

## Welfare-Based Optimal Monetary Policy with Unemployment and Sticky Prices: A Linear-Quadratic Framework<sup>†</sup>

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*We derive a linear-quadratic model that is consistent with sticky prices and search and matching frictions in the labor market. We show that the second-order approximation to the welfare of the representative agent depends on inflation and “gaps” that involve current and lagged unemployment. Our approximation makes explicit how welfare costs are generated by the presence of search frictions. These costs are distinct from the costs associated with relative price dispersion and fluctuations in consumption that appear in standard new Keynesian models. We show the labor market structure has important implications for optimal monetary policy. (JEL E24, E31, E52)*

The steep increases in unemployment associated with the financial crisis and global recession of 2008–2009, and the widespread focus on unemployment in both the popular press and in policy debates, is in sharp contrast to the canonical new Keynesian model in which unemployment is noticeably absent. In that model, workers are never unemployed, and only hours worked per worker vary over the business cycle. As a consequence, the basic new Keynesian (NK) model cannot shed light on whether monetary policy should respond to the unemployment rate or whether there is a role for stabilizing unemployment fluctuations that is distinct from stabilizing fluctuations in inflation and the consumption gap as in standard NK models.

Our objective in this paper is to explore the implications for monetary policy of a model with sticky prices and search-based unemployment. We first show how such a model can be reduced to a linear expectational-IS curve and a Phillips curve linking inflation and the gap between unemployment and its efficient level. The coefficients in these two relationships depend on the underlying structural parameters of the model that govern preferences, the degree of nominal price rigidity, and the search and bargaining processes in the labor market.

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We then derive a second-order approximation to the welfare of the representative household and show that, in addition to the standard inflation and consumption gap terms, a new term appears that involves labor market tightness. This new term captures all the welfare costs associated with labor market search inefficiency. In a standard new Keynesian model, inflation leads to an inefficient composition of market consumption because of the dispersion of relative prices inflation causes. Employment fluctuations can lead to an inefficient composition of total consumption between market-produced consumption goods, for which production requires employment matches that incur search costs, and home-produced consumption goods, for which production does not. This inefficiency is distinct from the inefficient composition of market consumption generated by inflation and so results in an additional objective in the loss function. The first best is attained when both inflation and the gap between unemployment and its efficient level are always equal to zero. However, because labor market frictions introduce a new state variable, optimal policy involves smoothing a quasi-difference in the level of this unemployment gap. Thus, neither the level of unemployment nor simply the level of the unemployment gap correctly measures the appropriate objective of monetary policy.

In a standard NK model, fluctuations in employment, consumption, and output all move in proportion to one another relative to their flexible-price counterparts. Thus, fluctuations in welfare can be expressed equivalently in terms of any one of these variables, together with inflation volatility. With search frictions, this equivalence does not hold, so the unemployment gap term we obtain in the welfare approximation cannot be replaced with a consumption gap term. Each gap plays a distinct role in affecting welfare, and by developing a quadratic approximation to welfare, we obtain explicit expressions for the relative weight on each, and can assess how this weight varies with structural characteristics of the labor market. Besides affecting the goals of the policy maker, search frictions also change the monetary transmission mechanism by adding a cost channel for the interest rate along with the traditional demand channel.<sup>1</sup>

Given the linear representation of the structural equations and a model-consistent quadratic loss function, the framework can be used to study monetary policy issues in the same way the standard NK model has been used. In light of the empirical evidence from DSGE models with labor market search frictions for the United States (Luca Sala, Ulf Söderström, and Antonella Trigari 2008) and for the Euro area (Kai Christoffel, Keith Kuester, and Tobias Linzert 2009), we allow for stochastic fluctuations in the relative bargaining power of workers and firms. This shock distorts the flexible-price equilibrium and generates policy trade-offs in our model, just as cost shocks do in standard NK models.

In a basic NK model, cost-push shocks can lead to large losses if the central bank pursues a single-minded focus on price stability. We find, however, that if cost-push shocks reflect random fluctuations in the relative bargaining power of workers and firms, price stability is nearly optimal. The reason is closely related to the argument

<sup>1</sup>A cost channel arises when firms' marginal cost depends directly on the interest rate as, for example, in Lawrence J. Christiano, Martin Eichenbaum, and Charles L. Evans (2005). The policy implications of a cost channel in a model without labor market frictions are discussed in Federico Ravenna and Walsh (2006).

made by Marvin Goodfriend and Robert G. King (2001) that the long-term nature of employment relationships reduces the welfare costs of temporary deviations of the contemporaneous marginal product of labor from the marginal rate of substitution between leisure and consumption. With efficient bargaining, but fluctuations in bargaining shares, price stability remains close to the optimal policy.

We find that a policy designed to minimize volatility in inflation and in the level of the unemployment gap—policy objectives used in some of the existing literature—targets the wrong measure of search inefficiency and can produce a significant reduction in welfare.<sup>2</sup> And, in contrast to the results obtained in the staggered price and wage adjustment model of Christopher J. Erceg, Dale W. Henderson, and Andrew T. Levin (2000), a simple Taylor rule results in a welfare loss that is much higher than that achieved under the optimal policy. In fact, the backward-looking policy rule estimated by Richard Clarida, Jordi Galí, and Mark Gertler (2000) for the Volcker-Greenspan era generates welfare loss equal to nearly 1.5 percent of steady-state consumption compared to essentially no loss under a policy of price stability.

A growing number of papers have incorporated unemployment into NK models. Examples include Arnaud Chéron and François Langot (1999), Walsh (2003, 2005); Christoffel, Kuester, and Linzert (2009); Olivier J. Blanchard and Galí (2007); Michael U. Krause and Thomas A. Lubik (2007), Ester Faia (2008); Krause, David J. López-Salido, and Lubik (2008); Ravenna and Walsh (2008); Sala, Söderström, and Trigari (2008); Carlos Thomas (2008); Gertler, Sala, and Trigari (2008); Gertler and Trigari (2009); and Trigari (2009). The focus of these earlier contributions has extended from exploring the implications for macro dynamics in calibrated models to the estimation of DSGE models with labor market frictions. Sala, Söderström, and Trigari (2008) evaluate monetary policy trade-offs and optimal policy in an estimated model with matching frictions in the labor market, but they use an ad hoc quadratic loss function rather than the model consistent welfare approximation we derive.

The papers closest in motivation to ours are Blanchard and Galí (2010) and Thomas (2008). Both of these papers make specific assumptions about how the wage-setting process generates inefficient fluctuations in the way the surplus from an employment match is shared between the worker and the firm. Our approach does not take a stand on the sources of these fluctuations, and instead assumes they are exogenous, a strategy already pursued by Robert Shimer (2005). Many authors, including Blanchard and Galí (2010), henceforth BG, and Thomas (2008), have assumed these fluctuations reflect some form of real wage rigidity, but the role of wage stickiness in accounting for macroeconomic fluctuations is a topic of active debate. Shimer (2005) demonstrated that matching models with wages set by Nash bargaining cannot generate the level of unemployment volatility seen in the data, and imposing wage rigidity increases the volatility of unemployment. However, Christian Haefke, Marcus Sonntag, and Rens (2008) and Christopher A. Pissarides

<sup>2</sup>While we focus on optimal policy, the presence of labor market search frictions also affects some of the standard properties of simple Taylor rules. For example, the conditions for determinacy do not generally satisfy the so-called Taylor principle. See Takushi Kurozumi and Willem Van Zandweghe (2008) for an analysis of determinacy in a model that is quite similar in structure to the model we develop here.

(2009) conclude that wage stickiness does not explain the unemployment volatility puzzle. To highlight the implications of search frictions in a model that is otherwise well known, we follow the standard NK model and do not impose constraints on wage adjustment. Instead, the stochastic fluctuations in worker-firm bargaining shares we assume can also be interpreted as deviations of the real wage from its efficient level, and so capture some of the same effects generated by assuming real wage rigidity.<sup>3</sup>

There are other differences between the model we employ and those developed by BG (2010) and Thomas (2008). In contrast to the Mortensen-Pissarides search model we employ (Dale T. Mortensen and Pissarides 1994), BG (2010) assume firms face hiring costs that are increasing in the degree of labor market tightness (measured as new hires relative to unemployment). BG (2010) assume offsetting income and substitution effects on labor supply, implying unemployment remains constant in the face of productivity shocks when prices are flexible. This implies that monetary policy should focus on stabilizing the *level* of unemployment, as well as inflation. Our model allows unemployment to fluctuate under flexible prices, but because productivity causes the efficient level of unemployment to fluctuate, the appropriate objective of policy is an unemployment *gap* that is more comparable to the output gap appearing in standard NK models. We also find that both the current unemployment gap and its lagged value are relevant for welfare; because of search frictions, the number of unemployed workers at the end of the previous period is an endogenous state variable.

In addition, the search and matching framework is, in our view, better able to link labor market characteristics to macroeconomic behavior than the hiring costs approach used by BG (2010). For example, the roles of vacancies, job turnover, unemployment benefits, and job-finding probabilities are explicit in our model. The welfare approximation in BG (2010) also relies on the assumption that hiring costs are of second order in magnitude, an assumption we can dispense with.

Thomas (2008) incorporates convex costs of posting vacancies with staggered real wage adjustment and derives a quadratic welfare approximation in terms of squared deviations of variables from their steady-state values. In contrast, our approach, besides yielding an expression for the welfare loss that is simpler in form, shows explicitly how each variable appearing in the objective function can be expressed in terms of a squared deviation from its *efficient* level. This helps to highlight that policy involves stabilizing real variables around time-varying efficient levels, not constant steady-state levels, and that optimal policy involves closing gaps.

Two further issues merit brief discussion before beginning our analysis. First, all the existing literature that incorporates unemployment into models with nominal rigidities has assumed households are able to insure against idiosyncratic consumption risk. Thus, an agent's consumption is independent of employment status. We too follow the literature in making this assumption. A full understanding of the welfare costs of unemployment will undoubtedly require a recognition of heterogeneity

<sup>3</sup>The implications of wage rigidity for optimal policy are discussed in Ravenna and Walsh (2009).

and imperfect consumption insurance. Doing so is beyond the scope of the present paper, but it is clearly an important topic for future research.

Second, even in the context of a model that deviates from the basic NK model along one dimension, the derivation of a second-order approximation to the welfare of the representative agent expressed in terms of efficiency gaps becomes quite complex. In our view, the benefits of such a derivation outweigh the costs as the linear-quadratic approach has proven immensely useful in providing insights relevant for monetary policy design. For example, the approach has helped highlight the role of distortions in affecting the relative weight placed on inflation versus output volatility, clarified the definition of output around which actual output should be stabilized, and facilitated the analysis of optimal policy design.<sup>4</sup>

The rest of the paper is organized as follows. Section I presents the basic model, derives a log-linearized version of the model, and discusses the connections between labor market structure and the Phillip curve. The model-consistent welfare approximation and optimal policy are studied in Section II. The impact of labor market structure on optimal policy is investigated in Section III, while conclusions are summarized in Section IV.

## I. The Model Economy

The model consists of households whose utility depends on the consumption of market and home produced goods, firms that employ labor to produce a wholesale good which is sold in a competitive market, and retail firms that transform the wholesale good into differentiated final goods sold to households in an environment of monopolistic competition. The labor market is characterized by search frictions. Households members are either employed (in a match) or searching for a new match. Retail firms adjust prices according to a standard Calvo specification. The modelling strategy of locating labor market frictions in the wholesale sector where prices are flexible and locating sticky prices in the retail sector among firms who do not employ labor provides a convenient separation of the two frictions in the model. A similar approach was adopted in Walsh (2003, 2005), Ravenna and Walsh (2008), Thomas (2008), and Trigari (2009).

