

**SEIGNIORAGE AND TAX SMOOTHING IN  
THE UNITED STATES  
1914–1986\***

**Bharat TREHAN**

*Federal Reserve Bank of San Francisco, San Francisco, CA 94105, USA*

**Carl E. WALSH**

*University of California, Santa Cruz, CA 95064, USA  
Federal Reserve Bank of San Francisco, San Francisco, CA 94105, USA*

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When monetary and fiscal authorities cooperate to minimize the distortionary costs of financing an exogenous stream of government expenditures, Barro's tax-smoothing model implies a long-run relationship between tax revenues and inflation. Previous empirical tests of this relationship are reinterpreted in light of recent work on cointegration and are shown to hold only when stochastic temporal variation in the excess burden of taxes and seigniorage is transitory in nature. A new test that holds in the presence of nonstationary disturbances is developed and applied. Annual U.S. data from the period 1914 to 1986 reject the revenue-smoothing hypothesis.

## **1. Introduction**

Theoretical models of seigniorage have generally been normative in nature, building on the initial analyses of Bailey (1956) and Friedman (1969) and the theory of optimal taxation [Diamond and Mirrlees (1971)]. Friedman showed that the optimal rate of deflation equaled the real rate of return on nonmonetary assets. A policy of achieving this optimal deflation rate has revenue implications for the government's budget since base money must be retired at an appropriate rate, but Friedman's analysis assumed that the revenue required to do this could be raised through lump-sum taxation of the private sector.

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The theory of optimal taxation considers situations in which nondistortionary taxes are unavailable. In a static framework, the policy that minimizes the total excess burden of raising a fixed amount of revenue calls for equating marginal distortionary costs across all available tax instruments. Bailey (1956) and Phelps (1975) applied this result to the optimal inflation question, showing that the optimal tax policy would involve raising revenues from both taxes and seigniorage.<sup>1</sup> Barro's (1979) well-known tax-smoothing hypothesis extends the optimal taxation result along the time axis by noting that the marginal distortionary costs of raising revenues should be equated across time periods. If marginal costs are linear in the tax rates, the condition for intertemporal optimality requires that tax rate changes be unpredictable. This is the sense in which taxes are smoothed.

In Barro's analysis, it was assumed that monetary policy was not varied to generate revenues. Mankiw (1987) combined the implications of the work by Phelps and Barro to note that if both fiscal and monetary policy are used to optimally finance government expenditures, tax rates and inflation will vary together over time. We refer to this augmented version of Barro's hypothesis as the 'revenue-smoothing hypothesis'. In the face of stochastic shocks to the government's revenue needs, all revenue instruments will be adjusted in the same direction in order to maintain the equality of marginal distortionary costs.

Both Mankiw and Barro view the revenue-smoothing model as providing a positive theory of tax setting and inflation. The empirical evidence on this issue has focused on the implied contemporaneous relationship between tax rates and inflation and has tested the revenue-smoothing model under some fairly restrictive auxiliary hypotheses. In this paper, we develop a hierarchy of tests that allow us to relax several restrictive auxiliary assumptions typically employed in testing the revenue-smoothing hypothesis. We show that previous studies of the relationship between taxes and inflation have not provided adequate tests of the hypothesis, and we implement a test that allows for possibly nonstationary stochastic shifts in the excess burdens generated by alternative revenue sources.

In section 2, we develop the revenue-smoothing hypothesis and discuss the testable restrictions the theory imposes on the data. These restrictions will depend on the nature of any underlying shifts in the excess burdens associated with different revenue sources, and we consider how the appropriate tests depend on the stochastic specification of the model. Section 3 reviews the previous empirical literature on seigniorage, while section 4 contains our empirical results. Conclusions are summarized in section 5.

<sup>1</sup>Recent work [Kimbrough (1986), for example] has shown that the optimal tax on money is zero when money is used as an intermediate good in the production of transaction services. This conclusion is based on the assumption that the government has  $n - 1$  commodity taxes available to it, where  $n$  is the total number of commodities.

## 2. Revenue smoothing as a theory of seigniorage

Define the real value of the outstanding stock of interest-bearing government debt at the end of period  $t$  as  $b_t$ . For convenience we will treat this debt as if it were comprised entirely of one-period bonds yielding a constant real return of  $r$ . Let  $R = 1 + r$  be the gross interest factor. The evolution of  $b_t$  is described by

$$b_t = Rb_{t-1} + g_t - \tau_t y_t - s_t, \quad (1)$$

where  $g_t$ ,  $\tau_t y_t$ , and  $s_t$  are real noninterest government expenditures, real tax revenue, and real seigniorage. Real output is denoted by  $y$  and  $\tau$  is the average (and marginal) tax rate. Seigniorage is defined as the real value of the change in the stock of base money:

$$s_t = (M_t - M_{t-1})/p_t = m_t - m_{t-1}(p_{t-1}/p_t),$$

where  $M_t$  is the nominal stock of base money at the end of period  $t$ ,  $p_t$  is the aggregate price level at time  $t$ , and  $m_t = M_t/p_t$  is the real, end-of-period stock of base money.

