l_{i,t}\left(\bar{a}^l_t\right) > s^l_{i,t}\left(\bar{a}^l_t\right) = 0. \tag{24}$$

That is, a type l worker who generates a zero surplus in the competitive equilibrium would, from the perspective of the social planner, still generate a positive surplus. Therefore, the cutoff productivity level in the efficient allocation (i.e., $\bar{a}^{sp}_t$ such that $\bar{s}^l_t(\bar{a}^{sp}_t) = 0$) is less that $\bar{a}^l_t$. Some type l workers who experience endogenous separation and become unemployed in the competitive equilibrium would remain employed by the social planner. Similarly, some unemployed type l workers who obtain interviews but are screened out in the competitive equilibrium would be hired by the social planner. This also translates into a higher share of low-efficiency workers among the unemployed and a lower expected benefit to posting vacancies in the competitive equilibrium.

It is not just low-efficiency workers that are affected. Because job posting is reduced, type h workers also experience a lower job finding rate and longer average duration of unemployment. Ceteris paribus, endogenous separations are too high in the competitive equilibrium, average unemployment is also too high, and average unemployment duration is inefficiently long.[3]

2.2 The social planner's problem with idiosyncratic shocks to all workers

This section presents the social planner's problem for the case in which the model is generalized to allow both worker types to experience idiosyncratic shocks to productivity. Both types are not subject to endogenous separation rates, denoted by $\rho^{n,j}_t$ for worker type $j = h, l$. Relative to the problem in which only type l workers received idiosyncratic productivity shocks and face an endogenous separation hazard, the following equations in the social planner's problem are affected:

- The planner's objective becomes

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \delta^i_t \left[D_t \frac{\mathcal{C}^{1-\sigma}_{t+i}}{1-\sigma} - (1-\xi_{t+i}) N_{t+i} \int_{\bar{a}^h_t}^{1} v(h^h_{i,t+i}) f_h(a) da - \xi_{t+i} N_{t+i} \int_{\bar{a}^l_t}^{1} v(h^l_{i,t+i}) f_l(a) da \right]$$

  as hours will across high-productivity workers depending on their idiosyncratic productivity shock.

- Both worker types may be screened out in the interview process, so hires are now given by

$$H_t = (1-\gamma_t)\left(1-\rho^{n,h}_t\right) k^w_t S_t + \gamma_t \left(1-\rho^{n,l}_t\right) k^w_t S_t = (1-\rho^n_t)\, k^w_t S_t,$$

[3] The appendix shows that this result can be extended to the case in which both worker types experience individual-specific i.i.d. productivity shocks and endogenous separations.

and the aggregate endogenous rate at which interviewees are screened out is

$$\rho^n_{\gamma,t} \equiv \gamma_t \rho^{n,l}_t + (1-\gamma_t)\,\rho^{n,h}_t.$$

The subscript γ is introduced to denote that this is the average screening out rate which depends on the composition of the composition of the unemployment pool.

- The average separation rate from among employed workers, denoted by $\rho^n_{\xi,t}$, is given by

$$\rho^n_{\xi,t} = \xi_{t-1}\rho^{n,l}_t + \left(1-\xi_{t-1}\right)\rho^{n,h}_t,$$

where

$$\xi_t = (1-\rho^n_t)\left[\frac{(1-\rho^x_t)\xi_{t-1}N_{t-1} + \gamma_t k^w_t S_t}{N_t}\right],$$

- Employment evolves according to

$$\begin{aligned} N_t &= (1-\rho^x_t)\left[\left(1-\xi_{t-1}\right)\left(1-\rho^{n,h}_{\xi,t}\right) + \xi_{t-1}\left(1-\rho^{n,l}_{\xi,t}\right)\right] N_{t-1} + H_t \\ &= (1-\rho^x_t)\left[\left(1-\rho^n_{\xi,t}\right)\right] N_{t-1} + H_t. \end{aligned}$$

- Production technology becomes

$$Y_t = \left\{\xi_t\phi^l\left[\frac{\int_{\bar{a}^l_t}^1 a^l_{i,t}h^l_{i,t}dF_l(a_i)}{1-F_l(\bar{a}^l_t)}\right] + (1-\xi_t)\phi^h_t\left[\frac{\int_{\bar{a}^h_t}^1 a^h_{i,t}h^h_{i,t}dF_h(a_i)}{1-F(\bar{a}^h_t)}\right]\right\} N_t.$$

- Goods clearing is

$$\phi^l\int_{\bar{a}^l_t}^1 a^l_{i,t}h^l_{i,t}di + \phi^h\int_{\bar{a}^h_t}^1 a^h_{i,t}h^h_{i,t} + w^u\left(L^h - N^h_t\right) + w^u\left(L^l - N^l_t\right) - \kappa V_t - \mathcal{C}_t L \geq 0.$$

All other equations are unaffected.

In deriving the solution,to the social planner's problem, let A^j_t be the set of employed type j workers and let $\#N^j_t$ be their number. The social planner's problem is described by

$$W_t(N^h_{t-1}, N^l_{t-1}) = \max_{\mathcal{C}_t, C_t, V_t, I_t, \bar{a}^j_t, N^h_t, \#N^l_t, h^h_{i,t}, h^l_{i,t}} \left[\begin{array}{c} U(\mathcal{C}_t)L - \int_{i\in A^h_t} v\left(h^h_{i,t}\right)di \\ -\int_{i\in A^l_t} v\left(h^l_{i,t}\right)di + \beta \mathrm{E}_t W_{t+1}(N^h_t, N^l_t) \end{array}\right]$$

$$\phi^l\int_{i\in A^l_t} a^l_{i,t}h^l_{i,t}di + \phi^h\int_{i\in A^h_t} a^h_{i,t}h^h_{i,t}di - \kappa V_t - C_t L \geq 0$$

$$C_t L + w^u\left(L^h - N^h_t\right) + w^u\left(L^l - N^l_t\right) - \mathcal{C}_t L \geq 0$$

$$\left[1-F(\bar{a}_t^h)\right]\left[(1-\rho_t^x)\,N_{t-1}^h+(1-\gamma_t)\,I_t\right]-N_t^h=0$$

$$\left[1-F(\bar{a}_t^l)\right]\left[(1-\rho_t^x)\,N_{t-1}^l+\gamma_t I_t\right]-N_t^l=0$$

$$\psi V_t^{1-\alpha}\left[1-(1-\rho_t^x)\left(N_{t-1}^h+N_{t-1}^l\right)\right]^{\alpha}-I_t=0$$

where

$$\gamma_t\equiv\frac{L^l-(1-\rho_t^x)N_{t-1}^l}{\left[1-(1-\rho_t^x)\left(N_{t-1}^h+N_{t-1}^l\right)\right]}=\frac{S_t^l}{S_t^h+S_t^l}.$$

Define $\lambda_{1,t}$, $\lambda_{2,t}$ τ_t^h, τ_t^l, and ζ_t as the Lagrangian multipliers on these constraints. Then, the first order conditions take the form:

$$\mathcal{C}_t\text{: } U'(\mathcal{C}_t)L-\lambda_{2,t}L=0$$

$$C_t\text{: }-\lambda_{1,t}L+\lambda_{2,t}L=0\Rightarrow\lambda_{1,t}=\lambda_{2,t}=U'(\mathcal{C}_t)$$

$$V_t\text{: }-\kappa\lambda_{1,t}+\psi\,(1-\alpha)\left(\frac{I_t}{V_t}\right)\zeta_t=0$$

$$I_t\text{: }(1-\gamma_t)\left[1-F(\bar{a}_t^h)\right]\tau_t^h+\gamma_t\left[1-F(\bar{a}_t^l)\right]\tau_t^l-\zeta_t=0$$

$$h_{i,t}^h\text{, for } a_{i,t}^h\geq\bar{a}_t^{sh}\text{: }-v'(h_{i,t}^h)+\lambda_{1,t}\phi^h a_{i,t}^h=0\Rightarrow\frac{v'(h_{i,t}^h)}{U'(\mathcal{C}_t)}=\phi^h a_{i,t}^h$$

$$h_{i,t}^l\text{, for } a_{i,t}^l\geq\bar{a}_t^{sl}\text{: }-v'\left(h_{i,t}^l\right)+\lambda_{1,t}\phi^l a_{i,t}^l=0\Rightarrow\frac{v'(h_{i,t}^l)}{U'(\mathcal{C}_t)}=\phi^l a_{i,t}^l$$

$$N_t^h\text{: of type } h\text{: }-v(h_{i,t}^h)+\lambda_{1,t}\phi^h a_{i,t}^h h_{i,t}^h-\lambda_{2,t}w^u-\tau_{i,t}^h+\beta\mathrm{E}_t\frac{\partial W_{t+1}(N_t^h,N_t^l)}{\partial N_t^h}=0$$

$$N_t^l\text{ of type } l\text{: }-v(h_{i,t}^l)+\lambda_{1,t}\phi^l a_{i,t}^l h_{i,t}^l-\lambda_{2,t}w^u-\tau_{i,t}^l+\beta\mathrm{E}_t\frac{\partial W_{t+1}(N_t^h,N_t^l)}{\partial N_t^l}=0$$