### A. Final Goods

The demand for final goods arises from two sources: households who purchase retail goods to form a consumption bundle and wholesale firms who must employ real resources to recruit and hire workers.

*Households.*—Households consist of a large number of members who can be either employed by wholesale firms in production activities or unemployed. In the former case, they receive a market real wage  $w_t$ ; in the latter case, they receive a fixed amount  $w^u$  of household production units. As is standard in the literature on

<sup>4</sup>Michael Woodford (2003) develops the linear-quadratic approach and illustrates its value for policy analysis.

matching frictions, we assume that consumption risks are fully pooled. The household's instantaneous utility at time  $t$  is given by the preference specification

$$U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma},$$

where total consumption  $C_t$  consists of market goods  $C_t^m$  and home production  $w^u(1 - N_t)$ :

$$(1) \quad C_t = C_t^m + w^u(1 - N_t),$$

where  $N_t$  is the number of household members employed during the period. Market consumption is an aggregate of goods purchased from the continuum of retail firms, indexed by  $j \in [0, 1]$ , that produce differentiated final goods:

$$C_t^m \leq \left[ \int_0^1 C_t^m(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

Intratemporal optimal choice across goods implies

$$(2) \quad C_t^m(j) = \left[ \frac{P_t(j)}{P_t} \right]^{-\varepsilon} C_t^m, \text{ where } P_t \equiv \left[ \int_0^1 P_t(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}.$$

Households maximize expected discounted utility, and the intertemporal first order condition for the households' decision problem yields the standard Euler equations:

$$(3) \quad \lambda_t = \beta E_t \left\{ i_t \frac{P_t}{P_{t+1}} \lambda_{t+1} \right\},$$

where  $i_t$  is the gross return on an asset paying one unit of currency in any state of the world, and  $\lambda_t \equiv C_t^{-\sigma}$  is the marginal utility of consumption.

*Wholesale Firms.*—Firms in the wholesale sector produce output using labor through the production function  $Y_t^w = Z_t N_t$ , where  $Z_t$  is an exogenous stationary productivity shock common to all firms. The production process also requires firms to pay a per period cost to post employment vacancies. To post  $v_t$  vacancies, wholesale firms buy individual final goods  $v_t(j)$  from each  $j$  final-goods-producing retail firm subject to the constraint

$$(4) \quad \left[ \int_0^1 v_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} \geq v_t.$$

Total expenditure on job posting costs is given by

$$\kappa \int_0^1 P_t(j) v_t(j) dj,$$

which wholesale firms minimize subject to (4) for any choice of  $v_t$ , where  $\kappa$  is the cost per vacancy. The demand by wholesale firms for the final goods produced by retail firm  $j$  is given by

$$(5) \quad v_t(j) = \left[ \frac{P_t(j)}{P_t} \right]^{-\varepsilon} v_t,$$

and at the optimum, the cost to keeping a vacancy open in period  $t$  can be written as  $P_t \kappa$ .

Total expenditure on final goods by households and wholesale firms is

$$\int_0^1 P_t(j) C_t^m(j) dj + \kappa \int_0^1 P_t(j) v_t(j) dj = \int_0^1 P_t(j) Y_t^d(j) dj = P_t (C_t^m + \kappa v_t),$$

where  $Y_t^d(j) \equiv C_t^m(j) + \kappa v_t(j)$  is total demand for final good  $j$ .

*Retail Firms.*—Retail firms purchase wholesale output at price  $P_t^w$  in a competitive market. The wholesale good is then converted into a differentiated final good that is sold to households and wholesale firms. Retail firms maximize profits subject to a CRS technology for converting wholesale goods into final goods, the demand functions (2) and (5), and a restriction on the frequency with which they can adjust their price.

Each period a retail firm can adjust its price with probability  $1 - \omega$ . A firm that can adjust its price in period  $t$  chooses  $P_t(j)$  to maximize

$$\sum_{i=0}^{\infty} (\omega\beta)^i E_t \left[ \left( \frac{\lambda_{t+i}}{\lambda_t} \right) \left( \frac{(1 + \tau)P_t(j) - P_{t+i}^w}{P_{t+i}} \right) Y_{t+i}(j) \right]$$

subject to

$$(6) \quad Y_{t+i}(j) = Y_{t+i}^d(j) = \left[ \frac{P_t(j)}{P_{t+i}} \right]^{-\varepsilon} Y_{t+i}^d,$$

where  $Y_t^d$  is aggregate demand for the final goods basket, and we assume the firm's output is subsidized at the fixed rate  $\tau$ . This subsidy will be employed when we wish to ensure the steady-state equilibrium is efficient. The real marginal cost for retail firms is the price of the wholesale good relative to the price of final output,  $P_t^w/P_t$ . The standard pricing equation obtains which, when linearized around a zero-inflation steady state yields a NK Phillips curve in which the retail price markup  $\mu_t \equiv P_t/P_t^w$  is the driving force for inflation. As in a standard Phillips curve, the elasticity of inflation with respect to real marginal costs will be  $\delta \equiv (1 - \omega)(1 - \beta\omega)/\omega$ .



*Market Clearing.*—Goods market clearing requires that household consumption of market produced goods, plus final goods purchased by wholesale firms to cover the costs of posting job vacancies, equal the output of the retail sector:

$$(7) \quad Y_t = C_t^m + \kappa v_t = C_t - w^u(1 - N_t) + \kappa v_t,$$

where  $v_t$  is the aggregate number of vacancies posted.

### B. Wholesale Goods, Employment and Wages

The labor market is characterized by search frictions. At the beginning of each period  $t$  a share  $\rho$  of the matches  $N_{t-1}$  that produced output in period  $t - 1$  breaks up. Workers not in a productive match at  $t - 1$  or who do not survive the exogenous separation hazard at the start of the period seek new matches.<sup>5</sup> Thus, the number of job seekers in period  $t$  is

$$(8) \quad u_t \equiv 1 - (1 - \rho)N_{t-1}.$$

Note that  $u_t$  is a predetermined variable as of time  $t$ .<sup>6</sup> Unemployed workers are matched stochastically with job vacancies. The matching process is represented by a CRS matching function

$$(9) \quad m_t = m(u_t, v_t) = \chi v_t^\alpha u_t^{1-\alpha} = \chi \theta_t^\alpha u_t,$$

where  $\theta_t \equiv v_t/u_t$  is the measure of labor market tightness, and  $0 < \alpha < 1$ . The number of matches that produce in period  $t$  is

$$(10) \quad N_t = (1 - \rho)N_{t-1} + m(u_t, v_t).$$

To hire workers, wholesale firms must post vacancies. Given that the size of the firm is indeterminate with constant returns to scale, we can focus on the firm's decision to hire a marginal worker. With free entry, the value of a vacancy is zero in equilibrium. This so-called job posting condition implies that the expected value of a filled job will equal the cost of posting a vacancy, or

$$(11) \quad q_t J_t = \kappa,$$

where  $J_t$  is the real value of a filled job and  $q_t \equiv m_t/v_t$  is the probability a firm with a vacancy will fill it. The value of a filled job is also equal to the firm's current period

<sup>5</sup>By incorporating only a constant rate of exogenous separations, we follow most of the literature that has embedded labor search into monetary policy models. There is, of course, an active debate on the relative importance of endogenous fluctuations in unemployment inflows and outflows at business cycle frequencies; see Steven J. Davis and John C. Haltiwanger (1992); Shimer (2007); and Michael W. L. Elsby, Ryan Michaels, and Gary Solon (2009). For a monetary model with endogenous job destruction, see Walsh (2003, 2005).

<sup>6</sup>We take the number of job seekers,  $u_t$ , as our measure of unemployment. The standard measure of unemployment would more closely match the number of workers not in a match at the end of the period,  $1 - N_t$ . The two are related since  $u_{t+1}$  is equal to  $1 - N_t$  plus the number of exogenous separations  $\rho N_t$ .



profit plus the discounted value of having a match in the following period. If a job produces output  $Z_t$  and  $w_t$  is the wage paid to the worker, then the value of a filled job in terms of final goods is

$$(12) \quad J_t = \left( \frac{P_t^w}{P_t} \right) Z_t - w_t + (1 - \rho) \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \right) J_{t+1}$$

or

$$(13) \quad \frac{Z_t}{\mu_t} = w_t + \frac{\kappa}{q_t} - (1 - \rho) \beta E_t \left( \frac{1}{R_t} \right) \left( \frac{\kappa}{q_{t+1}} \right),$$

where  $R_t^{-1} \equiv \beta(\lambda_{t+1}/\lambda_t)$  is the stochastic discount factor, and both wages and vacancies are measured in terms of the retail goods basket. The left side of (13) is the marginal product of a worker. The right side is the marginal cost of a worker to the firm. In the absence of labor market frictions, this cost would just be the real wage, and one would have  $Z_t/\mu_t = w_t$ , or  $1/\mu_t = w_t/Z_t$ ; this corresponds to the standard NK model, where the real marginal cost variable that drives inflation is the real wage divided by labor productivity. With labor market frictions, additional factors come into play. According to (13), the cost of labor also includes the search costs associated with hiring ( $\kappa/q_t$ ) and the discounted recruitment cost savings if an existing employment match survives into the following period.

The real wage appears in (13). A standard approach allowing for flexible wages is to assume Nash bargaining between firms and workers in which each participant receives a fixed share of the total surplus. In this case, Shimer (2005) pointed out that the real wage responds strongly to productivity shocks, leaving unemployment much less volatile than in the data. In light of the “Shimer puzzle,” many authors have introduced some form of real wage rigidity (see for example Robert E. Hall 2005, Gertler and Trigari 2009). Since our objective is to develop a simple framework that parallels the basic NK model yet incorporates unemployment, we follow the literature that assumes Nash bargaining over wages. This choice is consistent with the assumption of flexible wages underlying the basic NK model and allows a straightforward comparison of the policy implications of the two frameworks. We deviate from the standard assumption of fixed bargaining weights, however, by allowing the division of a match surplus to vary stochastically.<sup>7</sup>

For a worker, if  $p_t \equiv m_t/u_t$  denotes the job finding probability of an unemployed worker, the valuation equation for being in a match that produces in period  $t$  is

$$V_t^E = w_t + \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \right) \left\{ (1 - \rho) V_{t+1}^E + \rho [p_{t+1} V_{t+1}^E + (1 - p_{t+1}) V_{t+1}^U] \right\},$$

since a matched worker survives the exogenous separation hazard with probability  $1 - \rho$ , is exogenously separated with probability  $\rho$  but finds another match with

<sup>7</sup>Christoffel, Kuester, and Linzert (2009) show that in an estimated DSGE model of the Euro area with search friction, bargaining shocks play a significant role in output and inflation fluctuations, both in absolute terms and relative to other labor market disturbances. In our model, where wages are Nash-bargained in every period, bargaining shocks increase the volatility of employment relative to output.

probability  $p_{t+1}$ , and fails to find a match with probability  $1 - p_{t+1}$ . The valuation equation for being unmatched is

$$V_t^U = w^u + \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \right) [(1 - p_{t+1})V_{t+1}^U + p_{t+1}V_{t+1}^E],$$

since, with probability  $1 - p_{t+1}$ , the worker fails to find a match, and, with probability  $p_{t+1}$ , the worker finds a match. Thus, the surplus value of a match to a worker is

$$(14) \quad V_t^S \equiv V_t^E - V_t^U = w_t - w^u + \beta(1 - \rho)E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \right) (1 - p_{t+1})V_{t+1}^S.$$