If both  $\tau$  and  $s$  are set by a single policy authority, or if separate monetary and fiscal authorities act cooperatively, then one approach to a theory of seigniorage is to assume  $s$  is set on the basis of the same set of criteria emphasized by Barro (1979) in his theory of government debt. Barro argued that in the presence of distortionary costs of raising tax revenues, tax rates will be set on the basis of permanent government expenditures, with temporary fluctuations in expenditures reflected in budget deficits. Mankiw (1987) has applied this argument to seigniorage and provided some empirical evidence for the U.S. Poterba and Rotemberg (1988) focus on the role of commitment and examine a sample of O.E.C.D. countries. We will develop the implications of the revenue-smoothing hypothesis for optimal seigniorage within a version of the model of Poterba and Rotemberg (1988) modified to incorporate the role played by shifts in the relative cost of tax and seigniorage revenues. The extension allows us to develop more general tests of the revenue-smoothing hypothesis.

Taking expectations of (1) conditional on time  $t$  information and recursively solving forward yields

$$\sum R^{-i} E_t(\tau_{t+i} y_{t+i} + s_{t+i}) = Rb_{t-1} + \sum R^{-i} E_t g_{t+i}. \quad (2)$$

Note that (2) imposes the requirement of intertemporal budget balance by setting  $E_t \lim R^{-i} b_{t+i}$  equal to zero.

The distortionary costs associated with revenue generation are functions of the entire time path of the marginal tax rate  $\tau_{t+i}$  and the rate of inflation  $p_{t+i+1}/p_{t+i}$ . We assume that these costs are both time-separable in taxes and inflation and that they can be represented by constant elasticity functions. The excess burden of taxes is taken to equal  $\tau_t^{1+\alpha}\phi_t/(1+\alpha)$ , where  $\phi$  is a stochastic term that shifts the distortionary cost of tax revenues. We require  $\alpha > 0$  so that marginal costs are increasing in  $\tau$ . It will be convenient to follow Poterba and Rotemberg and write the costs of inflation in terms of the benefits of deflation:  $(p_{t-1}/p_t)^{1-\beta}\varepsilon_t/(1-\beta)$ . Marginal benefits will be decreasing in the rate of deflation as long as  $\beta > 0$ . We allow for stochastic shifts in the benefit function by including the term  $\varepsilon_t$ . The expected present discounted value of total revenue-collection costs is then given by

$$E_t \sum R^{-i} \left[ \tau_{t+i}^{1+\alpha} \phi_{t+i} / (1+\alpha) - (p_{t+i}/p_{t+i+1})^{1-\beta} \varepsilon_{t+i} / (1-\beta) \right]. \quad (3)$$

The presence of  $\phi$  and  $\varepsilon$  is meant to capture the impact of a wide variety of factors that might affect the resource costs of generating revenues. These can represent temporary or permanent shifts in labor supply or in the economy's transaction technology, or institutional changes such as the introduction of income-tax withholding or the use of a new tax instrument such as the introduction of the personal income tax in 1913.

We assume that the budget authority chooses paths for the tax rate and the rate of inflation in order to minimize the expected present discounted value of tax distortions while financing an exogenous path of real government expenditures. When the government can commit to a future path for its policy instruments, the optimal path has the property that no rearrangement of revenues either across time or between revenue sources can result in a lowering of total expected discounted costs.

Let  $d_t = b_t + m_t$  denote total government indebtedness at the start of period  $t$ . Then, variations in  $\tau_t$  and  $p_{t-1}/p_t$  that leave the path of  $d$  fixed must not lower the time  $t$  distortionary costs. Optimality requires that the marginal cost of raising an additional dollar of revenue by varying  $\tau$ ,  $\tau_t^\alpha \phi_t / y_t (1+\theta)$ , equal the marginal benefit from lowering inflation revenues by one dollar,  $(p_{t-1}/p_t)^{-\beta} \varepsilon_t / m_{t-1} (1+\mu)$ , where  $\mu$  is the elasticity of real money demand with respect to  $(p_{t-1}/p_t - R)$  and  $\theta$  is the elasticity of real income with respect to the marginal tax rate.<sup>2</sup> This optimality condition implies, together with (3), that

$$\tau_t^\alpha \phi_t / y_t (1+\theta) = (p_{t-1}/p_t)^{-\beta} \varepsilon_t / m_{t-1} (1+\mu). \quad (4)$$

<sup>2</sup>The assumption of a constant interest elasticity of money demand rules out a Laffer curve for seigniorage in which an increase in  $\pi$  would reduce revenues.

Eq. (4) summarizes the *intratemporal* optimality condition that characterizes the allocation of revenue across the two sources.