The first order conditions for V and I imply

$$\begin{aligned}\kappa\lambda_{1,t} &= \left\{(1-\gamma_t)\left[1-F(\bar{a}_t^h)\right]\tau_t^h+\gamma_t\left[1-F(\bar{a}_t)\right]\tau_t^l\right\}\psi\,(1-\alpha)\left(\frac{I_t}{V_t}\right)\\ &= \left\{(1-\gamma_t)\left[1-F(\bar{a}_t^h)\right]\tau_t^h+\gamma_t\left[1-F(\bar{a}_t)\right]\tau_t^l\right\}\psi\,(1-\alpha)\,\theta_t^{-\alpha}\end{aligned}$$

while the ones for N^h and N^l imply

$$\tau_{i,t}^h=U'(\mathcal{C}_t)\left[\phi^h a_{i,t}^h h_{i,t}^h-\frac{v(h_{i,t}^h)}{U'(\mathcal{C}_t)}-w^u\right]+\beta\mathrm{E}_t\frac{\partial W_{t+1}(N_t^h,N_t^l)}{\partial N_t^h}$$

$$\tau^l_{i,t} = U'(\mathcal{C}_t)\left[\phi^l a^l_{i,t}h^l_{i,t} - \frac{v(h^l_{i,t})}{U'(\mathcal{C}_t)} - w^u\right] + \beta \mathrm{E}_t \frac{\partial W_{t+1}(N^h_t, N^l_t)}{\partial N^l_t}$$

Integrating these last two over i yields

$$\tau^h_t \equiv \int_{i\in A^h_t} \tau^h_{i,t}di = U'(\mathcal{C}_t)\left\{\phi^h \int_{i\in A^h_t}\left[a^h_{i,t}h^h_{i,t} - v(h^h_{i,t})\right]di - w^u\right\} + \beta \mathrm{E}_t \frac{\partial W_{t+1}(N^h_t, N^l_t)}{\partial N^h_t}$$

$$\tau^l_t \equiv \int_{i\in A^l_t} \tau^l_{i,t}di = U'(\mathcal{C}_t)\left\{\phi^l \int_{i\in A^l_t}\left[a^l_{i,t}h^l_{i,t} - v(h^l_{i,t})\right]di - w^u\right\} + \beta \mathrm{E}_t \frac{\partial W_{t+1}(N^h_t, N^l_t)}{\partial N^l_t}$$

To evaluate the terms of the form $\beta \mathrm{E}_t\left[\partial W_{t+1}/\partial N^j_t\right]$, use the envelope conditions,

$$\begin{aligned}\frac{\partial W_t(N^h_{t-1}, N^l_{t-1})}{\partial N^h_{t-1}} \quad &= \quad (1-\rho^x_t)\,\tau^h_t - I_t\frac{\partial\,(1-\gamma_t)}{\partial N^h_{t-1}}\tau^h_t + \left[1-F(\bar{a}^h_t)\right] I_t \frac{\partial\,(1-\gamma_t)}{\partial N^h_{t-1}}\int_{i\in A^h_t}\frac{\tau^h_{i,t}}{1-F(\bar{a}^h_t)}di \\ &\quad -(1-\rho^x_t)\,\zeta_t\alpha\left(\frac{I_t}{S_t}\right)\end{aligned}$$

$$\begin{aligned}\frac{\partial W_t(N^h_{t-1}, N^l_{t-1})}{\partial N^h_{t-1}} \quad &= \quad \tau^h_t\left[1-F(\bar{a}^h_t)\right]\left\{(1-\rho^x_t) - I_t\frac{\partial\gamma_t}{\partial N^h_{t-1}}\right\} \\ &\quad +\tau^l_t\left[1-F(\bar{a}^l_t)\right]\left(I_t\frac{\partial\gamma_t}{\partial N^h_{t-1}}\right) - \zeta_t\,(1-\rho^x_t)\,\alpha\left(\frac{I_t}{S_t}\right)\end{aligned}$$

and

$$\begin{aligned}\frac{\partial W_t(N^h_{t-1}, N^l_{t-1})}{\partial N^l_{t-1}} \quad &= \quad \tau^l_t\left[1-F(\bar{a}^l_t)\right]\left\{(1-\rho^x_t) + I_t\frac{\partial\gamma_t}{\partial N^l_{t-1}}\right\} \\ &\quad +\tau^h_t\left[1-F(\bar{a}^h_t)\right]\left(-I_t\frac{\partial\gamma_t}{\partial N^l_{t-1}}\right) - \zeta_t\,(1-\rho^x_t)\,\alpha\left(\frac{I_t}{S_t}\right)\end{aligned}$$

together with

$$\frac{\partial\gamma_t}{\partial N^h_{t-1}} = \frac{(1-\rho^x_t)\left[L^l-(1-\rho^x_t)N^l_{t-1}\right]}{S^2_t} = (1-\rho^x_t)\,\frac{\gamma_t}{S_t}$$

$$\frac{\partial\gamma_t}{\partial N^l_{t-1}} = \frac{-(1-\rho^x_t)\,S_t + (1-\rho^x_t)\left[L^l-(1-\rho^x_t)N^l_{t-1}\right]}{S^2_t} = -(1-\rho^x_t)\,\frac{1-\gamma_t}{S_t} \leq 0.$$

Noting that $I/S = k^w$ and defining

$$x_t \equiv (1-\rho^x_t)\left\{\left[1-F(\bar{a}^h_t)\right]\tau^h_t - \left[1-F(\bar{a}^l_t)\right]\tau^l_t\right\}, \tag{25}$$

one obtains

$$\frac{\partial W_t(N^h_{t-1},N^l_{t-1})}{\partial N^h_{t-1}}=\tau^h_t\left(1-\rho^x_t\right)\left[1-F(\bar{a}^h_t)\right]\left(1-\alpha k^w_t\right)-\left(1-\alpha\right)\gamma_t k^w_t x_t$$

$$\frac{\partial W_t(N^h_{t-1},N^l_{t-1})}{\partial N^l_{t-1}}=\tau^l_t\left(1-\rho^x_t\right)\left[1-F(\bar{a}^l_t)\right]\left(1-\alpha k^w_t\right)+\left(1-\alpha\right)\left(1-\gamma_t\right)k^w_t x_t$$

So

$$\mathrm{E}_t\frac{\partial W_{t+1}(N^h_t,N^l_t)}{\partial N^h_t}=\mathrm{E}_t\left\{\begin{array}{c}\left(1-\rho^x_{t+1}\right)\tau^h_{t+1}\left[1-F(\bar{a}^h_{t+1})\right]\left(1-\alpha k^w_{t+1}\right)\\ -\left(1-\alpha\right)\gamma_{t+1}k^w_{t+1}x_{t+1}\end{array}\right\}$$

and

$$\mathrm{E}_t\frac{\partial W_{t+1}(N^h_t,N^l_t)}{\partial N^l_t}=\mathrm{E}_t\left\{\begin{array}{c}\left(1-\rho^x_{t+1}\right)\tau^l_{t+1}\left[1-F(\bar{a}^l_{t+1})\right]\left(1-\alpha k^w_{t+1}\right)\\ +\left(1-\alpha\right)\left(1-\gamma_{t+1}\right)k^w_{t+1}x_{t+1}\end{array}\right\}$$