Let  $b_t$  denote the worker's share of the job surplus in period  $t$ , where  $b_t$  is assumed to follow a stationary stochastic process. Under Nash bargaining, the sharing rule implies

$$(15) \quad (1 - b_t)V_t^S = b_t J_t = b_t \left( \frac{\kappa}{q_t} \right).$$

Using (15) to eliminate  $V_t^S$  from (14) yields the following expression for the wage:

$$(16) \quad w_t = w^u + \left( \frac{b_t}{1 - b_t} \right) \left( \frac{\kappa}{q_t} \right) - (1 - \rho)E_t \left( \frac{1}{R_t} \right) (1 - p_{t+1}) \left( \frac{b_{t+1}}{1 - b_{t+1}} \right) \left( \frac{\kappa}{q_{t+1}} \right).$$

Substituting (16) into (13), one finds that the relative price of wholesale goods in terms of retail goods is equal to

$$(17) \quad \frac{P_t^w}{P_t} = \frac{1}{\mu_t} = \frac{\xi_t}{Z_t},$$

where  $\xi_t$  is the effective cost of labor and is defined as

$$(18) \quad \xi_t \equiv w^u + \left( \frac{1}{1 - b_t} \right) \left( \frac{\kappa}{q_t} \right) - (1 - \rho)E_t \left( \frac{1}{R_t} \right) \left( \frac{1 - b_{t+1}p_{t+1}}{1 - b_{t+1}} \right) \left( \frac{\kappa}{q_{t+1}} \right).$$

Labor market tightness affects inflation through  $\xi_t$ . A rise in labor market tightness reduces  $q_t$ , the probability a firm fills a vacancy, and raises the value of a filled job ( $\kappa/q_t$ ). This increases wages in the wholesale sector and raises wholesale prices relative to retail prices. The resulting rise in the marginal cost of the retail firms and fall in the retail price markup increases inflation. Expectations of greater labor market tightness in the future increase the expected cost of hiring in the future. This increases the value of existing matches, since, with probability  $1 - \rho$ , an existing match survives to the following period and eliminates the need to incur future job posting costs. Hence, an increase in the expected cost of future job postings lowers the effective cost of current labor matches. This reduces wholesale prices relative to retail prices and lowers retail price inflation. Finally, because it is the discounted value of expected future labor market conditions that affects the firm's decision to post an extra vacancy, there is a cost channel effect, as the real interest rate has a direct impact on  $\xi_t$ , and therefore on inflation.

A rational expectations equilibrium satisfies (3), the optimal retail pricing condition, (7)–(10), (13), (16)–(18), and the definitions of  $\theta_t$ ,  $q_t$ ,  $p_t$ ,  $R_t$ , and  $\lambda_t$  described in the text. These equations jointly determine  $Y_t$ ,  $C_t$ ,  $\pi_t$ ,  $N_t$ ,  $u_t$ ,  $v_t$ ,  $m_t$ ,  $w_t$ ,  $\mu_t$ ,  $\xi_t$ ,  $\theta_t$ ,  $q_t$ ,  $p_t$ ,  $R_t$ ,  $\lambda_t$ , and the nominal interest rate  $i_t$  once a specification of monetary policy is added.

### C. The Linearized Model

In this section, we derive a log-linear approximation of the rational expectations equilibrium around the efficient steady state. We then show that the log-linearized model can be reduced to a system of two equilibrium conditions that correspond to the new Keynesian expectational IS and Phillips curves, expressed in terms of unemployment and inflation rather than in terms of output and inflation.

Let  $\hat{x}_t$  denote the log deviation of a variable  $x$  around its steady-state value  $\bar{X}$ , let  $\hat{x}_t^e$  be the stochastic, efficient equilibrium value of  $\hat{x}_t$ , and let  $\tilde{x}_t \equiv \hat{x}_t - \hat{x}_t^e$  denote the efficiency gap for  $\hat{x}_t$ , i.e., the gap between  $\hat{x}_t$  and its stochastic, efficient equilibrium counterpart. In the first step, we use the goods market clearing condition, the production function, and the labor market conditions to express consumption in terms of unemployment. Goods market clearing requires that

$$Y_t = C_t - w^u(1 - N_t) + \kappa v_t$$

as output is used for market consumption (total consumption minus home production, or  $C_t - w^u(1 - N_t)$ ) and vacancy posting costs. Log linearizing this condition yields

$$(19) \quad \hat{y}_t = \left(\frac{\bar{C}}{\bar{Y}}\right)\hat{c}_t + w^u\hat{n}_t + \left(\frac{\kappa\bar{v}}{\bar{Y}}\right)(\hat{\theta}_t + \hat{u}_t),$$

where use has been made of the fact that  $\hat{v}_t = \hat{\theta}_t + \hat{u}_t$ . From the CRS production function,  $\hat{y}_t = \hat{n}_t + z_t$ , so (19) implies

$$(20) \quad \left(\frac{\bar{C}}{\bar{Y}}\right)\hat{c}_t = (1 - w^u)\hat{n}_t + z_t - \left(\frac{\kappa\bar{v}}{\bar{Y}}\right)(\hat{\theta}_t + \hat{u}_t).$$

Log linearizing (8), which links the number of employed workers and the number of job seeking workers, and (10), which governs the evolution of employment, yields

$$(21) \quad \hat{u}_t = -\eta\hat{n}_{t-1}, \text{ where } \eta \equiv (1 - \rho)\left(\frac{\bar{N}}{\bar{u}}\right)$$

and

$$\hat{n}_t = (1 - \rho)\hat{n}_{t-1} + \alpha\rho\hat{\theta}_t + \rho\hat{u}_t.$$

These two equations then imply

$$(22) \quad \hat{u}_{t+1} = \rho_u\hat{u}_t - \alpha\rho\eta\hat{\theta}_t,$$

where  $0 < \rho_u \equiv (1 - \rho)(1 - \rho\bar{N}/\bar{u}) < 1$ ; higher labor market tightness reduces unemployment as more job seekers find employment matches. Combining (21) and (22) with (20) gives

$$(23) \quad \hat{c}_t = \varphi_1 \hat{u}_{t+1} - \varphi_2 \hat{u}_t + \left( \frac{\bar{Y}}{\bar{C}} \right) z_t,$$

where  $\varphi_1 \equiv -(\bar{Y}/\eta\bar{C})[1 - w^u - (\kappa\bar{v}/\alpha\rho\bar{Y})]$  and  $\varphi_2 \equiv (\kappa\bar{v}/\alpha\rho\eta\bar{C})(\alpha\rho\eta + \rho_u)$ .

Since the representative household's optimal consumption plan will satisfy a standard log-linearized Euler condition, equation (23) can be used to eliminate  $\hat{c}_t$  and obtain an Euler condition expressed in terms of the current and lagged number of job seekers, the real interest rate, and current and expected future productivity:

$$(24) \quad \hat{u}_{t+1} = \left( \frac{1}{\varphi_1 + \varphi_2} \right) \times \left[ \varphi_1 E_t \hat{u}_{t+2} + \varphi_2 \hat{u}_t - \left( \frac{1}{\sigma} \right) (i_t - E_t \pi_{t+1}) + \left( \frac{\bar{Y}}{\bar{C}} \right) (E_t z_{t+1} - \hat{z}_t) \right].$$

The online Appendix shows that at an efficient steady state,  $\varphi_1/(\varphi_1 + \varphi_2) = \beta/(1 + \beta)$ , so subtracting the stochastic efficient equilibrium conditions to express variables in terms of gaps and letting  $\tilde{r}_t \equiv \tilde{i}_t - E_t \pi_{t+1}$ , the Euler condition takes the form

$$(25) \quad \tilde{u}_{t+1} = \left( \frac{\beta}{1 + \beta} \right) E_t \tilde{u}_{t+2} + \left( \frac{1}{1 + \beta} \right) \tilde{u}_t - \left( \frac{1}{\hat{\sigma}} \right) \tilde{r}_t,$$

where  $\hat{\sigma} = \sigma(1 + \beta)(\kappa\bar{v}/\alpha\bar{C})(\alpha - 1 + \bar{U}/\rho\bar{N})$ .

In a standard NK model, the Euler condition is forward looking, containing no lagged endogenous variables. Often, the optimal monetary policy literature assumes habit persistence on the part of households, resulting in a lagged output gap term in the IS relationship. In our model,  $\tilde{u}_t$ , which is predetermined at time  $t$ , appears because search frictions cause equilibrium production to be affected by the number of workers who enter the period without matches or are displaced from existing matches. This leads to the presence of a backward-looking component in the IS relationship when expressed in terms of unemployment, even with standard household preferences. The weights on  $E_t \tilde{u}_{t+2}$  and  $\tilde{u}_t$  in (25) are, respectively,  $\beta/(1 + \beta)$  and  $1/(1 + \beta)$ , each approximately equal to one-half.

Given the Calvo-specification for price adjustment, the linearized Phillips curve takes the standard form given by

$$\pi_t = \beta E_t \pi_{t+1} - \delta \hat{\mu}_t,$$

since the marginal cost for retail firms is  $\mu_t^{-1}$ . Equation (17) implies  $\hat{\mu}_t = z_t - \hat{\xi}_t$  and (18) can be linearized to allow  $\hat{\xi}_t$  to be written in terms of current and expected

labor market tightness, the real interest rate, and the bargaining disturbance. The retail price markup  $\hat{\mu}_t$  can then be expressed as

$$(26) \quad \hat{\mu}_t = z_t - \hat{\xi}_t = z_t - A(1 - \alpha)\hat{\theta}_t + A\beta(1 - \rho)[1 - \alpha - b\theta q(\theta)] \\ \times E_t\hat{\theta}_{t+1} - A\beta(1 - \rho)[1 - b\theta q(\theta)]\hat{r}_t - B\hat{b}_t,$$

where  $A \equiv \mu\kappa/(1 - b)q(\theta)$ ,  $B \equiv A[b/(1 - b)][1 - \beta(1 - \rho)(1 - p)\rho_b]$ , and we have assumed  $\hat{b}_t$  follows an AR(1) process with serial correlation coefficient  $\rho_b$ . A rise in labor market tightness increases wages and reduces the retail price markup, increasing the marginal cost of retail firms. This leads to a rise in inflation. For a given  $\hat{\theta}_t$ , a rise in  $E_t\hat{\theta}_{t+1}$  increases the markup  $\hat{\mu}_t$  and reduces current inflation.<sup>8</sup> Expectations of future labor market tightness imply a lower expected future job filling probability, raising the expected cost of filling a job in the future. This increases the value of an existing match and reduces the effective cost of labor (see (18)). This fall in the labor costs of wholesale firms reduces wholesale prices relative to retail prices and lowers the marginal cost of retail firms. A rise in the real interest rate lowers the present value of the future vacancy cost savings associated with an existing match, increases the effective cost of labor, and increases wholesale prices relative to retail prices. This leads to a rise in inflation. Finally, a rise in the bargaining power of workers raises labor costs and wholesale prices relative to retail prices, leading to a fall in the retail price markup.