*Intertemporal* optimality requires that expected marginal distortionary revenue costs be equalized across time. This requires that the following two conditions hold:

$$E_t \tau_{t+1}^\alpha \phi_{t+1} / y_{t+1} = \tau_t^\alpha \phi_t / y_t, \quad (5)$$

$$E_t (p_t / p_{t+1})^{-\beta} \varepsilon_{t+1} / m_t = (p_{t-1} / p_t)^{-\beta} \varepsilon_t / m_{t-1}. \quad (6)$$

Taking natural logs of (4)–(6) and using a first-order Taylor approximation for (5) and (6) allows these optimality conditions to be written as

$$\begin{aligned} \ln \tau_t = & a_0 + (\beta/\alpha) \pi_t + (1/\alpha) [\ln y_t - \ln m_{t-1}] \\ & + (1/\alpha) [\ln \varepsilon_t - \ln \phi_t], \end{aligned} \quad (7)$$

$$\begin{aligned} E_t \ln \tau_{t+1} = & \ln \tau_t + (1/\alpha) [E_t \ln y_{t+1} - \ln y_t] \\ & - (1/\alpha) [E_t \ln \phi_{t+1} - \ln \phi_t], \end{aligned} \quad (8)$$

$$\begin{aligned} E_t \pi_{t+1} = & \pi_t + (1/\beta) [E_t \ln m_t - \ln m_{t-1}] \\ & - (1/\beta) [E_t \ln \varepsilon_{t+1} - \ln \varepsilon_t], \end{aligned} \quad (9)$$

where  $a_0 = (1/\alpha) \ln(1 + \theta) / (1 + \mu)$  and we have defined the rate of inflation as  $\pi_t = \ln(p_t / p_{t-1})$ .<sup>3,4</sup>

Eq. (8) yields the familiar random-walk result for the optimal tax rate when the expected growth rate of income is zero and distortionary tax costs are a time-invariant function of the tax rate ( $\phi \equiv 1$ ). More generally, if  $\ln y$  and  $\ln \phi$  are difference-stationary processes, tax-rate changes may contain a predictable component. For example, if  $\ln y_{t+1} = \ln y_t + u_{t+1} - k u_t$  while  $\ln \phi_{t+1} = \ln \phi_t + v_{t+1}$  and  $u$  and  $v$  white-noise processes, then  $E_t \ln \tau_{t+1} = \ln \tau_t - (1/\alpha) k u_t$ . A positive  $u_t$  signals slower real income growth next period, so the optimal path of taxes calls for higher current taxes relative to next-period taxes.

<sup>3</sup>It is common in dealing with dynamic optimization problems to directly estimate the Euler conditions; if these take the form  $E_t f(x_{t+1}) = f(x_t)$ , where  $x$  is a decision variable, one estimates  $f(x_{t+1}) = f(x_t) + u_{t+1}$ , where  $u_{t+1} = f(x_{t+1}) - E_t f(x_{t+1})$ . Due to the presence of  $\varepsilon_{t+1}$  and  $\phi_{t+1}$ , however, the realized values of the left-hand side of both (5) and (6) are unobservable.

<sup>4</sup>As is clear from (7), we can view  $[\ln \varepsilon - \ln \phi]$  as capturing the effects of any stochastic variation in  $\theta$  and  $\mu$  as well as shifts in the distortionary costs associated with seigniorage and taxes.

Optimal revenue smoothing implies that the rate of inflation and the tax rate should exhibit similar behavior. Eq. (9) shows that  $\pi$  will be nonstationary, with its first difference a function of growth in real money holdings and expected changes in the distortionary costs of inflationary finance.

The intratemporal relationship between inflation and taxes is given by eq. (7). Expenditure changes that effect  $\ln \tau$  also move  $\pi$  in the same direction. Increases in permanent government expenditures are financed by raising additional revenue from all sources, so both  $\tau$  and  $\pi$  are increased. The optimal trade-off between revenue sources depends on their relative tax bases ( $\ln y_t - \ln m_{t-1}$ ) and their relative costs in terms of current economic distortions ( $\ln \varepsilon_t - \ln \phi_t$ ).

If the income velocity of money is constant and the distortionary cost functions are time-invariant, (7) reduces to

$$\ln \tau_t = a'_0 + (\beta/\alpha)\pi_t, \quad (10)$$

where  $a'_0 = a_0 + (1/\alpha)k$  and  $k$  is the natural log of velocity. As stated, the theory predicts an  $R^2$  of one for eq. (10). However, it seems reasonable to believe that both inflation and taxes will be subject to short-term disturbances that will cause some slippage in the relationship. For example, if  $\ln \phi$  and  $\ln \varepsilon$  follow stationary moving-average processes, then (7) becomes

$$\ln \tau_t = a'_0 + (\beta/\alpha)\pi_t + A(L)e_t, \quad (11)$$

where  $A(L)e_t = (1/\alpha)[\ln \varepsilon_t - \ln \phi_t]$ .

Since (8) and (9) imply that both  $\ln \tau$  and  $\pi$  are integrated of order one, eq. (11) implies that  $\ln \tau$  and  $\pi$  are cointegrated with cointegrating vector  $(1 - \beta/\alpha)$ . I.e., the linear combination of  $\ln \tau$  and  $\pi$  given by  $\ln \tau_t - (\beta/\alpha)\pi_t$  is a stationary stochastic process. Revenue smoothing implies that both  $\ln \tau$  and  $\pi$  contain unit roots; however, since both are responding to shifts in permanent government spending, they will contain a common stochastic trend.