Hence, using these results in the expressions for $\tau^l_{i,t}$ and $\tau^h_{i,t}$,

$$\begin{aligned}\tau^l_{i,t} &= U'(\mathcal{C}_t)\left[\phi^l a^l_{i,t}h^l_{i,t}-\frac{v(h^l_{i,t})}{U'(\mathcal{C}_t)}-w^u\right]+\beta\left(1-\rho^x_{t+1}\right)\mathrm{E}_t\left(1-\alpha k^w_{t+1}\right)\left[1-F(\bar{a}^h_{t+1})\right]\tau^l_{t+1}\\ &\quad+\beta\left(1-\alpha\right)\mathrm{E}_t\left(1-\gamma_{t+1}\right)k^w_{t+1}x_{t+1}\end{aligned}$$

$$\begin{aligned}\tau^h_{i,t} &= U'(\mathcal{C}_t)\left[\phi^h a^h_{i,t}h^h_t-\frac{v(h^h_t)}{U'(\mathcal{C}_t)}-w^u\right]\\ &\quad+\beta\left(1-\rho^x_{t+1}\right)\mathrm{E}_t\left[1-F(\bar{a}^h_{t+1})\right]\left(1-\alpha k^w_{t+1}\right)\tau^h_{t+1}\\ &\quad-\beta\left(1-\alpha\right)\mathrm{E}_t\gamma_{t+1}k^w_{t+1}x_{t+1}\end{aligned}$$

Define $\bar{s}^j_t\equiv\tau^j_t/\lambda_{1,t}=\tau^j_t/U'\left(\mathcal{C}_t\right)$ and $X_t\equiv x_t/\lambda_{1,t}$. Then for $j=h,l$ and using (14),

$$\begin{aligned}\bar{s}^h_{i,t} &= \left[\frac{\chi}{1+\chi}\left(\frac{\lambda_t}{\ell}\right)^{\frac{1}{\chi}}\left(a^h_{i,t}\phi^h\right)^{\frac{1+\chi}{\chi}}-w^u\right]+\beta\left(1-\rho^x_{t+1}\right)\mathrm{E}_t\left(\frac{\lambda_{1,t+1}}{\lambda_{1,t}}\right)\left(1-\alpha k^w_{t+1}\right)\left[1-F(\bar{a}^{sh}_{t+1})\right]\bar{s}^h_{t+1}\\ &\quad-\beta\left(1-\alpha\right)\mathrm{E}_t\left(\frac{\lambda_{1,t+1}}{\lambda_{1,t}}\right)\gamma_{t+1}k^w_{t+1}X_{t+1},\end{aligned}\tag{26}$$

and

$$\begin{aligned}\bar{s}^l_{i,t} &= \left[\frac{\chi}{1+\chi}\left(\frac{\lambda_t}{\ell}\right)^{\frac{1}{\chi}}\left(a^l_{i,t}\phi^l\right)^{\frac{1+\chi}{\chi}}-w^u\right]+\beta\left(1-\rho^x_{t+1}\right)\mathrm{E}_t\left(\frac{\lambda_{1,t+1}}{\lambda_{1,t}}\right)\left(1-\alpha k^w_{t+1}\right)\left[1-F(\bar{a}^{sl}_{t+1})\right]\bar{s}^l_{t+1}\\ &\quad+\beta\left(1-\alpha\right)\left(1-\rho^x_{t+1}\right)\mathrm{E}_t\left(\frac{\lambda_{1,t+1}}{\lambda_{1,t}}\right)\left(1-\gamma_{t+1}\right)k^w_{t+1}X_{t+1}.\end{aligned}\tag{27}$$

To focus on average productivity differences while holding the dispersion of idiosyncratic productivity realizations within types the same, assume the productivity of worker i of type j is $a^j_{i,t} = \phi^j + e_{i,t}$ for $j = h, l$, where ϕ^j is the expected productivity of a worker of type j. By definition, $\phi^h > \phi^l$; on average, type h workers are more productivity than type l workers. In turn, $e_{i,t}$ is an idiosyncratic component of worker productivity drawn from the same distribution F for both types, with mean zero and variance σ^2_e. We assume $F(-\phi^l) = 0$ to ensure $a^j_{i,t} \geq 0$.[4] The cutoff productivity realization that triggers endogenous separation (or screening out in the interview process) for type j is $\bar{a}^j_t$. The endogenous separation rates, denoted $\rho^{n,j}_t$ for type $j = l, h$, are given by the probability $a^j_{i,t} = \phi^j + e_{i,t} < \bar{a}^j_t$, or $\rho^{n,j}_t = F(\bar{a}^j_t - \phi^j)$.

The joint surplus to the worker-firm in the competitive equilibrium for a type j worker is a function of the worker's $a^j_{i,t}$ and equals

$$s^j_{i,t}(a^j_{i,t}) = \left[\frac{\chi}{1+\chi}\left(\frac{\lambda_t}{\ell}\right)^{\frac{1}{\chi}}\left(a^j_{i,t}\right)^{\frac{1+\chi}{\chi}} - w^u\right] + \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right)(1-\rho^x_{t+1})\left(1 - F\left(\bar{a}^j_{t+1}\right)\right)\left(1 - \alpha k^w_{t+1}\right) s^j_{t+1} \tag{28}$$

when $\mu_t = 1$ and $\eta = a$ and hours have been eliminated by using (14).[5]. Comparing this expression to the surplus valuations (26) and (27) again show that when evaluated at the same allocation, the composition externality captured by X_t drives a wedge between the valuations of the worker types in the market equilibrium and the efficient allocation. The difference between the market and efficient match surpluses depends on the expected discounted future values of X_{t+i}, where

$$X_{t+i} = \left(1 - \rho^x_{t+1}\right)\left[\left(1 - \rho^{n,h}_{t+i}\right)\bar{s}^h_{t+i} - \left(1 - \rho^{n,j}_{t+i}\right)\bar{s}^l_{t+i}\right] \geq 0.$$

Thus, the market and efficient equilibria differ. The only difference from the case considered in the paper is that X_t now also accounts for the endogenous separation probability of a type h worker (see 25).

Given the lower expected separation rate and higher average productivity of type h workers, $s^h_{i,t} - \bar{s}^h_{i,t} \geq 0$ and $s^l_{i,t} - \bar{s}^l_{i,t} \leq 0$ as before for workers with the same current productivity. From an efficiency perspective, the market equilibrium overvalues type h workers and undervalues workers of type l. Firms retain and hire too many high-efficiency workers and separate from and screen out too many low-efficiency workers.

From (28), the joint surplus of a worker of type j in the market equilibrium can be written as

$$s^j_{i,t}(a^j_{i,t}) = s_{1,t}(a^j_{i,t}) + \mathrm{E}_t s^j_{2,t+1}, \tag{29}$$

[4] In the text of the paper, we assumed productivity of a type l was $a^l_{i,t}\phi^l$. It will be more convenient to allow for differences in average productivity between types by incorporating it into differences in the distributions functions of $a^l_{i,t}$ and $a^h_{i,t}$. This will allow us to focus on average productivity differences while holding the variance of productivity the same across worker types.