To obtain a Phillips curve in terms of unemployment gaps, we use (22) to express  $\hat{\theta}$  in terms of  $\hat{u}_{t+1}$  and  $\hat{u}_t$ , and then use (26) to obtain

$$(27) \quad \pi_t = \beta E_t\pi_{t+1} + (\delta a_1/\alpha\rho\eta)[\rho_u\beta(E_t\tilde{u}_{t+2} - \rho_u\tilde{u}_{t+1}) - (\tilde{u}_{t+1} - \rho_u\tilde{u}_t)] \\ + \beta\delta a_3\tilde{r}_t + \delta B\hat{b}_t,$$

where

$$a_1 = [(1 - \alpha)/(1 - b)](\kappa\bar{v}/\rho\bar{N}) \\ a_2 = a_1[(1 - \rho)/(1 - \alpha)](1 - \alpha - \rho\bar{N}/\bar{u}) \\ a_3 = a_1[(1 - \rho)/(1 - \alpha)](1 - b\rho\bar{N}/\bar{u}).$$

A final simplification is obtained if (25) is used to eliminate  $E_t\tilde{u}_{t+2}$  from (27), yielding

$$(28) \quad \pi_t = \beta E_t\pi_{t+1} - a_1\delta\left[\frac{(1 - \beta\rho_u)(1 - \rho_u)}{\alpha\rho\eta}\right]\tilde{u}_{t+1} \\ + \delta\left[\beta a_3 + \left(\frac{a_1\rho_u}{\sigma\alpha\rho\eta}\right)\right]\tilde{r}_t + \delta B\hat{b}_t.$$

<sup>8</sup> In our baseline calibration,  $1 - \alpha = b$ , so  $1 - \alpha - b\theta q(\theta) = (1 - \alpha)[1 - \theta q(\theta)] > 0$ .

Equation (28) is isomorphic to a NK Phillips curve with an unemployment rate gap replacing an output gap and with a cost channel operating through the real rate of interest rather than through the nominal rate as in Ravenna and Walsh (2006).<sup>9</sup>

## II. Optimal Monetary Policy

To study optimal monetary policy, we assume the monetary authority's objective is to maximize the expected present discounted value of the utility of the representative household. A rich and insightful literature has developed from the initial contributions of Julio J. Rotemberg and Woodford (1997) and Woodford (2003) employing policy objectives based on a second order approximation to the welfare of the representative agent. As is well known, the appropriate welfare approximation depends on the exact structure of the model. In this section, we discuss the quadratic objective function that arises in our model with sticky prices and labor market frictions. Mathematical details are provided in the online Appendix.

### A. The Quadratic Approximation to Welfare

Efficiency requires that three conditions hold: prices must be flexible so that the markup is constant; the fiscal subsidy  $\tau$  must ensure the steady-state markup equals 1; and the Arthur J. Hosios (1990) condition must hold ( $b = 1 - \alpha$ ). The second order approximation to welfare when the steady state is efficient is

$$(29) \quad \sum_{i=0}^{\infty} \beta^i U(C_{t+i}) = \frac{U(\bar{C})}{1 - \beta} - \frac{\varepsilon}{2\delta} U_c \bar{C} \sum_{i=0}^{\infty} \beta^i L_{t+i} + t.i.p.$$

where *t.i.p.* denotes terms independent of policy, and the period-loss function is

$$(30) \quad L_t = \pi_t^2 + \lambda_0 \tilde{c}_t^2 + \lambda_1 \tilde{\theta}_t^2,$$

where  $\lambda_0 = \sigma(\delta/\varepsilon)$  and  $\lambda_1 = (1 - \alpha)(\delta/\varepsilon)(\kappa\bar{v}/\bar{C})$ . The weight on  $\tilde{c}_t^2$  is exactly the same as that obtained in a standard NK model if utility is linear in hours worked. That is, in the basic NK model, the relative weight on the output gap in the loss function is, in terms of the present notation,  $\delta(\sigma + \eta_N)/(1 + \eta_N\varepsilon)\varepsilon$ , where  $\eta_N$  is the inverse of the wage-elasticity of labor supply (see Woodford 2003 or Walsh 2010, 386). If  $\eta_N = 0$ , one obtains  $\sigma\delta/\varepsilon$ , which is the value of  $\lambda_0$  in (30).

To understand this loss function, recall that in a standard NK model, utility is reduced by inefficient volatility of consumption, yet inflation also reduces utility because it generates a dispersion of relative prices that leads to an inefficient composition of consumption. That is, even if total consumption is equal to its efficient

<sup>9</sup>Note that conditional on  $\tilde{r}_t$  and  $\tilde{u}_{t+1}$ , the IS relationship (25) implies that  $\tilde{u}_t + \beta E_t \tilde{u}_{t+2}$  must be constant. A higher value of  $\tilde{u}_t$ , again conditional on  $\tilde{u}_{t+1}$ , implies greater labor market tightness  $\tilde{\theta}_t$ , as vacancies must be higher to prevent the higher  $\tilde{u}_t$  from leading to a rise in  $\tilde{u}_{t+1}$ . Greater labor market tightness in period  $t$  raises real marginal cost at  $t$  and would tend to increase inflation. But at the same time,  $\beta E_t \tilde{u}_{t+2}$  must be lower to maintain  $\tilde{u}_t + \beta E_t \tilde{u}_{t+2}$  constant, consistent with the Euler condition. The fall in  $\beta E_t \tilde{u}_{t+2}$  implies an increase in expected future labor market tightness, and this acts to lower inflation. The two effects exactly offset leaving inflation independent of lagged unemployment.

level, up to first order, the composition of consumption across individual goods is inefficient in the presence of inflation. Because of diminishing marginal utility with respect to leisure, inefficient fluctuations in hours also reduce welfare in the standard NK model. However, from the aggregate production function, hours can be expressed in terms of consumption so that total loss can be written solely as a function of inflation volatility and consumption (output) volatility.

The standard distortion arising from inflation is also present in the model with labor search frictions. Therefore, as in the NK model, welfare is decreasing in inflation volatility. And because of diminishing marginal utility, volatility of consumption reduces welfare. The marginal disutility of working is constant in our framework, but to transfer workers from home production to market production involves the matching function, which is characterized by diminishing marginal productivity with respect to labor market tightness as long as  $0 < \alpha < 1$ . The costs of job posting rise more when vacancies increase than they fall when vacancies decrease. Thus, volatility in vacancies relative to their efficient level reduces welfare and accounts for the separate term in labor market tightness that appears in the loss function (30).<sup>10</sup>

Even if inflation is zero, so that market consumption is obtained through an efficient combination of the differentiated market goods, the composition of total consumption between market goods and home production can be inefficient if vacancy postings, and thus the aggregate cost of search (equal to the wedge between output and market consumption), deviate from their efficient value. This result does not hinge on our particular specification of home production or search frictions (as long as they are not linear), but simply on the fact that an alternative way of generating utility (home production) is available to unemployed agents, and this alternative does not suffer from the search friction necessary to produce matches and market consumption. This source of inefficient resource allocation would continue to be present if the model were extended to allow for variable hours in production and disutility in hours worked. Expressed alternatively,  $\tilde{c}_t^2$  captures the welfare loss due to fluctuations in total consumption when households are risk averse, while  $\pi_t^2$  represents losses arising from an inefficient composition of market goods, and  $\tilde{\theta}_t^2$  represents losses due to an inefficient composition of market and nonmarket consumption goods.

In (30), the weight on  $\tilde{\theta}$  gap volatility relative to consumption gap volatility is equal to  $(1 - \alpha)\kappa\bar{v}/\bar{C}$ . Rewriting this as  $(1 - \alpha)(\bar{C}^m/\bar{C})(\kappa\bar{v}/\bar{C}^m)$  shows that as vacancy costs associated with producing market consumption rise or market consumption's share of total consumption rises, the welfare cost of  $\theta$ -gap fluctuations increases. From the matching function,  $\alpha$  is the elasticity of the value of a filled job with respect to  $\theta$ . If  $\alpha = 1$ , the matching technology displays constant returns to scale with respect to  $\theta$ , and volatility in  $\tilde{\theta}$  does not generate a welfare loss. However, for  $0 < \alpha < 1$ , the matching function is characterized by decreasing returns to  $\theta$ . The additional costs of vacancies when  $\tilde{\theta} > 0$  exceeds the cost savings that occur

<sup>10</sup> Search frictions also affect the equilibrium movements of the consumption gap by changing the propagation mechanism and thus optimal policy. The change in the propagation mechanism does not, however, result in a change on the *weight* on the consumption gap in the loss function.



when  $\tilde{\theta} < 0$ . The overall welfare loss from volatility in  $\tilde{\theta}$  is greater when  $1 - \alpha$  is large. Changes in  $\alpha$  will also affect the steady-state cost of vacancy posting relative to consumption. A rise in the elasticity of matches with respect to vacancies (a rise in  $\alpha$ ) increases the level of vacancies in the steady state and leads to a rise in  $\kappa\bar{v}/\bar{C}$ . This acts to increase the welfare cost of volatility in the  $\tilde{\theta}$ -gap. Whether a rise in  $\alpha$  increases or decreases the cost of inefficient fluctuations in labor market tightness will depend on the calibration of the model's parameters. We return to this point in the following section after discussing our baseline calibration.

In a similar model, Thomas (2008) derives a second-order approximation to the utility of the representative agent that consists of a term that is quadratic in inflation, reflecting the loss from price dispersion, and additional terms made up of squares of a number of endogenous variables, including consumption, employment, and labor market tightness. These terms cannot be written in terms of *gaps* relative to the efficient equilibrium, so they do not provide a way to disaggregate the inefficiencies created by the search frictions from those created by nominal price stickiness. In contrast, our approximation expresses the loss function in terms of inefficiency gaps that the policy maker would want to minimize and provides the weights that the policy maker should attach to each inefficiency gap.

Because policy concerns about the labor market are normally expressed in terms of unemployment, and not labor market tightness, it is useful to replace the  $\theta$ -gap in the loss function using (22). Making this substitution, the quadratic loss function becomes

$$(31) \quad L_t = \pi_t^2 + \lambda_0 \tilde{c}_t^2 + \bar{\lambda}_1 (\tilde{u}_{t+1} - \rho_u \tilde{u}_t)^2,$$

where  $\bar{\lambda}_1 = \lambda_1 (1/\alpha\rho\eta)^2 = (1 - \alpha)(\delta/\varepsilon)(\kappa\bar{v}/\bar{C})(1/\alpha\rho\eta)^2$ . Both  $\tilde{u}_{t+1}$  and  $\tilde{u}_t$  matter because of the persistence exhibited by employment matches. If all matches dissolved at the end of every period as in a standard NK model, so that  $\rho = 1$  and  $\rho_u = 0$ , log-linearization of (8), (9), and (10) implies  $\hat{n}_t = \alpha\hat{\theta}_t$ . With  $\hat{n}_t$  and  $\hat{\theta}_t$  moving proportionally, the consumption gap and labor market tightness gap could be combined into a single consumption gap variable. When matches persist (i.e., when  $\rho < 1$ ), current employment depends on current labor market tightness, but it also depends on the stock of matches that survived from the previous period.

Since (23) implies  $\tilde{c}_t = \varphi_2(\beta\tilde{u}_{t+1} - \tilde{u}_t)$ ,<sup>11</sup> we could also write the loss function in the form

$$L_t = \pi_t^2 + \bar{\lambda}_0 (\beta\tilde{u}_{t+1} - \tilde{u}_t)^2 + \bar{\lambda}_1 (\tilde{u}_{t+1} - \rho_u \tilde{u}_t)^2$$

with  $\bar{\lambda}_0 = \lambda_0 \varphi_2^2$ . If the initial unemployment gap  $\tilde{u}_t$  is zero, maintaining  $\tilde{u}_{t+i} = 0$  for all  $i > 0$  also ensures that  $\tilde{c}_{t+i} = 0$  for all  $i \geq 0$ . However, if  $\tilde{u}_t \neq 0$ , then the central bank must trade-off efficient labor market tightness, which would require setting  $\tilde{u}_{t+1} = \rho_u \tilde{u}_t$ —against volatility in the consumption gap, which would call for setting  $\tilde{u}_{t+1} = \tilde{u}_t/\beta$ .

<sup>11</sup> In the efficient steady state,  $\varphi_1 = -\beta\varphi_2$  (see the online Appendix).