Eq. (11) was derived under the assumption that velocity remained constant. When this is not the case, (11) becomes

$$\ln \tau_t = a_0 + (\beta/\alpha)\pi_t + (1/\alpha)[\ln y_t - \ln m_{t-1}] + A(L)e_t. \quad (12)$$

As long as velocity follows a stationary stochastic process, the theoretical prediction of cointegration between  $\pi$  and  $\ln \tau$  will continue to hold. However, a variety of researchers have suggested that velocity is in fact nonstationary.<sup>5</sup> If this is the case, eq. (12) continues to predict a cointegrating relationship,

<sup>5</sup>See, for example, Gould et al. (1978).

this time between the three variables  $\pi$ ,  $\ln \tau$ , and  $\ln y_t - \ln m_{t-1}$ . This implication of the theory also holds if  $\ln \phi$  and  $\ln \varepsilon$  contain deterministic trends.

Finally, if either  $\ln \varepsilon$  or  $\ln \phi$  is not stationary around a deterministic trend, eq. (7) no longer implies any cointegrating relationships among inflation, tax rates, or velocity. When distortionary costs are subject to permanent shifts, the testable implications of the theory then involve restrictions across the joint processes for  $\pi$  and  $\ln \tau$  implied by the entire set of first-order conditions given by (7)–(9). These restrictions arise because the coefficients in all three equations are functions solely of  $\alpha$  and  $\beta$ .

Thus, we interpret the revenue-smoothing hypothesis as suggesting a hierarchy of testable implications about the behavior of inflation. First, the rate of inflation should be nonstationary in its level. This follows directly from (9). Second, under the auxiliary hypotheses that velocity and shifts in relative distortionary costs are trend-stationary, inflation and the log tax rate should be cointegrated. Third, if velocity is difference-stationary, inflation and the tax variable will not be cointegrated, but a cointegrating relationship will exist among inflation, the log of the tax rate, and velocity. Rejection of this cointegrating relationship is still consistent with the revenue-smoothing hypothesis if the shocks in the cost functions contain permanent components. A final test of the model is obtained by testing the cross-equation restrictions implied by the theory. This is the most general test in that we can allow for unit roots in the processes for  $\ln \phi$  and  $\ln \varepsilon$ .

### 3. Review of previous empirical literature

The existing empirical literature on the relationship between fiscal policy, usually summarized by the fiscal deficit, and inflation or seigniorage is voluminous. The bulk of this literature has been atheoretical in nature and has focused on the estimation of monetary-policy reaction functions. These attempt to determine whether the Federal Reserve tends to monetize large budget deficits. Such an outcome might represent the direct response of the Fed to deficits or might result from Fed attempts to prevent interest-rate increases generated by the deficits.

In general, researchers using post-WWII data from the U.S. have not found strong evidence to suggest that fiscal deficits and money growth are linked. For example, for the period 1954–1983, Joines (1985) found no statistically significant effect of nonwar deficits or war spending on base money growth. This is consistent with the findings of King and Plosser (1985) who attempt to distinguish between the reaction-function approach and fiscal dominance in the sense of Sargent and Wallace (1981). The reaction-function approach argues that the Fed can pursue independent policies in the short run, but that base money growth may be positively related to deficits because the Fed either reacts directly to deficits or attempts to stabilize interest rates that would

otherwise rise in the face of deficits. Under fiscal dominance, monetary policy is determined residually from eq. (2) after the fiscal authority has set the paths for taxes and expenditures. Thus a pattern of fiscal deficits implies revenues to finance expenditures must eventually be raised via seigniorage. King and Plosser conclude that the data seem consistent with either view.

One problem with this literature is that the post-WWII period may not exhibit sufficient variation to cast much light on the seigniorage–deficit relationship. Joines (1985) examines the deficit–base money growth relationship over the period 1915 to 1953 and finds that nonwar-related deficits had only transitory effects on base money growth. War spending, however, did have a significant positive effect on the growth rate of the monetary base.

The literature on direct tests of the tax-smoothing hypothesis is more limited. Kochin et al. (1985), using data over the 1929–1979 period, found that they could not reject the random-walk implication of the tax-smoothing hypothesis; that is, they could not reject the hypothesis that changes in the tax rate are unpredictable.<sup>6</sup> By contrast, Sahasakul (1986) found evidence suggesting the opposite, using data over the period from 1937 to 1982. However, as we have shown in section 2 above, predictability of tax-rate changes is not inconsistent with the tax-smoothing hypothesis. In Trehan and Walsh (1988), we suggested that appropriate tests of the tax-smoothing hypothesis should focus on the long-run relationships between government spending and taxes, since various other factors could influence the short-run relationships. It is not sufficient to demonstrate that the tax-rate process is a random walk (or more generally, that it contains a unit root); instead, it is necessary to show that any permanent component of tax changes is related to the permanent component of government spending. We showed that the data easily rejected any such long-run relationship between the two variables for the 1890–1986 period. However, using cointegration tests to test the tax-smoothing hypothesis requires that certain auxiliary hypotheses hold; for instance, it is necessary that the process governing the evolution of tax-collection costs be stationary.