[5] This modifies (18) to reflect the new specification of productivity and that it applies for both labor types.

where

$$s_{1,t}(a^j_{i,t}) \equiv \left[\frac{\chi}{1+\chi}\left(\frac{\lambda_t}{\ell}\right)^{\frac{1}{\chi}}\left(a^j_{i,t}\right)^{\frac{1+\chi}{\chi}} - w^u\right]$$

is an increasing function of $a^j_{i,t}$ that depends on the individual worker only through $a^j_{i,t}$, while the continuation value

$$\mathrm{E}_t s^j_{2,t+1} \equiv \beta \mathrm{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(1-\rho^x_{t+1}\right)\left(1-\alpha k^w_{t+1}\right)\left[1-F\left(\bar{a}^l_{t+1}\right)\right]s^j_{t+1}$$

is independent of $a^j_{i,t}$ and depends only on the worker's type. From the definition of $\bar{a}^j_t$,

$$s^j_{i,t}(\bar{a}^j_t) = 0 \Rightarrow s_{1,t}(\bar{a}^j_t) = -\mathrm{E}_t s^j_{2,t+1}. \tag{30}$$

Consider initially a situation in which average productivity of each group is the same: $\phi^h = \phi^l$. Then $\bar{a}^h_t = \bar{a}^l_t$, $\bar{a}^h_t - \phi^h = \bar{a}^l_t - \phi^l$, $\rho^{n,h}_t = \rho^{n,l}_t$, and $\mathrm{E}_t s^h_{2,t+1} = \mathrm{E}_t s^l_{2,t+1}$. Suppose the average productivity of the type h workers, ϕ^h, increases. This has two effects. First, the expected surplus from a type h employed worker, s^h_{t+1}, increases. Second, at the initial $\bar{a}^h_{t+1} = \bar{a}^l_{t+1}$, the probability $e_{i,t+1} < \bar{a}^h_{t+1} - \phi^h$ falls when ϕ^h increases and the endogenous separation rate for type h workers therefore falls. Both factors act to increase $\mathrm{E}_t s^h_{2,t+1}$. Therefore, for a high-efficiency worker whose current productivity is $\bar{a}^l_t$,

$$s^h_{i,t}(\bar{a}^l_t) = s_{1,t}(\bar{a}^l_t) + \mathrm{E}_t s^h_{2,t+1} = \mathrm{E}_t s^h_{2,t+1} - \mathrm{E}_t s^l_{2,t+1} > 0,$$

where (30) has been used. The joint surplus for a type h worker with current productivity equal to $\bar{a}^l_t$ is positive. Consequently, a type h worker with productivity equal to $\bar{a}^l_t$ would be retained by the firm and a type h job seeker who obtains an interview would be hired, while a type l with the same current productivity would not. It follows that $\bar{a}^h_t < \bar{a}^l_t$; the cutoff level of productivity that governs endogenous separation and screening is lower for type h workers than it is for type l workers. With $\bar{a}^h_t < \bar{a}^l_t$ and $\phi^h > \phi^l$, $\bar{a}^h_t - \phi^h < \bar{a}^l_t - \phi^l$. Hence,

$$\rho^{n,h}_t = F\left(\bar{a}^h_t - \phi^h\right) \leq F\left(\bar{a}^l_t - \phi^l\right) = \rho^{n,l}_t.$$

The endogenous separation rate for type h workers is lower than for type l workers.

3 The model of temporary layoffs

Workers on temporary layoff are counted as unemployed but are not included in the pool of job searchers. Let T^j_t be the pool of type j workers on temporary layoff. T^h_t and T^l_t evolve according to the laws of motion:

$$T^h_t = (1-r)\left[T^h_{t-1} + \Gamma^h_t \rho^x_t\left(1-\xi_{t-1}\right)N_{t-1}\right] \tag{31}$$

$$T_t^l = (1-r)\left[T_{t-1}^l + \Gamma_t^l \rho_t^x \xi_{t-1} N_{t-1}\right], \tag{32}$$

where Γ_t^j is the share of worker type j exogenous separations flowing into the temporary unemployment pool and r is the constant recalls rate.

Let Γ^j equal the steady state share of type j exogenously separating workers on temporary layoff. The shock processes Γ_t^j are given by

$$\Gamma_t^j = (1-\rho_\Gamma)\Gamma^j + \rho_\Gamma \Gamma_t^j + \varepsilon_{\Gamma,t}^j,$$

for $j = h, l$. In the parameterization we assume $\Gamma_t^h = \Gamma_t^l = \Gamma_t$ and $\Gamma^h = \Gamma^j = \Gamma$. In this case, letting $T_t = T_t^h + T_t^l$, (31) and (32) imply

$$T_t = (1-r)\left(T_{t-1} + \rho_t^x \Gamma_t N_{t-1}\right). \tag{33}$$

$r\left(T_{t-1}^l + \rho_t^x \xi_t N_{t-1}\right)$ type l workers are recalled and exit the stock of workers on temporary layoffs at time t, but only $1-\rho_t^n$ of these will be rehired. The remaining fraction ρ_t^n are those whose with $a_{i,t}^l < \bar{a}_t^l$; given our timing assumptions, these workers end period t among the unmatched and join the pool of searching workers in the following period.

The aggregate number of workers seeking employment S_t is the total labor force minus those still employed after the exogenous separation hazard and those still on temporary layoff is

$$S_t = 1 - T_t - (1-\rho_t^x) N_{t-1}. \tag{34}$$

Total new matches equal new hires plus recall hires:

$$H_t = (1-\gamma_t \rho_t^n) k_t^w S_t + r\left[T_{t-1}^h + \rho_t^x \Gamma_t N_{t-1}^h + (1-\rho_t^n)\left(T_{t-1}^l + \rho_t^x \Gamma_t N_{t-1}^l\right)\right].$$

Employment in period t consists of surviving matches and newly created matches:

$$\begin{aligned} N_t \quad = \quad & (1-\rho_t^x)\left[(1-\xi_{t-1}) + (1-\rho_t^n)\xi_{t-1}\right] N_{t-1} \\ & + \left(1-\gamma_t \rho_t^{n,l}\right) k_t^w S_t + r\left[T_t^h + \rho_t^x \Gamma_t (1-\xi_{t-1}) N_{t-1}\right] \\ & + r\left(1-\rho_t^{n,l}\right)\left(T_t^l + \rho_t^x \Gamma_t \xi_{t-1} N_{t-1}\right). \end{aligned} \tag{35}$$

Finally, the share ξ_t of type l workers among the employed is given by:

$$\xi_t = (1-\rho_t^n)\left[\frac{\xi_{t-1}(1-\rho_t^x) N_{t-1} + \gamma_t k_t^w S_t + r\left(T_{t-1}^l + \Gamma_t \rho_t^x \xi_{t-1} N_{t-1}\right)}{N_t}\right],$$

where the last two terms in numerator consist of those type l who are interviewed and are not screened out, $(1-\rho_t^n)\gamma_t k_t^w S_t$, and those recalled but not screened out, $(1-\rho_t^n) r\left(T_{t-1}^l + \Gamma_t \rho_t^x \xi_{t-1} N_{t-1}\right)$.

4 Calibration and robustness

4.1 Parameters for steady state unemployment

In our parameterization of the steady state unemployment for the unobservable variables U^l_{ss}, U^h_{ss} we compute a baseline calibration relying on estimates by Gregory et al. (2021), which are based on US LEHD data, and propose two alternative calibrations. Gregory et al. (2021) identify three groups of workers with different employment spells characteristics: α, β and γ workers, with unemployment rates respectively equal to 4.2%, 12.5% and 28.8% and labor force shares equal to 0.55, 0.25 and 0.2.

The authors warn that these unemployment rates, obtained from establishment data, return an average unemployment rate of 11.2%, much higher than the unemployment rate from the BLS's CPS based on household data. To make The Gregory, et. al. rates consistent with the U.S. average unemployment rate of 5.6% targeted in our model, we rescale the α, β, γ unemployment rates so that their weighted average unemployment rate is equal to the one we targeted.

4.1.1 Baseline

We assume α workers represent the set of h-workers, and the group including both β and γ workers represents the set of l-workers. This gives unemployment rates $U^h_{ss} = 2.1\%$, $U^l_{ss} = 9.87\%$ and a labor force share of l-worker equal to 0.45. The model does not have an equilibrium for this set of parameters, and we target $U^h_{ss} = 2.97\%$, $U^l_{ss} = 9.87\%$, which is the minimum value of U^l_{ss} for which the given U^h_{ss} and all other targeted steady state moments an equilibrium can be found. To ensure the average unemployment rate is unchanged and equal to 5.6%, the implied share of l-worker is adjusted to 0.38.

We identify the key parameter that characterizes this equilibrium and the strength of the selection effect on aggregate variables with the ratio U^l_{ss}/U^h_{ss},equal to 3.32. We provide results for two alternative parameterizations of the market equilibrium. These alternatives produce a higher and a lower ratio U^l_{ss}/U^h_{ss} relative to the baseline for which results are provided in the main text.