With our baseline calibration discussed in the following section;  $\lambda_1$  is small, reflecting the fact that vacancy costs are small relative to total output. In fact, if we assume terms of the form  $(\kappa\bar{v}/\bar{N})\hat{x}_t\hat{y}_t$  are third-order, then the loss function for a second-order approximation to welfare would take the form

$$(32) \quad \pi_t^2 + \lambda_0 \tilde{c}_t^2$$

and involve only inflation and the consumption gap.<sup>12</sup> However, when expressing loss in terms of the unemployment gap as in (31),  $(1/\alpha\rho\eta)^2$  is approximately equal to 13 under our baseline calibrations, so even when  $\lambda_1$  is small, we do not drop this term when we derive optimal policy.

### B. Responses Under Optimal Monetary Policy

In this section, we examine equilibrium under the optimal timeless perspective form of commitment policy (Woodford 2003), assuming the central bank acts to minimize the loss function given by our quadratic approximation to welfare. The constraints on policy are given by (25) and (27). Since the productivity shock does not appear explicitly in either the objective function or the constraints of the policy problem, optimal policy insulates inflation and the unemployment gap from productivity shocks and lets actual unemployment move with efficient, flexible-price unemployment. The central bank simply adjusts policy to keep the real interest rate gap  $\tilde{r}_t$  equal to zero whenever productivity shocks move the efficient level of the real interest rate. This result, however, is the consequence of our assumption that the Hosios condition holds in the steady state so that wage setting under Nash bargaining yields the efficient outcome. If  $b$  were to differ from  $1 - \alpha$ , a productivity shock would present the policymaker with a trade-off between moving the interest rate, so as to stabilize inflation, or moving the interest rate to steer firms' incentive to post vacancies toward the efficient level (Ravenna and Walsh 2010 discuss this trade-off).

Even when  $b$  equals  $1 - \alpha$  on average, stochastic fluctuations in bargaining shares present the central bank with a trade-off between stabilizing inflation and stabilizing variability in the unemployment gap. A positive realization of  $\hat{b}_t$  pushes up wages and the price of wholesale goods relative to retail goods. This increases the marginal costs of the retail firms and leads to a rise in inflation. It also leads wholesale firms to post fewer vacancies, leading to a decline in employment. The policymaker would want to raise interest rates to offset the inflationary impact of this shock, but doing so worsens the decline in labor market tightness through a standard aggregate demand channel and, from (12), by reducing the present value of a filled match.<sup>13</sup> Essentially, the bargaining shock enters (28) as a cost-push shock since it

<sup>12</sup> BG (2010) also assume hiring costs are small, leading them to drop cross-product terms with hiring costs, so (32) would represent the loss in our model under assumptions similar to those used by BG (2010).

<sup>13</sup> From (12),

$$J_t = (P_t^w/P_t)Z_t - w_t + (1 - \rho)E_t(1/R_t)J_{t+1},$$

so conditional on the wage, an increase in the real interest rate reduces  $J_t$ . Equation (18) shows how the effective cost of labor is increasing in the real interest rate.

TABLE 1—PARAMETER VALUES

Exogenous separation rate	$\rho$	0.1
Vacancy elasticity of matches	$\alpha$	0.5
Replacement ratio	$\phi$	0.54
Steady-state vacancy filling rate	$q$	0.9
Labor force	$\bar{N}$	0.9416
Discount factor	$\beta$	0.99
Relative risk aversion	$\sigma$	2
Markup	$\mu$	1.2
Price adjustment probability	$1 - \omega$	0.25

is associated with inefficient fluctuations in unemployment. When bargaining shares fluctuate, stabilizing inflation and stabilizing labor market variables become conflicting objectives even if the initial unemployment gap is zero.

Our approach does not take a stand on the sources of these fluctuations in  $\hat{b}_t$  and simply assumes they are exogenous. Other micro-founded policy objective functions make stronger assumptions on the source of the inefficiency by modeling explicitly deviations of the wage and of the surplus share from the efficient equilibrium. Clearly, we could replicate any endogenous wage sequence generated by a productivity shock by building an appropriate sequence of  $\hat{b}_t$  shocks.

To evaluate policy outcomes, we calibrate the model. The baseline values for the model parameters are set to standard values in the literature and are given in Table 1. We assume the period length is one quarter and set  $\beta = 0.99$ . We impose the Hosios condition in the steady state by setting  $b = 1 - \alpha$ . Estimates of  $\alpha$ , the elasticity of matches with respect to vacancies, generally fall in the 0.4 to 0.6 range, so we set  $\alpha = 0.5$ . The US unemployment rate averaged 5.84 percent over the 1983–2007 period, so we set  $\chi$  in the matching function (9) to ensure steady-state employment is equal to  $1 - 0.0584 = 0.9416$ . We calibrate the replacement ratio  $\phi \equiv w^u/w$  at 0.54 for the United States based on estimates from the Organisation for Economic Co-operation and Development (OECD) database on benefits and wages. From the estimated monthly separation rate of 3.4 percent reported in Shimer (2005), we set the quarterly separation rate  $\rho$  equal to 10 percent.<sup>14</sup> This is consistent with the estimates of Davis, Haltiwanger, and Scott Schuh (1996) and is the value employed in the related literature.<sup>15</sup> Estimates of the steady-state job finding probability  $q$  vary widely in the literature. den Haan, Ramey, and Watson (2000) cite data from Davis, Haltiwanger, and Scott Schuh (1996) to calibrate  $q$  equal to 0.71. Thomas F. Cooley and Vincenzo Quadrini (1999) and Walsh (2005) also set  $q = 0.7$ . This value may be too low, as Davis, R. Jason Faberman, and Haltiwanger (2009) estimate a daily job-filling probability of around 5 percent. Following their assumption of an average of 26 working days per month, or three times that per quarter, a daily rate of  $f$  would imply the probability a vacancy is filled over a quarter to be roughly

<sup>14</sup> We translate the monthly rate into a quarterly rate following the method of Blanchard and Galí (2010).

<sup>15</sup> For example, Wouter J. den Haan, Garey Ramey, and Joel Watson (2000); Walsh (2003, 2005); and Blanchard and Galí (2010).

$1 - (1 - f)^{3 \times 26} = 0.98$ .<sup>16</sup> Because we use steady-state conditions to solve for the job posting cost  $\kappa$  and the wage  $w$ , variations in  $q$  have little effect on our results, so we set  $q = 0.9$ .<sup>17</sup>

The volatility of the bargaining and productivity shocks are chosen so that, conditional on a policy of price stability, the standard deviation of output is  $\sigma_Y = 1.82$  percent and the standard deviation of employment is  $\sigma_N = 1.71$  percent. These values are in line with US business cycle dynamics over the postwar period, and result in a volatility of the innovations for  $\hat{b}_t$  and  $\hat{Z}_t$  of 3.87 percent and 0.32 percent and an output-to-employment volatility equal to 0.94. The high ratio between the volatility of the bargaining and productivity shocks needed to match the data is an implication of the well known Shimer puzzle: in a search labor market model with flexible wages and constant surplus shares, the relative volatility of unemployment and output would be one order of magnitude too small. In a model where the wage adjustment process results in time-varying surplus shares, bargaining shocks would not be needed to match the data. We assume a first-order autocorrelation coefficient of 0.8 for both exogenous shocks. Sala, Söderström, and Trigari (2008) show that, in an estimated DSGE model with labor market search frictions, where the policy maker aims at stabilizing  $\pi_t$  and  $\tilde{u}_t$ , markup shocks generate a much more severe trade-off between stabilizing inflation and the unemployment gap, compared to bargaining shocks. This is especially so when wages are flexible, a case where, as in our model, technology shocks generate no trade-off. As bargaining shocks are the only cost-push shocks in our model, they are responsible for all the deviations of unemployment and inflation from the efficient equilibrium under the optimal policy.

The behavior of inflation and unemployment in the face of a bargaining shock under optimal policy will depend on the relative weights on the policy objectives. These weights, in turn, depend on the parameters such as  $\alpha$ , the elasticity of matches with respect to vacancies, and  $\rho$ , the rate of exogenous job destruction, that characterize the labor market. Figures 1 and 2 provide some evidence on how these parameters affect the trade-offs between policy objectives. Figure 1 shows  $\lambda_1/\lambda_0 = (1 - \alpha)\kappa\bar{v}/\bar{C}$ , the weight on the labor market tightness gap relative to the consumption gap in the loss function (30) as a function of  $\alpha$  and  $\rho$ . As noted earlier,  $\alpha$  has two effects on this ratio. First, an increase in  $\alpha$  reduces the convexity of matches as a function of labor market tightness and lowers the cost of inefficient fluctuations in  $\tilde{\theta}$ ; this is the role of the  $1 - \alpha$  term in the  $\lambda_1/\lambda_0$  ratio. Second, an increase in  $\alpha$  raises steady-state job posting costs relative to consumption  $\kappa\bar{v}/\bar{C}$  and this increases the cost of inefficient fluctuations in  $\tilde{\theta}$ . As Figure 1 illustrates, this second effect dominates and  $\lambda_1/\lambda_0$  is increasing in  $\alpha$ . In our calibration, we impose the Hosios condition that labor's share of the match surplus equals  $1 - \alpha$  to ensure the steady-state equilibrium is efficient. Hence, as  $\alpha$  increases, labor's share of the match surplus falls. Thus, the costs of labor market volatility increase as labor's

<sup>16</sup>This simple calculation ignores that fact that some vacancies within a quarter are closed without being filled and new vacancies are posted within the quarter. Davis, Faberman, and Haltiwanger (2009) account for these flows in obtaining their daily job-filling rate.

<sup>17</sup>Results for alternative values of  $q$  are available from the authors. To find  $\kappa$  and  $w$ , assume  $w^u = \phi w$ , where  $\phi$  is the wage replacement rate. Then (13) and (16) can be jointly solved for  $\kappa$  and  $w$ .

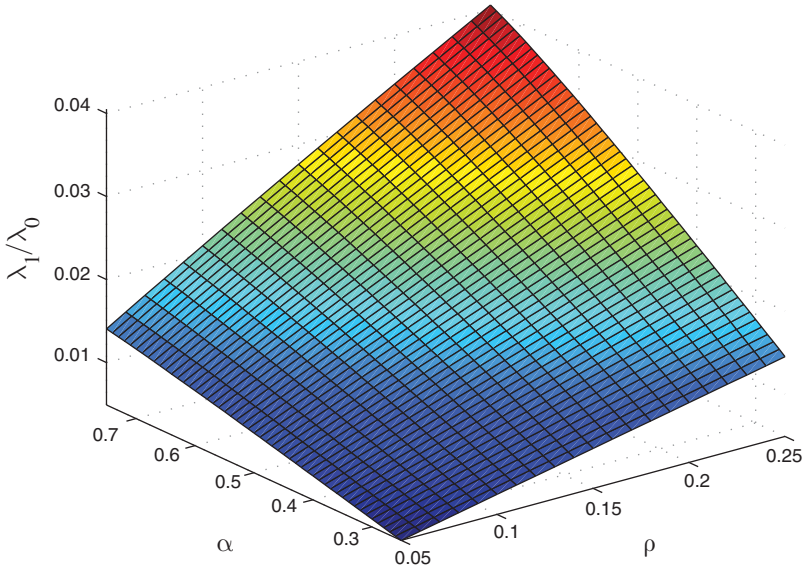


FIGURE 1. RELATIVE WEIGHTS ON  $\theta$  AND  $c$  GAPS ( $\lambda_1/\lambda_0$ ) IN LOSS FUNCTION (LOSS) AS FUNCTIONS OF  $\alpha$  AND  $\rho$