Mankiw (1987) tested the revenue-smoothing hypothesis by estimating a version of eq. (7) (the intratemporal first-order condition), implicitly assuming that velocity is constant and the cost functions are time-invariant. Using data over the 1951–1985 period, Mankiw regressed the rate of inflation on tax rates (in levels and first differences) and obtained a statistically significant positive coefficient.<sup>7</sup>

<sup>6</sup> However, they used only a limited set of variables, consisting of outstanding government debt, the level of government expenditures, and the size of the government deficit, to forecast tax rate changes.

<sup>7</sup> Mankiw's specification of excess burden leads to a positive relationship between  $\pi$  and  $\tau$ . Unfortunately, his measure of tax rates (federal government receipts divided by GNP) includes seigniorage revenue, since prior to 1982 Federal Reserve interest earnings returned to the Treasury were included in federal tax receipts.



The specification of the revenue-smoothing hypothesis that Mankiw uses implies that his equation is a cointegrating relationship, so the correct procedure is to employ the levels version of the regression and test the residuals for stationarity.<sup>8</sup> However, a basic implication of the theory is that both inflation and (log) tax rates should be nonstationary. Thus, before attempting any interpretation of an equation such as (11) in terms of the theory of cointegrated processes, it is necessary to determine the order of integration of the individual inflation and tax-rate series.

Poterba and Rotemberg (1988) extend Mankiw's analysis by allowing velocity to vary over time in estimating eq. (12). They also examine the post-war data for a variety of OECD countries and, for the U.S., extend the sample period back to 1890. For the U.S., a statistically significant positive relationship between inflation and tax rates is found only in the post-war period. Poterba and Rotemberg do not test  $\pi$  and  $\ln \tau$  for stationarity nor do they test the cointegrating implications of the theory.<sup>9</sup>

Of the other countries Poterba and Rotemberg examine, only the post-war Japanese data yield a positive relationship between inflation and taxes. However, if the error term in (7) is nonstationary, OLS estimates of equations of the type reported by both Mankiw and Poterba and Rotemberg are likely to yield inconsistent estimates of  $\beta/\alpha$  (the coefficient on the inflation rate in the tax-rate equation), due to the correlation between  $\pi_t$  and the error term  $(1/\alpha) [\ln \varepsilon_t - \ln \phi_t]$  in (7). A positive realization of  $\ln \varepsilon_t$ , for example, leads to a reduction in seigniorage. Since permanent government expenditures are unaffected, the revenue mix shifts toward taxes, generating a negative correlation between  $[\ln \varepsilon_t - \ln \phi_t]$  and  $\pi_t$ . The resulting downward bias in the OLS coefficient estimates may partially explain some of the insignificant and negative coefficient estimates Poterba and Rotemberg report.<sup>10</sup>

#### 4. Empirical results

Our focus is on the long-term relationship between taxes and inflation as summarized by the cointegrating implications of the revenue-smoothing hypothesis. Consequently, the nature of the statistical tests we carry out indicates that we work with long sample periods. Our data set consists of annual

<sup>8</sup>Mankiw's reported Durbin-Watson statistic is 0.67 which would allow the null hypothesis of no cointegration to be rejected [Engle and Granger (1987)]. Because Mankiw's level regressions include a time trend, however, critical values reported by Engle and Granger may not apply.

<sup>9</sup>The U.S. data Poterba and Rotemberg use includes both pre- and post-Federal Reserve years. The introduction of the Fed in 1914 represents a major institutional change, and in our empirical work, we focus solely on the period since 1914.

<sup>10</sup>This bias will not be present if  $[\ln \varepsilon_t - \ln \phi_t]$  is stationary while  $\tau_t$  and  $\pi_t$  are nonstationary and cointegrated. See West (1988).

Table 1  
Tests for unit roots.<sup>a</sup>

	Phillips	Said–Dickey	Stock–Watson
A. Sample period: 1914–1986			
Gov't expenditure <sup>b</sup>	-2.3	-2.0	-5.7
Tax rate	-2.6 <sup>c</sup>	-2.1	-3.6
Inflation	-4.4 <sup>e</sup>	-2.3	-7.5
Velocity	-1.0	-1.7	-2.9
B. Sample period: 1914–1947			
Gov't expenditure <sup>b</sup>	-1.1	-1.1	-9.0
Tax rate	-1.4	-0.6	-1.4
Inflation	-2.7 <sup>c</sup>	-0.1	-2.5
Velocity	-0.2	-0.4	-2.1
C. Sample period: 1948–1986			
Gov't expenditure <sup>b</sup>	-3.1 <sup>c</sup>	-1.6	-8.5
Tax rate	-4.9 <sup>c</sup>	-0.5	-1.4
Inflation	-2.3	-1.7	-3.9
Velocity	-6.1 <sup>c</sup>	-3.1 <sup>d</sup>	-0.9

<sup>a</sup>All variables except the inflation rate are in natural logs.