4.1.2 Parameterization with low U^l_{ss}/U^h_{ss}

This parameterization assumes the 16 to 24 year old group of workers represents the set of l-workers, and the over-24 age-group represents the set of l-workers. For the U.S.1948-2019 period, this gives unemployment rates of $U^h_{ss} = 4.5\%$ and $U^l_{ss} = 11.48\%$ and a labor force share of l-worker equal to 0.16. The ratio U^l_{ss}/U^h_{ss} is equal to 2.54. Figure 1 shows the behaviour of unemployment, conditional on the parameterization of unemployment, on the alternative implied parameters, and on the alternative filtered set of shocks d_t and ρ^x_t in the permanent layoff model. Note that l-workers unemployment volatility is higher than h-workers, and

l-workers are over-represented as share of unemployment relative to their labor force share during the pandemic recession.

4.1.3 Parameterization with high U_{ss}^l/U_{ss}^h

We assume γ workers as defined in Gregory et al. (2021) represent the set of l-workers, and the group consisting of both α and β workers represents the set of h-workers. This gives unemployment rates for the two groups of $U_{ss}^h = 3.4\%$ and $U_{ss}^l = 14.4\%$ and a labor force share of l-worker equal to 0.2. The ratio U_{ss}^l/U_{ss}^h is equal to 4.23. Figure 2 shows the behaviour of unemployment, conditional on the parameterization of unemployment, on the alternative implied parameters, and on the alternative filtered set of shocks d_t and ρ_t^x in the permanent layoff model. As in the earlier parameterizations, l-workers unemployment volatility is higher than h-workers, and l-workers are over-represented as unemployment rate share relative to their labor force share during the pandemic recession. Interestingly, keeping the other targeted parameters constant, parameterizations with a higher ratio of the unemployment rates between l and h workers imply a larger steady state reallocation through a higher level of selection. This in turns makes the **cyclicality** of selection smaller, and the time-variation of the unemployment pool composition less volatile. While this reduces the cyclicality of the productivity in the pool of unemployed, it affects only one of the channels through which selection operates (the composition effect), since it still allows firms to be selective in their hiring. Thus the quantitative results on unemployment volatility during the pandemic recession still hold.

4.1.4 Shocks parameterizations

Figure 3 shows both the targeted values and the filtered shock used in the baseline parameterization.

5 Monetary policy

5.1 Policy that responds to the separation rate

Figures 4 shows the impulse responses to the COVID shock in the temporary layoff model when monetary policy is given by

$$\ln(1+i_t) = -\ln\beta + 1.5\pi_t - 0.021\rho_t.$$

The value of the coefficient on the separation rate is calibrated so that the initial impact on unemployment is the same as under the unemployment rule with $\omega_U = -0.8$. For comparison, the figure also shows the responses under the policy of price stability and under the rule with $\omega_U = -0.8$.

Figure 5 shows the responses of the markup and the cutoff productivity level $\bar{a}_t^l$ under the same three policies as in the previous figure.

5.2 Policy that responds to type-l unemployment

Table A.1 expands on Table 2 of the text to include results under a monetary policy rule of the form

$$\ln(1+i_t) = -\ln\beta + 1.5\pi_t + \omega_{U^l} U_t^l \tag{36}$$

that reacts directly to the labor market conditions of type l workers by responding to U_t^l. The value of ω_{U^l} is set equal to -0.5 which achieves a 50% reduction in the initial impact of the COVID pandemic on the unemployment rate of type l workers. Note that under the assumptions of the model U_t^l is not directly observable. Thus, our hypothetical experiment under the rule given by (36) can be thought of a best case scenario to illustrate the what would happen if the central bank could respond directly to U_t^l to dampen the impact of the pandemic on this group of workers.

Table A.1: Outcome for Alternative Policies

				Alternative policies		
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
Variables in policy rule		π	π, **U**	π, **U**	π, ρ	π, **U**^{l}
Response coefficients ω_π and ω_U		**1.5, 0**	**1.5, −0.4**	**1.5, −0.8**	**1.5, −0.021**	**1.5, −0.5**
Target reduction	$\pi = 0$		**25% (U)**			**50%(U^l)**
1) Output loss	**27.09%**	**32.78%**	**26.02%**	**22.31%**	**23.63%**	**20.03%**
2) l-unemployment loss	**19.01%**	**24.56%**	**16.95%**	**12.85%**	**14.88%**	**10.53%**
3) h-unemployment loss	**2.56%**	**3.04%**	**2.47%**	**2.16%**	**2.27%**	**1.97**
4) Inequality ratio	**7.42**	**8.07**	**6.85**	**5.95**	**6.55**	**5.35**
5) Sacrifice Ratio	–	–	**1.97**	**1.41**	**1.31**	**1.28**

This policy is the most successful at reducing the inequality ratio. It delivers a 50% reduction of in the unemployment rate of type l workers at the onset of the recession (by our choice of calibration) and produces an approximately equal fall in the aggregate unemployment rate. Table A.1 shows that this policy will also considerably lower the unemployment inequality ratio. Figures 6 and 7 also show that the policy will be more inclusive; limiting the rise U^l is sufficiently expansionary that the markup actually falls. As a result, the rise in the cutoff productivity level $\bar{a}_t^l$ is reduced by about 50% relative to the benchmark policy. With $\bar{a}_t^l$ rising less, endogenous separations rises less, implying fewer type l workers are screened out and more who are in matches are retained. However, this improvement in the labor market outcomes for type l workers comes at a considerable cost in terms of inflation volatility. The protracted fall in μ_t below its steady state leads to the highest level of inflation among all the rules in Table A.1. However the sacrifice ratio (1.28) is approximately equal to the one obtained with the rule (5) responding to the separation rate - a rule intended as well to support unemployment among the l workers.

5.3 Policy that stabilizes aggregate unemployment

It is interesting to examine the implications of a policy that keeps the aggregate unemployment at its steady state level and whether such a rule also stabilizes U^l and U^h. The outcome of this policy is shown in figure 8. As expected, the policy can only reduce unemployment by aggressively lowering the markup and thus generating a very high level of inflation.

The policy results in U^l falling by about 0.6 percentage points relative to steady state (while in the benchmark policy it increases by 16 percentage points), while U^h increases by about 0.3 percentage points relative to steady state (while in the benchmark policy it increases by about 1 percentage point). This policy leads to U^l and U^h moving in opposite directions relative to the steady state, but the extent of the deviations are small compared to the unemployment movements under the benchmark policy. We conclude that stabilizing aggregate U stabilizes nearly completely both U^l and U^h.[6]

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Ravenna, F. and C. E. Walsh (2012). Screening and Labor Market Flows in a Model with Heterogeneous Workers. *Journal of Money, Credit and Banking 44*(2), 31–71.

[6] Note that the wedge between U^l and U^h decreases with a policy targeting $U_t = U_{steady\ state}$ conditional on both a preference shock and a separation shock.

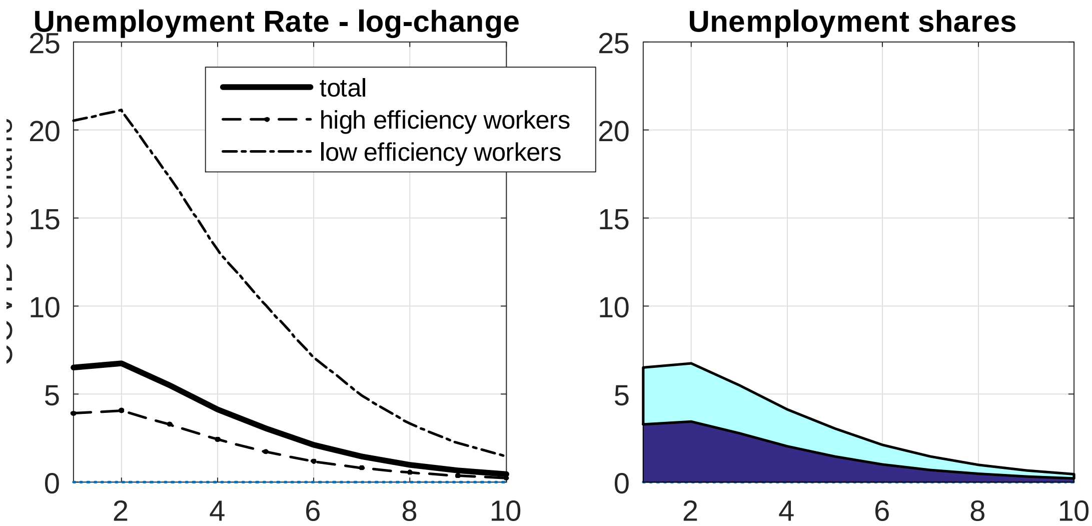

Figure 1: Permanent layoff model, Pandemic recession: Labor market outcomes with *low* unemployment ratio U^l_{ss}/U^h_{ss} parameterization across worker types.