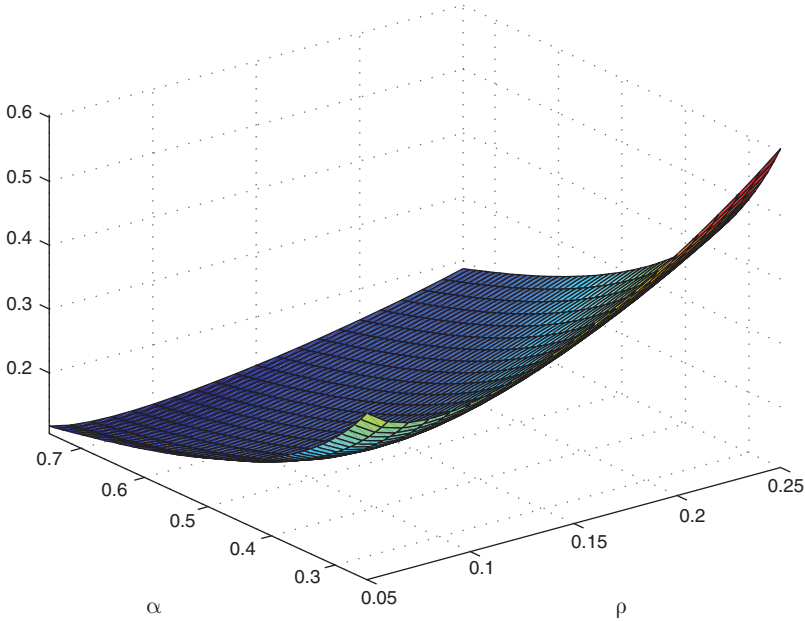


FIGURE 2. RELATIVE WEIGHTS ON  $(\tilde{u}_{t+1} - \rho_u \tilde{u}_t)^2$  AND  $\tilde{c}_t^2 (-1/\lambda_0)$  IN LOSS FUNCTION (ULOSS) AS FUNCTIONS OF  $\alpha$  AND  $\rho$

share of the surplus declines. This cost also increases with  $\rho$ , the rate of exogenous job separation as greater turnover raises the steady-state value of  $\kappa \bar{v}/\bar{C}$ .

As shown earlier, the  $\theta$ -gap could be replaced in the loss functions by  $(\rho_u \tilde{u}_t - \tilde{u}_{t+1})/\alpha \rho \eta$  so that the quadratic loss function becomes

$$\pi_t^2 + \lambda_0 \tilde{c}_t^2 + \bar{\lambda}_1 (\tilde{u}_{t+1} - \rho_u \tilde{u}_t)^2.$$

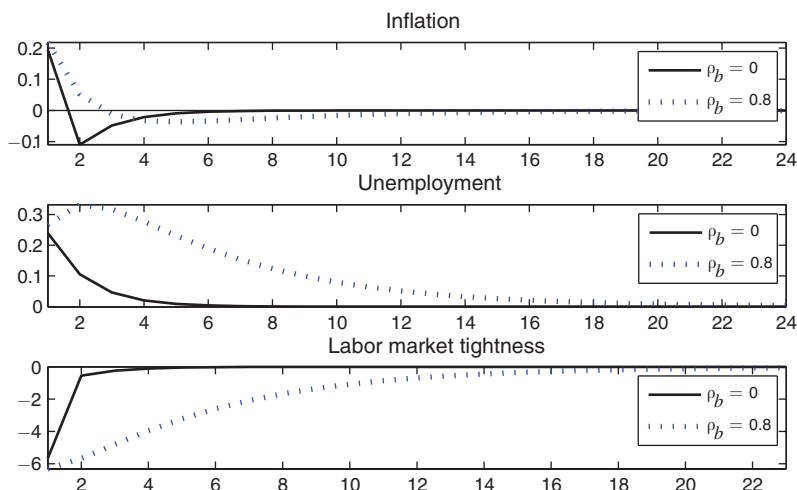


FIGURE 3. RESPONSE TO A ONE STANDARD DEVIATION BARGAINING SHOCK UNDER OPTIMAL COMMITMENT FOR  $\rho_b = 0$  AND  $\rho_b = 0.8$  ( $\pi$  AND  $\theta$  SCALED IN PERCENTAGE POINT DEVIATIONS FROM STEADY STATE; UNEMPLOYMENT SCALED AS PERCENTAGE POINTS OF TOTAL LABOR FORCE)

To compare the weight on the consumption gap to the weight on unemployment, Figure 2 plots  $\bar{\lambda}_1/\lambda_0$  as a function of  $\alpha$  and  $\rho$ . While stabilizing the unemployment term becomes more important relative to stabilizing consumption as  $\rho$  increases, it becomes less important as  $\alpha$  increases. This may seem at odds with the results in Figure 1, but as  $\alpha$  increases, volatility in  $\tilde{u}_{t+1} - \rho_u \tilde{u}_t$  is associated with less volatility in  $\tilde{\theta}_t$ . Thus, even though an increase in  $\alpha$  calls for stabilizing labor market tightness more, as shown in Figure 1, this can actually be achieved with greater volatility in  $\tilde{u}_{t+1} - \rho_u \tilde{u}_t$ .

The dynamic responses of inflation, the unemployment gap, and  $\tilde{\theta}_t$  to a one standard deviation innovation to the bargaining shock under optimal policy for the parameters in Table 1 are shown in Figure 3 for both a serially uncorrelated process (i.e.,  $\rho_b = 0$ ) and a persistent one ( $\rho_b = 0.8$ ).<sup>18</sup> The rise in labor's share due to the positive shock pushes up costs and leads to a rise in inflation. It also leads to an inefficient drop in vacancies and rise in the unemployment gap. Labor market tightness declines. This is the result of both a decrease in the job finding probability and an increase in the probability of filling a vacancy. The shock to the bargaining share generates dynamic behavior akin to a cost-push shock in the NK model. The dynamic process of adjustment in the labor market leads to a gradual return of unemployment to its efficient level. *Under the optimal policy*, the impact of the shock on inflation is small; inflation rises by only 20 basis points at an annual rate.

<sup>18</sup> Since the standard deviation of  $\hat{b}_t$  is 3.87 percent, the positive innovation corresponds to a shock that changes  $b_t$  from its steady-state value of 0.5 to  $0.5 \times 1.0387 = 0.5193$ .



### C. The Role of the Loss Function

In this section, we investigate the consequences of policies that are optimal for a misspecified objective function. In particular, we consider the welfare costs of designing policies to minimize an objective function that corresponds to the quadratic loss functions commonly employed in the literature on optimal monetary policy. We consider two alternatives to the welfare-based loss function. The first alternative simply drops the  $\tilde{\theta}_t^2$  term, yielding a loss function that more closely parallels a standard NK quadratic loss function:

$$(33) \quad L_t^{nk} \equiv \pi_t^2 + \lambda_0 \tilde{c}_t^2.$$

In this case, policy aims to stabilize inflation volatility and the volatility of the consumption gap. We employ the welfare-based value of  $\lambda_0$ , since, as noted earlier, this is equal to the same value that would arise in a standard NK model in which utility depends linearly on hours worked. This loss function ignores the inefficiencies arising from search costs in the labor market.

A second loss function previously employed in the literature includes inflation and the unemployment gap:

$$(34) \quad L_t^u(\lambda) \equiv \pi_t^2 + \lambda \tilde{u}_t^2.$$

Such a loss function has been employed by Athanasios Orphanides and John C. Williams (2007) and is used by Sala, Söderström, and Trigari (2008) in a model with search and matching frictions in the labor market. Because (34) represents an ad hoc specification of policy objectives, theory offers no guidance as to the value to assign to  $\lambda$ , the relative weight placed on unemployment objectives. For our baseline, we set  $\lambda$  so that the standard deviation of the unemployment gap under commitment is the same when minimizing either (34) or the welfare-based loss function (30). In this case,  $\lambda = 0.0035$ . Sala, Söderström, and Trigari (2008) derive optimal policy for various values of  $\lambda$  and find that a value of 0.0521 matches the standard deviation of unemployment in their model.<sup>19</sup> Therefore, we also report results for  $\lambda = 0.0521$ . Since this value of  $\lambda$  is nearly 15 times the one that would deliver the same unemployment gap volatility as the optimal policy, it will imply a very high volatility of inflation in our model. This experiment is useful though to provide a measure of the sensitivity of the loss to the relative weight placed on competing objectives. Orphanides and Williams (2007), for example, employ an even larger weight of 0.25 on unemployment in their analysis.

Results when policy is based on minimizing (under commitment) the alternative loss functions (33) and (34) are reported in Table 2. The first column of the table reports the percentage increase in the welfare-based loss function given by (30) when policy minimizes one of the alternative loss functions. Minimizing

<sup>19</sup>Because they express inflation at an annual rate, the actual value of  $\lambda$  they use is  $16 \times 0.0521 = 0.833$ .



TABLE 2—ALTERNATIVE POLICY OBJECTIVES: COMMITMENT

	Quadratic loss relative to opt. commitment (percent)	Welfare cost*	$\sigma_\pi$	$\sigma_{\tilde{u}}$	$\sigma_{\tilde{\theta}}$	$\sigma_\pi/\sigma_{\tilde{u}}$
Welfare-based loss						
(1)	0	0	0.24	0.72	11.82	0.33
Std. Loss in $\pi$ and $\tilde{c}$ -gap, $\lambda = \lambda_0$						
(2)	4.59	0.0011	0.02	0.75	12.36	0.03
Std. Loss in $\pi$ and $\tilde{u}$ -gap, $\lambda = 0.0035$						
(3)	0.34	0.0001	0.22	0.72	11.86	0.32
Std. Loss in $\pi$ and $\tilde{u}$ -gap, $\lambda = 0.0521$						
(4)	275.93	0.0683	1.96	0.51	8.27	3.83

\*Relative to welfare-based optimal commitment, as percent of steady-state consumption.

(33), for example, increases the loss by 4.59 percent (row 2). When policy minimizes inflation and unemployment volatility, the weight placed on the unemployment gap is crucial; minimizing (34) increases the loss by 0.34 percent (row 3) when  $\lambda = 0.0035$ , but by 275.93 percent (row 4) when the value  $\lambda = 0.0521$  is used.