<sup>b</sup>Equation for government expenditures allows for a linear trend. All others allow for a nonzero mean.

<sup>c</sup>Significant at 10%.

<sup>d</sup>Significant at 5%.

<sup>e</sup>Significant at 1%.

observations over the period 1914–1986.<sup>11</sup> We have excluded the pre-Federal Reserve period on the grounds that the institutional change represented by the introduction of the Fed altered the ability of the Federal government to collect seigniorage.<sup>12</sup> To allow comparison with earlier studies (that have mostly focused on the post-war period), we also present results for two subsamples spanning the years 1914–1947 and 1948–1986. However, the results from these subsamples should be treated with some caution, since they may be too short to permit reliable inference.

Table 1 gives the results of unit root tests for  $\ln g_t$ ,  $\ln \tau_t$ ,  $\pi_t$ , and  $\ln(y_t/m_{t-1})$ . Since tests for unit roots often lead to conflicting results, we present the results from three different tests: the Phillips (1987) test, the Said–Dickey (1984) test,

<sup>11</sup>Data on real GNP and the GNP deflator for the 1914–1928 period are from Balke and Gordon (1989), while the rest are from NIPA sources. Our average tax rate equals adjusted nominal federal receipts divided by nominal GNP. Data on High Powered Money before 1975 is from Friedman and Schwartz (1982); we have followed their methodology to extend these data to 1986. The rest of the data are described in Trehan and Walsh (1988).

<sup>12</sup>Mankiw and Miron (1986) have shown that the introduction of the Fed had the immediate effect of altering the time-series properties of nominal rates of interest. They find that short-term nominal rates were random walks after 1914. This evidence suggests that the post-Fed era may well reflect revenue-smoothing behavior by the federal government.

and the Stock–Watson (1989) test. Schwert (1987) points out that unit-root tests are adversely affected by the presence of moving-average errors, and suggests using long lag lengths to avoid this problem. Accordingly, we use his  $l_{12}$  formula to calculate lag lengths for all three tests; this leads to a lag length of eleven for our full sample, and a lag length of nine for each of the subsamples.

The Phillips test results for the 1914–86 period are shown in column 1 of table 1, panel A. Over the entire sample period, we cannot reject the null hypothesis of a unit root in the government expenditures variable at the 10% level even after allowing for a linear time trend. However, the null of a unit root in the  $\ln \tau_t$  process can be rejected at the 10% level, and at the 1% level for the  $\pi_t$  process. The strong rejection of nonstationarity in the inflation process is troublesome for the revenue-smoothing hypothesis since it implies that permanent changes in government expenditures did not lead to correspondingly permanent changes in inflation over our sample period.

While the Phillips test is robust to the presence of conditional heteroscedasticity, Schwert points out that the test rejects the null hypothesis of a unit root too often when the process under consideration contains moving-average errors. Schwert shows that a procedure suggested by Said and Dickey (1984) is much less susceptible to this problem, and the results from this test are presented in column 2 of table 1. On the basis of this test, we are unable to reject the null of a unit root in either  $\ln \tau_t$ ,  $\pi_t$ , or  $\ln g_t$ . The conflict between the Phillips and Said–Dickey tests in the case of the inflation process led us to employ the procedure developed by Stock and Watson (1989). Column 3 of table 1 shows that the results of the Stock–Watson test agree with those of the Said–Dickey test.

The lower two panels show the results for the subsamples. There is some conflict between the tests here as well. For instance, the Phillips test strongly rejects nonstationarity of the tax rate over the post-war period, while the other two do not. Similarly, the Phillips test also rejects nonstationarity of the pre-war inflation process at the 10% level, while the other two tests fail to do so. Nevertheless, for each of the three variables under consideration, at least two out of the three tests fail to reject nonstationarity. While we tentatively interpret this evidence as implying that the univariate representations of the tax rate and the inflation rate are consistent with the revenue-smoothing model, the sensitivity of the results to the specific test employed significantly weakens the support this evidence gives to the model.

Nonstationarity of both processes is necessary, but not sufficient, for the revenue-smoothing hypothesis. The hypothesis also implies specific restrictions on the multivariate representations of  $\ln \tau$  and  $\pi$ . We now present a series of tests to examine whether these restrictions are satisfied, successively relaxing a series of auxiliary constraints on the behavior of velocity and the excess burdens generated by alternative revenue sources.