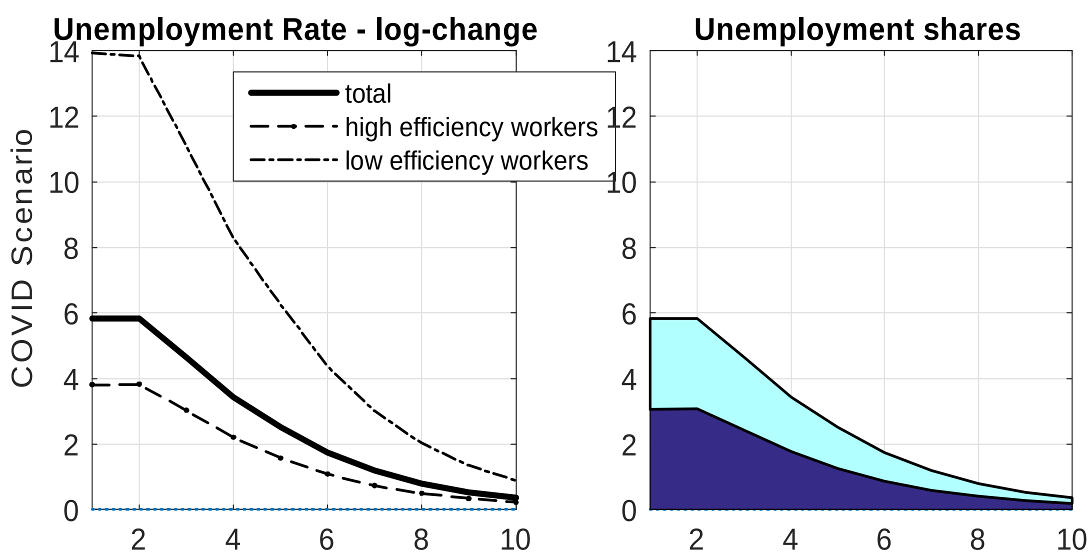

Figure 2: Permanent layoff model, Pandemic recession: Labor market outcomes with *high* unemployment ratio U^l_{ss}/U^h_{ss} parameterization across worker types.

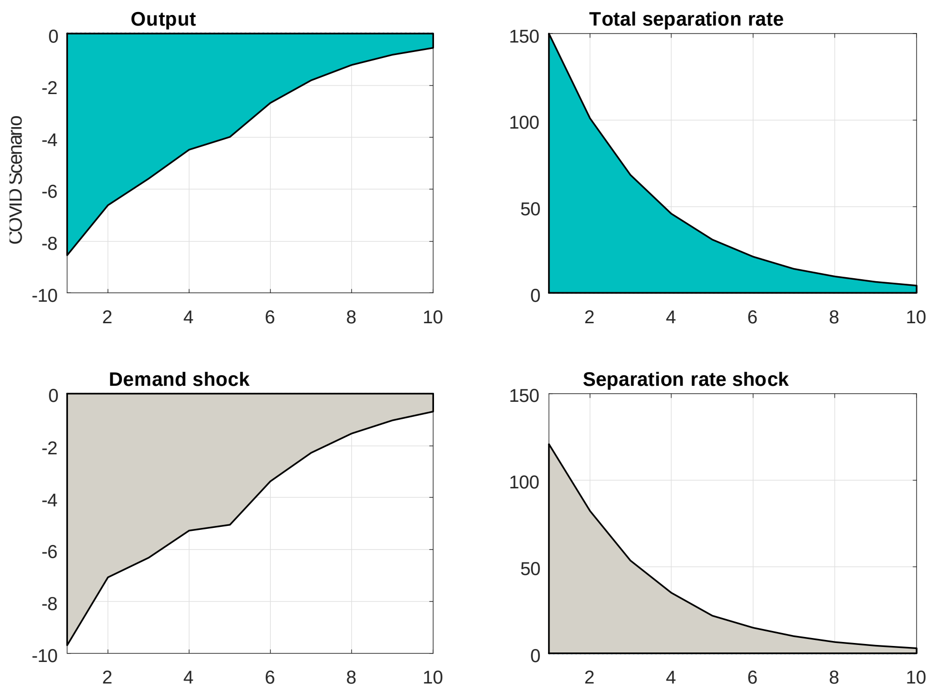

Figure 3: Permanent layoff model, Pandemic recession: Targeted values and filtered shock, baseline parameterization

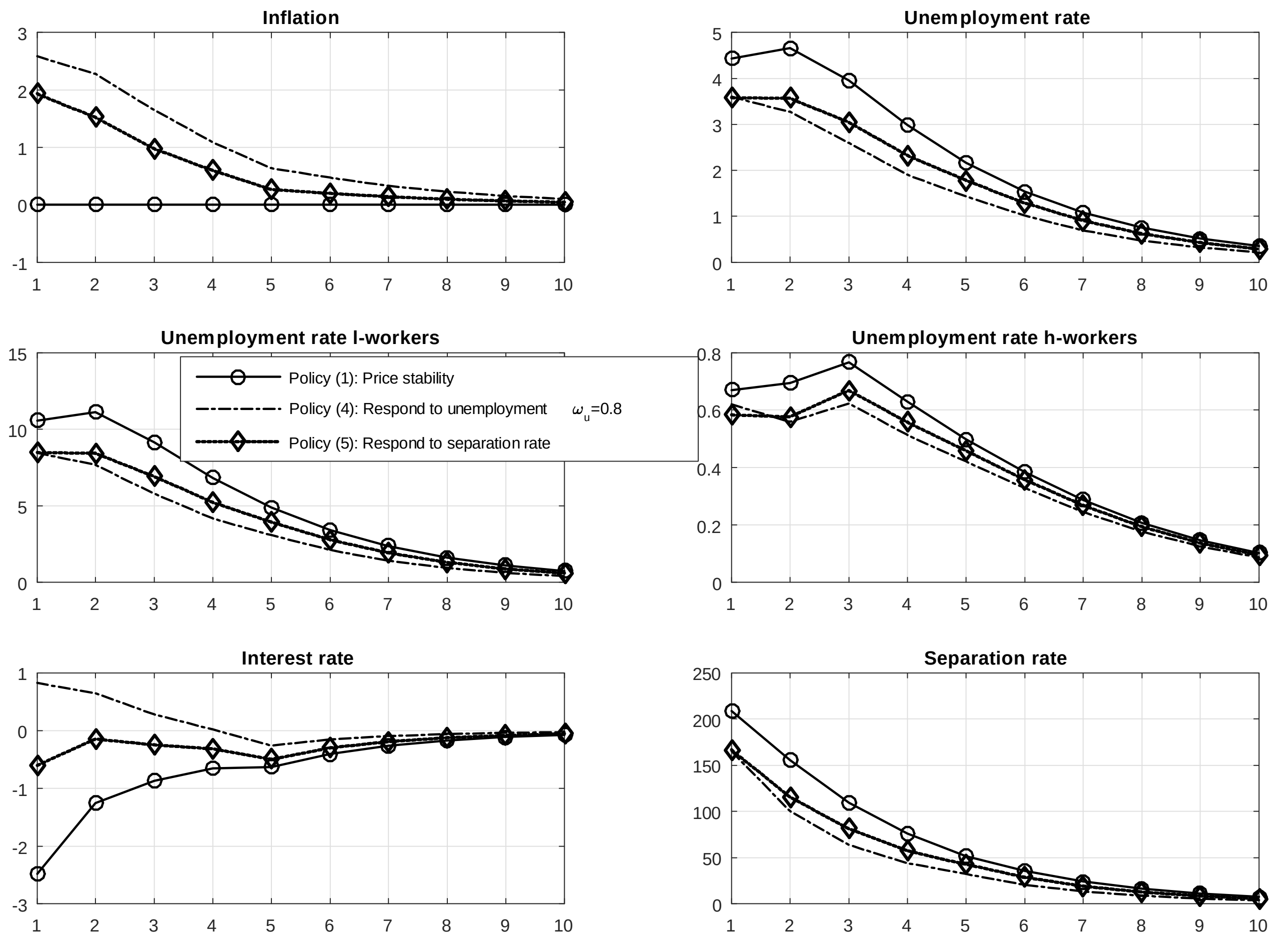

Figure 4: Temporary layoffs model. Impulse resonses to the COVID-19 shock under alternative policy rules. Inflation and interest rate are shown at quarterly rates.