To measure the welfare loss in terms of steady-state consumption, note that in (29) only the term involving the quadratic loss  $L_{t+i}$  differs under alternative policies. Let

$$L^p \equiv -\frac{\varepsilon}{2\delta} U_c \bar{C} E \sum_{i=0}^{\infty} \beta^i L_{t+i}^p$$

denote the loss under a policy  $p$ , and

$$L^r \equiv -\frac{\varepsilon}{2\delta} U_c \bar{C} E \sum_{i=0}^{\infty} \beta^i L_{t+i}^r$$

denote the loss under the welfare-based optimal commitment (Ramsey) policy. The welfare cost of implementing policy  $p$  can be measured as the percent increase in steady-state consumption  $\lambda^p$  that would make the representative agent indifferent between policy  $p$  and the Ramsey policy, where  $\lambda^p$  solves

$$\frac{U((1 + \lambda^p)\bar{C})}{1 - \beta} + L^p + t.i.p. = \frac{U(\bar{C})}{1 - \beta} + L^r + t.i.p.$$

The resulting measure is reported in column 2 of Table 2. Consistent with the comparison based on the quadratic loss itself, the welfare costs of deviating from the optimal commitment policy are small in terms of steady-state consumption equivalents except when a large weight is placed on the volatility of the unemployment gap. In fact, when  $\lambda = 0.0521$  in (34), performance deteriorates significantly (see row 4, Table 2). With this parameterization, policy is much more aggressive in stabilizing deviations of unemployment from the efficient level; the standard deviation

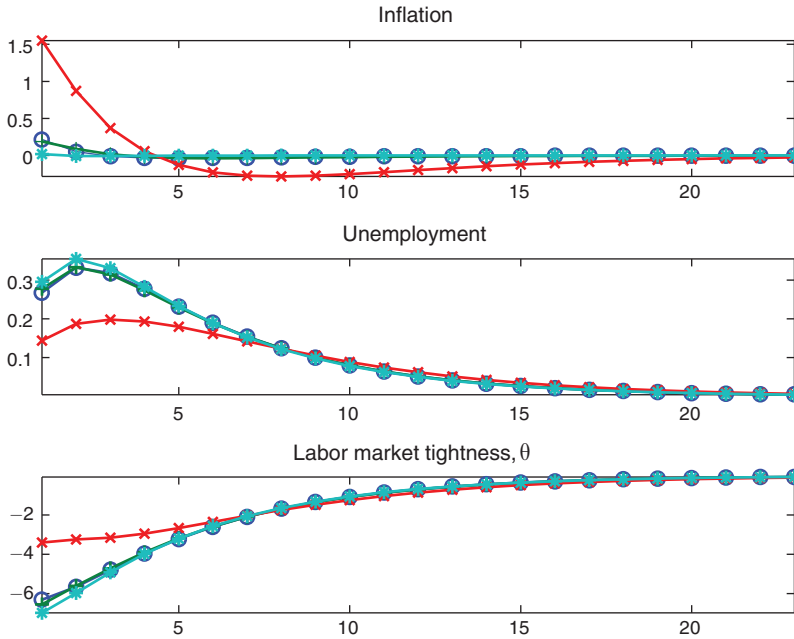


FIGURE 4. IMPULSE RESPONSES TO A ONE STANDARD DEVIATION BARGAINING SHOCK, OPTIMAL COMMITMENT POLICIES MINIMIZING DIFFERENT LOSS FUNCTIONS:  $\circ$  WELFARE-BASED LOSS;  $*$  LOSS BASED ON (33) WITH  $\lambda = \lambda_0$ ;  $+$  LOSS BASED ON (34) WITH  $\lambda = 0.0035$ ;  $x$  LOSS BASED ON (34) WITH  $\lambda = 0.0521$  ( $\pi$  AND  $\theta$  SCALED IN PERCENTAGE POINT DEVIATIONS FROM STEADY STATE; UNEMPLOYMENT SCALED AS PERCENTAGE POINTS OF TOTAL LABOR FORCE)

of inflation increases by a factor of eight, while the standard deviation of the unemployment gap falls by about one-third. The monetary authority would do much better by focusing on stabilizing inflation and ignoring altogether the impact of bargaining shocks on employment, as the second row of Table 2 shows.

The responses of inflation, the unemployment gap, and labor market tightness to a one standard deviation, serially correlated bargaining shock for the different policy objectives are shown in Figure 4. For comparison, the lines marked by circles give the impulse responses under the welfare-based optimal commitment policy and are the same as those shown in Figure 3. The responses are quite similar across the different loss functions with the exception of (34), when a large weight is placed on unemployment fluctuations, as this case allows a much greater response of inflation and, correspondingly, allows much less movement in the labor market variables. The policy based on the consumption gap loss given by (33) allows the most labor market volatility and almost completely neutralizes the impact of the bargaining shock on inflation. Both the welfare-based policy and the policy that minimizes (34) with  $\lambda = 0.0035$  produce almost identical impulse responses in reaction to the bargaining shock.

#### D. Discretion versus Commitment

In this section, we examine outcomes when policy is conducted in a discretionary regime. Results are reported in Table 3, which parallels the cases considered in Table 2 for optimal commitment. Several points are worth noting. First, the welfare

TABLE 3—ALTERNATIVE POLICY OBJECTIVES: DISCRETION

	Quadratic loss relative to opt. commitment (percent)	Welfare cost <sup>a</sup>	$\sigma_\pi$	$\sigma_{\tilde{u}}$	$\sigma_{\tilde{\theta}}$	$\sigma_\pi/\sigma_{\tilde{u}}$
Welfare-based loss						
(1)	10.50	0.0026	0.39	0.72	11.93	0.54
Std. Loss in $\pi$ and $\tilde{c}$ -gap, $\lambda = \lambda_0$						
(2)	4.55	0.0011	0.02	0.75	12.36	0.03
Std. Loss in $\pi$ and $\tilde{u}$ -gap, $\lambda = 0.0035$						
(3)	16.75	0.0041	0.45	0.72	12.04	0.62
Std. Loss in $\pi$ and $\tilde{u}$ -gap, $\lambda = 0.0521$						
(4)	1936.12	0.4815	4.83	0.43	7.35	1.13

<sup>a</sup>Relative to welfare-based optimal commitment as percent of steady-state consumption.

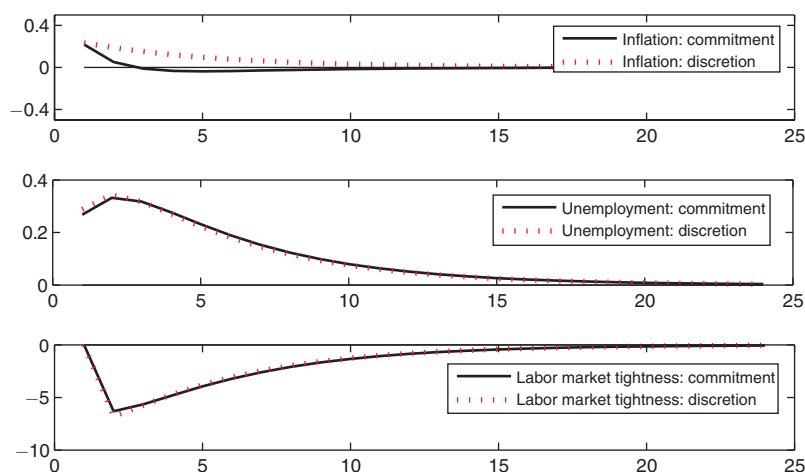


FIGURE 5. RESPONSES TO A ONE STANDARD DEVIATION BARGAINING SHOCK UNDER OPTIMAL COMMITMENT AND DISCRETION ( $\pi$  AND  $\theta$  SCALED IN PERCENTAGE POINT DEVIATIONS FROM STEADY STATE; UNEMPLOYMENT SCALED AS PERCENTAGE POINTS OF TOTAL LABOR FORCE)

cost of bargaining shocks under optimal discretion is 10.5 percent higher than obtained under the optimal commitment policy. This cost arises primarily from greater volatility of inflation under discretion. In fact, labor market outcomes are quite similar under either commitment or discretion, as shown in Figure 5 which compares the impulse responses under the two policies. The path of inflation differs under commitment and discretion primarily because of the different paths followed by expected inflation under the alternative policy regimes. This is illustrated in Figure 6, which graphs separately the expected inflation, unemployment, and the real interest rate cost components of (27). Finally, under discretion, the policy obtained using the standard NK model objective function results in an allocation closer to the commitment case and generates a higher welfare than the optimal discretionary policy.

Comparing Tables 2 and 3 shows that outcomes are fairly similar under either commitment or discretion except when policy minimizes a loss function that puts a large weight on unemployment gap variability. In this case, loss under discretion deteriorates significantly relative to the welfare-based commitment policy, rising to

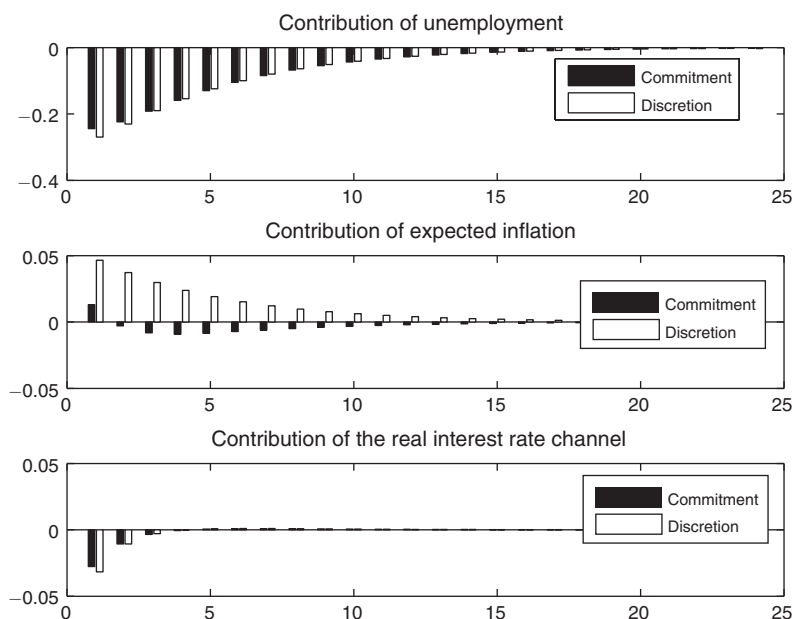


FIGURE 6. CONTRIBUTION OF LABOR MARKET, EXPECTED INFLATION, AND THE REAL INTEREST RATE TO THE PATH OF INFLATION NET OF THE DIRECT EFFECT OF THE BARGAINING SHOCK WHEN POLICY IS BASED ON THE WELFARE APPROXIMATION

0.4815 percent of steady-state consumption. With  $\lambda = 0.0521$ , discretion results in the welfare-based loss function being almost 2,000 percent higher than the value achieved under optimal commitment. The increase in loss occurs because discretion smooths labor market variables to a much greater degree than is done under commitment with a corresponding increase in the volatility of inflation.

### E. The Role of the Transmission Mechanism

Above we assumed policy was optimal, conditional on the wrong objective. That tells us how important deviations from the correct objective function are in generating welfare losses relative to the optimal plan (Ramsey allocation). Such an exercise is silent, however, on the implications of an optimal policy that is conditional on the wrong constraints—that is, on the wrong transmission mechanism for policy. In this section, we investigate the performance of simple targeting rules and instrument rules. Since a central bank is likely to target the wrong objective if its knowledge of the transmission mechanism is inaccurate, we examine the performance of policy rules that would be optimal conditional on an incorrect objective function *and* on an incorrect transmission mechanism. This exercise provides insights into whether the central bank generates large losses by ignoring the existence of search frictions and search unemployment.<sup>20</sup>

<sup>20</sup>To provide a measure of how sensitive the loss is to alternative policies in a given economic environment, we keep constant all parameters, including the exogenous shocks' volatility, implying the resulting volatility of the

TABLE 4—ALTERNATIVE POLICIES

	Quadratic loss relative to opt. commitment (percent)	Welfare cost <sup>a</sup>	$\sigma_\pi$	$\sigma_{\tilde{u}}$	$\sigma_{\tilde{\theta}}$	$\sigma_\pi/\sigma_{\tilde{u}}$
Baseline NK model optimal commitment policy (1)	6.62	0.002	0.04	0.75	12.47	0.05
Baseline NK model optimal discretionary policy (2)	6.89	0.002	0.03	0.75	12.48	0.040
Strict inflation targeting (3)	5.41	0.001	0	0.74	12.41	0
Taylor Rule: $i_t = 0.5\tilde{c}_t + 0.7\pi_t$ (4)	3,989	0.98	6.89	0.25	3.96	27.56
Estimated policy rule CGG Volker-Greenspan era: $i_t = 0.71i_{t-1} + 0.29(1.72\tilde{c}_{t-1} + 0.34\pi_{t-1})$ (5)	5,855	1.43	8.33	0.22	1.92	37.86

<sup>a</sup>Relative to optimal commitment utility level, as percent of steady-state consumption.