If one is willing to assume that velocity and the distortionary costs of tax collection are stationary, the revenue-smoothing hypothesis implies that  $\ln \tau$  and  $\pi$  are cointegrated. We test for cointegration using the Augmented Dicky–Fuller test [discussed in Engle and Granger (1987)] and the Stock–Watson test. A regression of  $\ln \tau_t$  on  $\pi_t$  over the 1914–1986 period leads to a coefficient of 0.02 on  $\pi_t$ , with an adjusted  $R^2$  of only 0.03. The computed value of the Augmented Dicky–Fuller (ADF) statistic is  $-1.6$ , well below the 10% critical value of  $-2.9$ .<sup>13</sup> The computed value of the Stock–Watson (SW)  $q_f(2, 1)$  statistic is  $-5.5$ , compared to the 10% critical value of  $-19.5$ . When the equation is estimated over the 1914–1947 period, we obtain an ADF statistic of  $-1.0$  and a SW statistic of  $-2.4$ . Similarly, for the post-war period (i.e., 1948–1986) we obtain an ADF statistic of  $-1.63$  and a SW statistic of  $-2.5$ . Thus, in no case do we even come close to rejecting the null hypothesis that the two-variable system is driven by two stochastic trends in favor of the hypothesis of a common trend as implied by the revenue-smoothing hypothesis.

Following the discussion in section 2 above, we now allow velocity to be nonstationary. The revenue-smoothing hypothesis then implies that  $\ln \tau_t$ ,  $\pi_t$ , and  $\ln(y_t/m_{t-1})$  are cointegrated. As a preliminary, the last row of panel A in table 1 shows that none of the three tests reject the null that  $\ln(y_t/m_{t-1})$  is nonstationary over the 1914–1986 period. Thus, permanent shifts in velocity have the potential to explain the lack of cointegration between taxes and inflation. For the 1914–1986 sample, the estimated cointegrating regression corresponding to eq. (12) is

$$\ln \tau_t = -2.41 + 0.02\pi_t + 0.06\ln(y_t/m_{t-1}), \quad (13)$$

with an adjusted  $R^2$  of 0.02, an ADF statistic of  $-1.69$ , and a SW statistic of  $-7.46$ . Estimating this equation over the two subsamples also does not lead to any evidence suggesting that the three variables are cointegrated.<sup>14</sup>

Finally, consider what happens when tax-collection costs are subject to nonstationary stochastic shifts. First, if  $[\ln \varepsilon_t - \ln \phi_t]$  is stationary (i.e.,  $\ln \varepsilon$  and  $\ln \phi$  are cointegrated), then the error in eq. (7) is stationary,  $\ln \tau_t$ ,  $\pi_t$ , and  $\ln(y_t/m_{t-1})$  should be cointegrated, and (13) is still the relevant regression for testing the revenue-smoothing hypothesis. If  $[\ln \varepsilon_t - \ln \phi_t]$  is stationary around a deterministic trend, testing for a cointegrating relationship can still be carried out in a straightforward manner. We tested for this possibility using the Stock–Watson test for the three-variable system consisting of  $\ln \tau_t$ ,  $\pi_t$ , and

<sup>13</sup>The number of lags used in the ADF and the SW tests were also determined using Schwert's  $l_{12}$  formula.

<sup>14</sup>For the 1914–1947 period, we obtain an ADF test statistic of  $-2.0$  (while the 10% critical value is  $-3.4$ ) and a SW statistic of  $-3.9$  (the 10% critical value is  $-19.5$ ). Over 1948–1986, we obtain an ADF statistic of  $-1.5$  and a SW statistic of  $-6.2$ .

$\ln(y_t/m_{t-1})$ .<sup>15</sup> For the 1914–1986 period, the computed value of the SW statistic was  $-9.20$ , which is not significant even at the 95% level. Neither of the SW statistics for the two subsamples is significant at the 50% level.

Matters become considerably more complicated if the trend in  $[\ln \varepsilon_t - \ln \phi_t]$  is stochastic. Excluding this term from estimated versions of (7) implies that it is no longer a cointegrating regression. Thus, cointegration tests can no longer provide evidence on the revenue-smoothing hypothesis. Our solution is to jointly estimate versions of (7) and (8) and to test whether the cross-equation restrictions implied by the theory are satisfied, since estimating (7) alone is no longer sufficient to test the model.<sup>16</sup>

More specifically, we rewrite (7) and (8) in first-difference form and estimate

$$\Delta \pi_t = (\alpha/\beta) \Delta \ln \tau_t - (1/\beta) \Delta \ln(y_t/m_{t-1}) + v_{1t}, \quad (7')$$

$$\Delta \ln \tau_t = (1/\alpha) \Delta \ln y_t + v_{2t}, \quad (8')$$

where the tax-collection costs are now contained in the residual terms of these equations. Specifically,  $v_{1t} = -(1/\beta) \Delta [\ln \varepsilon_t - \ln \phi_t]$  and  $v_{2t} = -(1/\alpha) \times [E_{t-1} \ln \phi_t - \ln \phi_{t-1}] - (1/\alpha) [\ln y_t - E_{t-1} \ln y_t] + [\ln \tau_t - E_{t-1} \ln \tau_t]$ . Under the assumption that  $\ln \varepsilon$  and  $\ln \phi$  are integrated of order at most equal to one, both  $v_{1t}$  and  $v_{2t}$  will be stationary.