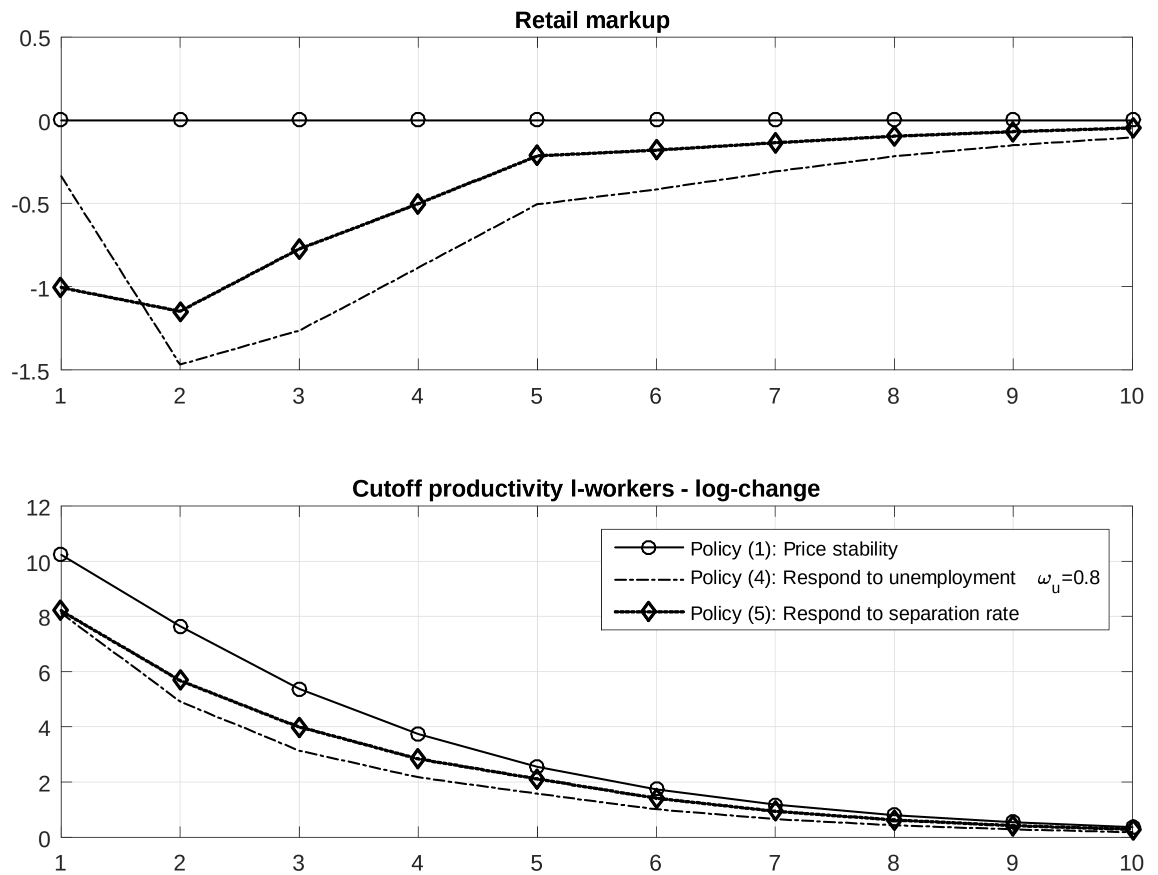

Figure 5: Temporary layoff model with alternative policies rules. Panels show the responses of μ_t (top) and $\bar{a}_t^l$ (bottom) to the pandemic shocks under different rules.

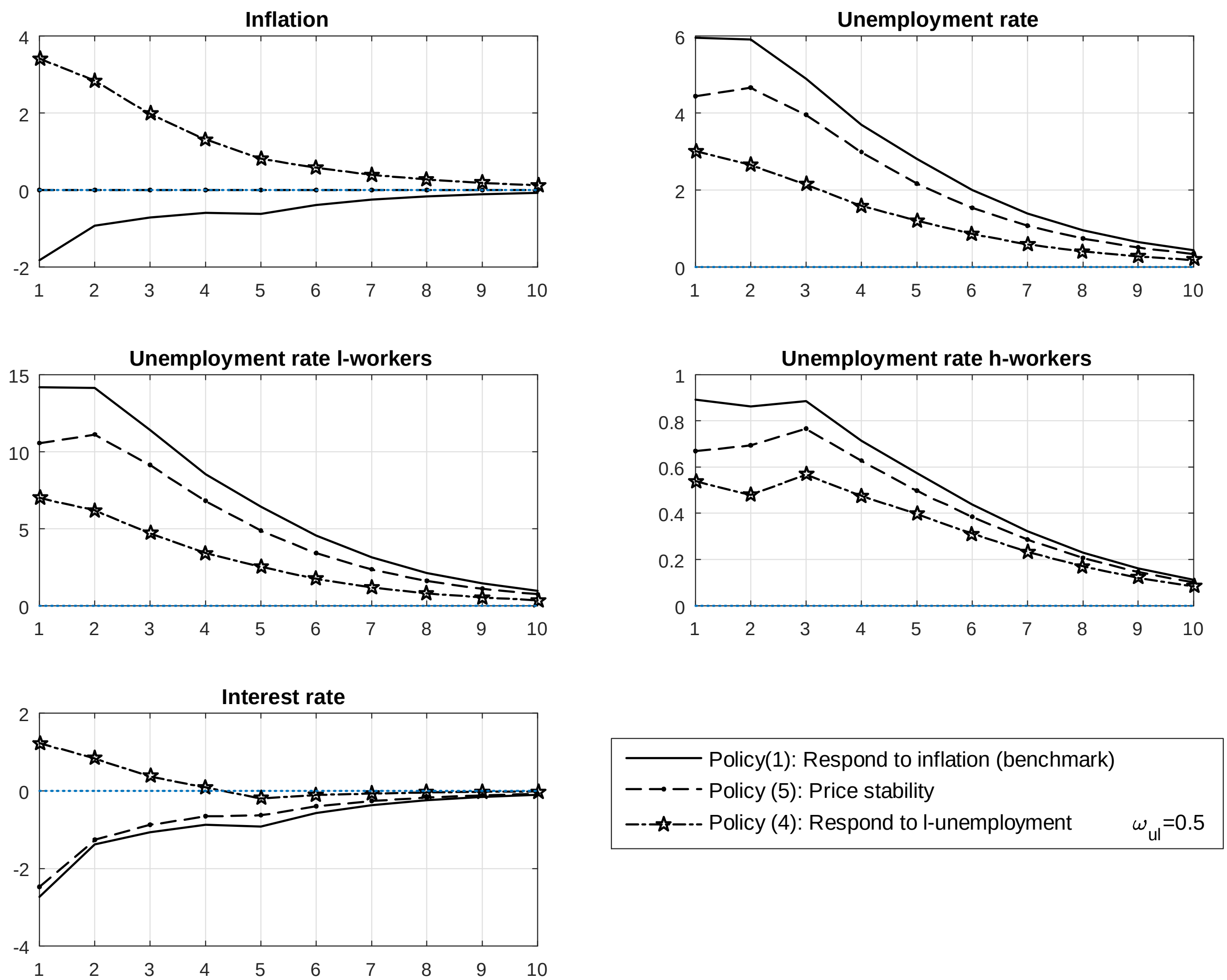

Figure 6: Temporary layoffs model. Impulse resonses to the COVID-19 shock under alternative policy rules. Inflation and interest rate are shown at quarterly rates.

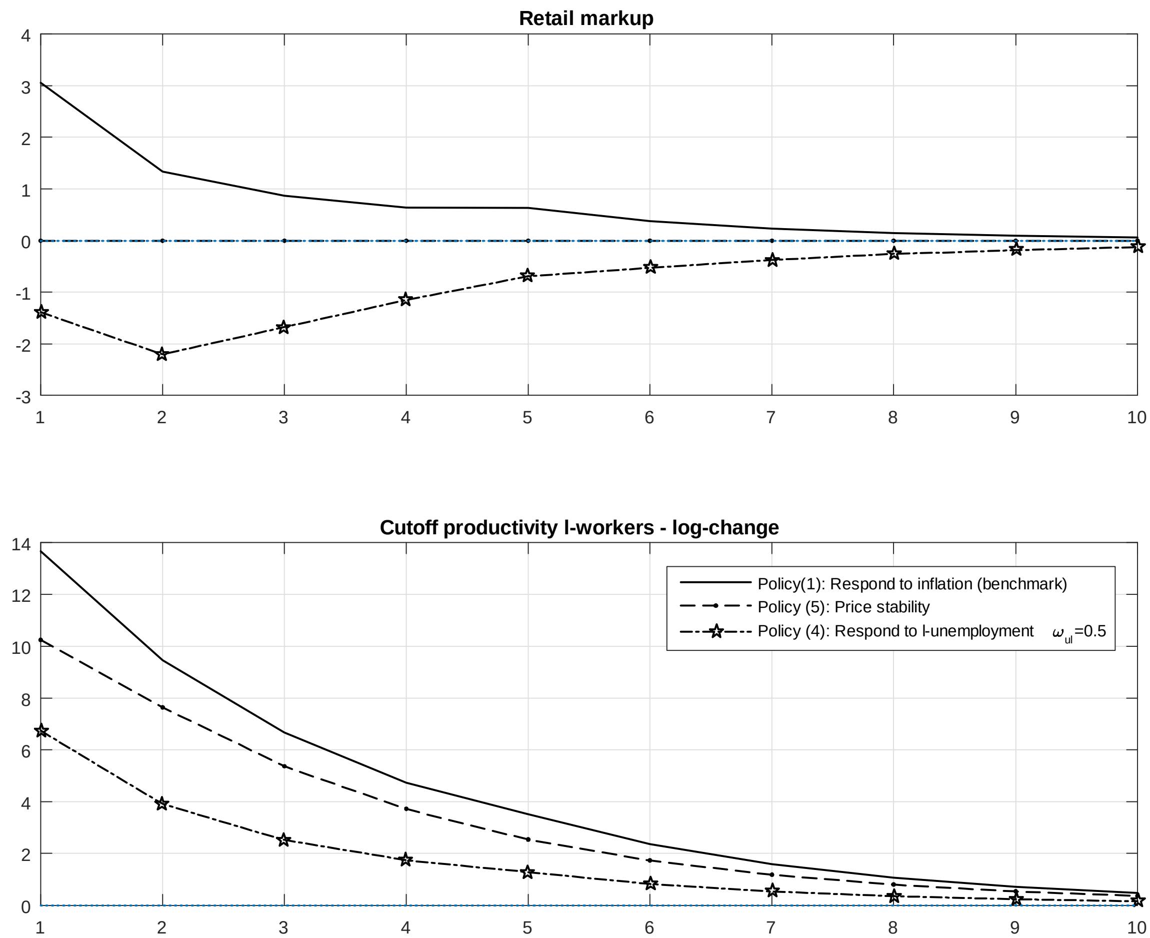

Figure 7: Temporary layoff model with alternative policies rules. Panels show the responses of μ_t (top) and $\bar{a}_t^l$ (bottom) to the pandemic shocks under different rules.

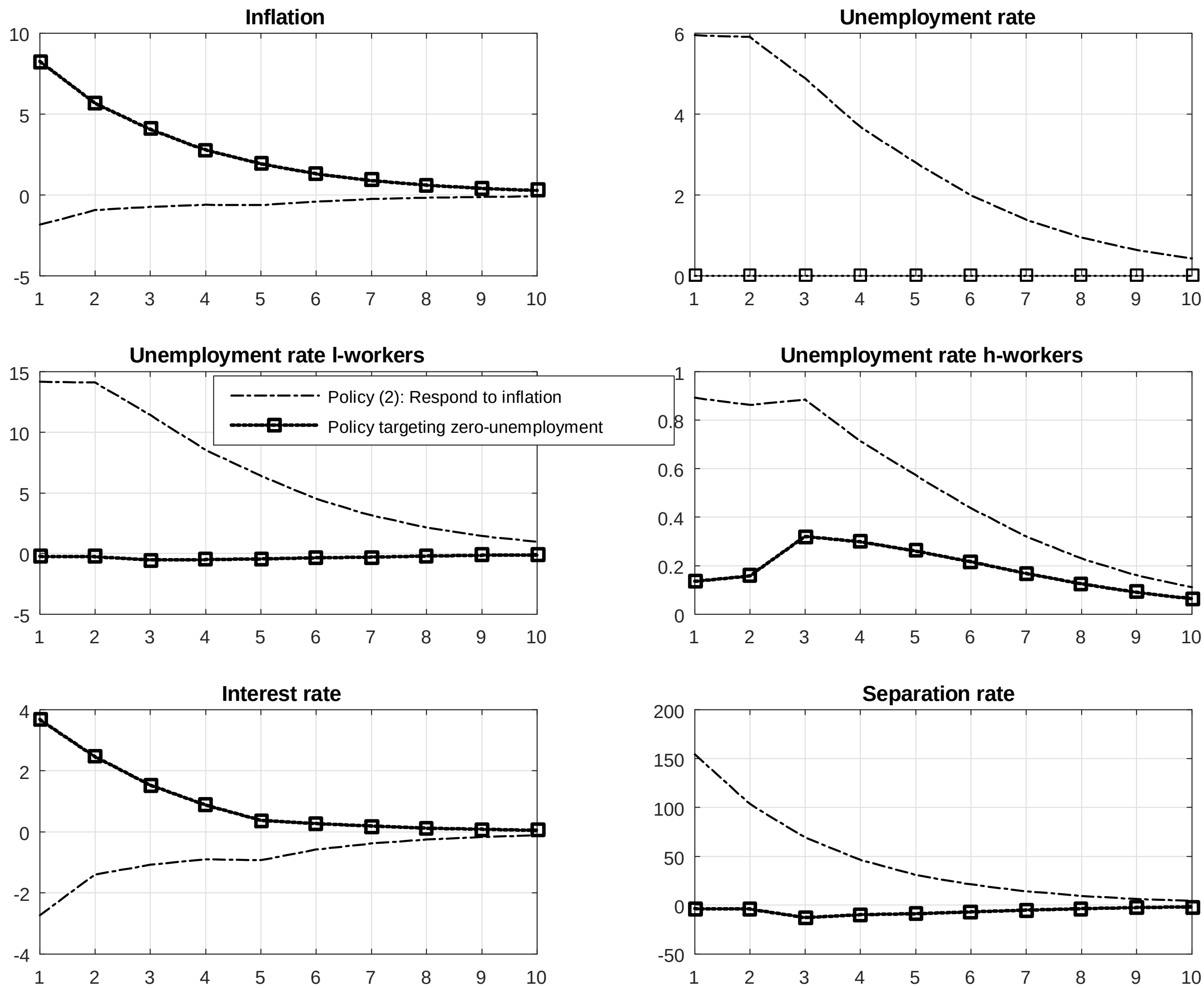

Figure 8: Temporary layoffs model. Impulse resonses to the COVID-19 shock under alternative policy rules. Inflation and interest rate are shown at quarterly rates.