Notice that it is necessary to use instrumental-variable techniques to estimate (7') and (8'), because the residuals in these equations will be correlated with all the explanatory variables. We have already shown that shocks to  $[\ln \varepsilon_t - \ln \phi_t]$  will alter both  $\ln \tau_t$  and  $\pi_t$ . Similarly, changes in distortionary costs will also affect both output and money holdings, and  $v_{2t}$  contains a forecast-error term that is correlated with  $\ln y_t$ . In addition,  $v_{1t}$  and  $v_{2t}$  are likely to be serially correlated. For example, even in the simplest case where  $[\ln \varepsilon_t - \ln \phi_t]$  is white noise,  $v_{1t}$  will contain a first-order moving-average term. Consequently, we use Hansen's (1982) Generalized Methods of Moments technique to jointly estimate (7') and (8') above. We assume innovations to  $\ln \varepsilon$  and  $\ln \phi$  are uncorrelated with lagged output and velocity.

We use lagged values of the regressors in these equations as instruments. Further, we assume that the first difference of  $[\ln \varepsilon_t - \ln \phi_t]$  is at most a fourth-order moving-average process, so we use the fifth and sixth lags of the regressors as instruments. Our set of instruments also includes the contemporaneous and one lagged value of the change in noninterest government expenditures ( $\Delta \ln g_t$ ), since the exogeneity of these expenditures is a maintained hypothesis of the revenue-smoothing model.

<sup>15</sup>For both the Engle–Granger and Stock–Watson tests, the appropriate procedure involves detrending the variables before carrying out the tests.

<sup>16</sup>We estimate only two of the first-order conditions (7)–(9), since the third can be derived from the other two.

The test we used is based on a comparison of the residuals from the restricted and unrestricted versions of (7') and (8'), and is described in Amemiya (1985). Unrestricted estimation of this system leads to

$$\begin{aligned}\Delta\pi_t &= \underset{(0.01)}{0.06} \Delta \ln \tau_t - \underset{(0.19)}{0.36} \Delta \ln(y_t/m_{t-1}), \\ \Delta \ln \tau_t &= \underset{(0.12)}{0.99} \Delta \ln y_t,\end{aligned}$$

where the standard errors are shown in parentheses. Thus, unrestricted instrumental-variables estimation does lead to a statistically significant positive relationship between  $\pi_t$  and  $\ln \tau_t$ .

We next estimated the two equations subject to the nonlinear restrictions shown in (7') and (8') above. This gave us an estimate of 0.70 for  $\alpha$  (with a standard error of 0.09) and 4.98 for  $\beta$  (the standard error is 3.29), which suggests that the elasticity of distortionary costs with respect to inflation is considerably higher than the elasticity with respect to taxes. Implementing the test described above leads to a rejection to the restriction implied by the revenue-smoothing hypothesis – the computed value of the  $\chi^2(1)$  statistic is 20.3, which is significant at the 1% level.

In contrast to this rejection for the whole sample period, we are unable to reject the restrictions for either the 1914–1947 or the 1948–1986 periods at the 5% level. (For the earlier period, the hypothesis can be rejected at the 10% level.) However, the inability to discriminate between the restricted and unrestricted versions seems to reflect the inability of either version to explain the data. For example, while we are unable to clearly reject the hypothesis over the 1914–1947 period, unrestricted estimation of (7') over this period leads to a significant, negative coefficient on the tax variable, directly contradicting the hypothesis. Only the post-war period appears to provide some support for the revenue-smoothing hypothesis, although even here the support is tempered by the low explanatory power of the model.<sup>17</sup>

## 5. Conclusions

In this paper we have derived and tested the restrictions obtained from a fairly general version of a model in which monetary and fiscal authorities cooperate to minimize the distortionary costs of taxation. We have shown that if both taxes and inflation are being determined by revenue smoothing considerations, they both should be nonstationary. In our empirical analysis we find that while the evidence generally supports the unit-root implications, it is by no means unambiguous. In any event, the existence of nonstationarity by itself does not prove that the hypothesis under consideration is correct; we

<sup>17</sup>The low power is not the result of our choice of instrumental variables. OLS estimation of (7') over the post-war period leads to an adjusted  $R^2$  of only 0.03.

also need to show that nonstationarity arises because of revenue-smoothing considerations. We showed that these considerations would imply cointegration between inflation and the log of the tax rate – assuming that velocity and the distortionary costs of raising revenue are stationary. The data reject this implication of the theory. A possible cause of this rejection may be that velocity is nonstationary. We showed that in this case there should exist at least one linear combination of inflation, the log of the tax rate and the log of velocity that is stationary. The data reject this implication of the theory as well.

Finally, when collection costs were assumed to contain stochastic trends we did find a positive short-run relationship between inflation and taxes. However, the set of variables suggested by the revenue-smoothing hypothesis explained very little of the variation in inflation. Further, the revenue-smoothing hypothesis implies restrictions that go beyond the existence of a positive relationship between movements in inflation and taxes. These restrictions are clearly rejected over the entire sample period (1914–1986). While we were unable to reject the hypothesis over the post-war period, the low explanatory power of the variables indicated by the theory suggests that revenue-smoothing considerations have not been significant elements in determining the behavior of seigniorage in the U.S.

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