We consider five alternative policy rules that have been widely employed in standard NK models: (1) the optimal targeting rule under commitment from a timeless perspective— $\pi_t = -(\lambda/\delta)(\tilde{c}_t - \tilde{c}_{t-1})$ , where  $\lambda = \delta(\sigma + \eta_N)/(1 + \eta_N\varepsilon)\varepsilon$  and  $\eta_N = 1$  is the parameterized labor supply elasticity in a model with separable utility in consumption and labor hours; (2) the optimal targeting rule under the time-consistent, discretionary policy— $\pi_t = -(\lambda/\delta)\tilde{c}_t$ ; (3) strict inflation targeting— $\pi_t = 0$ ; (4) a Taylor rule— $i_t = 0.5\tilde{c}_{t-1} + 0.7\pi_t$ ; (5) and a policy rule estimated by Clarida, Galí, and Gertler (2000) over the Volcker-Greenspan years 1979–1997— $i_t = 0.71 \times i_{t-1} + 0.29(1.72\tilde{c}_{t-1} + 0.34\pi_{t-1})$ .<sup>21</sup>

Implementing the optimal commitment targeting rule from the standard NK model leads to a 6.62 percent increase in the welfare loss (see the first column of Table 4, row 1) relative to the commitment policy based on minimizing the welfare-based loss function. Interestingly, strict inflation targeting outperforms both the commitment and time-consistent targeting rules. This simply reflects the fact that when policy employs the wrong model, there is no presumption that an optimized policy will lead to better outcomes. In some NK frameworks, such as the Erceg, Henderson, and Levin (2000) staggered wages-staggered prices model, or the baseline NK model with no cost-push shock in Galí (2002), a simple Taylor rule performs nearly as well as the optimal policy. This is not the case when search frictions generate a welfare cost. The fourth row of Table 4 shows that the Taylor rule does very poorly, leading to much more inflation variability and a loss 40 times larger than achieved under the optimal policy. The policy rule estimated by Clarida, Galí, and Gertler (2000) over the Volcker-Greenspan years of 1979–1997 leads to an even worse result (row 5), generating a welfare loss equal to 1.43 percent of steady-state consumption relative to the Ramsey policy.

endogenous variables does not necessarily match the data we used in our calibration.

<sup>21</sup>Because inflation and the interest rate in our model are expressed at quarterly rates, we divide the coefficient on  $\tilde{c}_t$  by four in the actual simulations. However, we present the Taylor rule here in its more familiar form since we always express inflation at annual rates in the tables and figures. To ensure determinacy of the equilibrium, we use a Taylor rule with a coefficient on inflation smaller than 1 for case (4), and the Clarida, Galí, and Gertler (2000) estimate on US data of a backward-looking policy rule for case (5). See Kurozumi and van Zandweghe (2008) for determinacy conditions in a model akin to our specification.

### III. The Role of Labor Market Structure

An important advantage of our model is its ability to characterize the optimal policy according to different assumptions about the structure of the labor market. For example, BG (2010) argue that Europe is characterized by a lower separation rate and level of steady-state employment than the United States. They suggest setting  $\rho = 0.025$  to capture the lower turnover in Europe. Over the period 1983–2007, Euro Area unemployment has averaged 10.11 percent, so we set  $\chi$  to ensure  $\bar{N} = 1 - 0.1011 = 0.8989$ . Based on the OECD evidence on average unemployment benefit replacement ratios for Euro area countries, we set  $\phi = 0.65$ .<sup>22</sup>

Based on these calibration differences, the inflation adjustment equation for the United States expressed in the form given by (28) is

$$(35) \quad \pi_t = \beta E_t \pi_{t+1} - 0.087 \tilde{u}_{t+1} + 0.103 \tilde{r}_t + 0.081 \hat{b}_t,$$

while that for the EU calibration is

$$(36) \quad \pi_t = \beta E_t \pi_{t+1} - 0.065 \tilde{u}_{t+1} + 0.845 \tilde{r}_t + 0.099 \hat{b}_t.$$

Two differences are apparent. First, the interest rate channel on inflation is much larger in the EU calibration. Second, inflation is less sensitive to the unemployment gap under the EU calibration. In large part, these effects reflect the higher persistence of unemployment under the EU calibration, for which  $\rho_u = 0.798$  versus a value of only 0.345 under the US calibration. The higher degree of persistence reduces the impact of  $\tilde{u}_{t+1}$  on inflation. If  $\rho_u$  is large, both current and future labor market conditions move together, so the impact of current conditions is offset to some degree by the co-movement of expected future conditions. In more flexible labor markets,  $\rho_u$  is smaller and current unemployment conditions induce a smaller co-movement in expected future conditions. The greater persistence of matches also explains why changes in the interest rate, which affect the present value of the match surplus, have a bigger impact under the EU calibration (see 27). When employment is more persistent, the expected discounted future labor market conditions have a bigger impact on current inflation (see 18), so changes in the rate used to discount the future have a correspondingly larger impact on current inflation.

Figure 7 plots the responses to a serially correlated shock to labor's bargaining share for the US and EU calibrations. The EU calibration leads to less volatility in the inflation rate and in labor market variables. This greater stability is the result of two factors. First, with the EU calibration reflecting a less flexible labor market, the steady-state share of output devoted to vacancy posting costs is smaller, reducing the welfare loss from inefficient fluctuations in the labor market and resulting in a

<sup>22</sup>This is in line with Stephen Nickell (1997) who reports an average replacement ratio for EU countries of 0.6 and 0.5 for the United States. The average duration of unemployment benefits is also shorter in the United States than in EU countries. It is important to note that the differences between our US and EU calibrations is restricted *only* to different values of the labor market parameters. Other parameters, including the frequency of price adjustment, are held constant across the two calibrations.

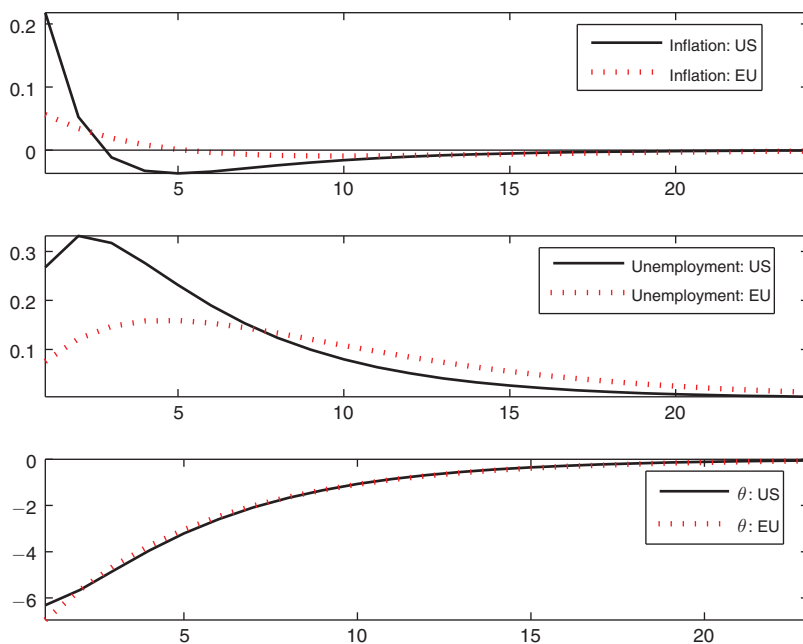


FIGURE 7. RESPONSES TO A ONE STANDARD DEVIATION BARGAINING SHOCK FOR US (SOLID LINE) AND EU (DOTTED LINE) CALIBRATIONS. ( $\pi$  AND  $\theta$  SCALED IN PERCENTAGE POINT DEVIATIONS FROM STEADY STATE; UNEMPLOYMENT SCALED AS PERCENTAGE POINTS OF TOTAL LABOR FORCE)

TABLE 5—ALTERNATIVE POLICY OBJECTIVES: EU CALIBRATION

	Quadratic loss relative to opt. commitment (percent)	Welfare cost*	$\sigma_\pi$	$\sigma_{\tilde{u}}$	$\sigma_{\tilde{\theta}}$	$\sigma_\pi/\sigma_{\tilde{u}}$
(1) Commitment	0	0	0.07	0.47	11.96	0.16
(2) Discretion	0.59	0.0001	0.09	0.47	11.96	0.20
Std. Loss in $\pi$ and $\tilde{c}$ -gap, $\lambda = \lambda_0$						
(3) Commitment	0.70	0.0001	0.01	0.47	12.04	0.01
(4) Discretion	0.66	0.0001	0.01	0.47	12.05	0.02
Std. Loss in $\pi$ and $\tilde{u}$ -gap, $\lambda = 0.0035$						
(5) Commitment	0.21	< 0.0001	0.13	0.47	11.96	0.16
(6) Discretion	2.08	0.0003	0.13	0.47	11.99	0.27
Std. Loss in $\pi$ and $\tilde{u}$ -gap, $\lambda = 0.0521$						
(7) Commitment	133.62	0.0191	1.02	0.43	10.89	2.38
(8) Discretion	434.18	0.0621	1.76	0.42	11.12	4.15

\*Cost of bargaining shocks as percent of steady-state consumption.

smaller  $\lambda_1$ .<sup>23</sup> To understand why the reduction in the rate of exogenous job destruction  $\rho$  from 0.1 under the US calibration to 0.025 for the EU calibration leads to greater stabilization of inflation, consider the limit as  $\rho \rightarrow 0$ . With no employment turnover and no vacancies, unemployment is constant, and optimal policy reduces to stabilizing the inflation rate at zero. Thus, optimal policy under the EU calibration assigns relatively more weight to achieving inflation stability than would be the case

<sup>23</sup> Since  $\lambda_0$ , the weight on the consumption gap, is independent of  $\rho$ , Figure 1 implies that the weight placed on stabilizing labor market gaps falls as  $\rho$  declines.



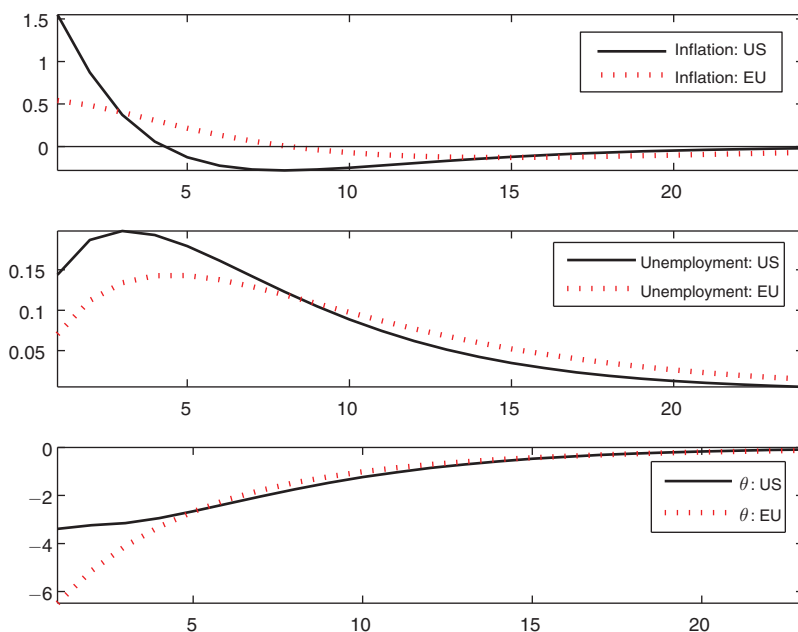


FIGURE 8. RESPONSES TO A ONE STANDARD DEVIATION BARGAINING SHOCK FOR US AND EU CALIBRATIONS WHEN POLICY MINIMIZES THE STANDARD LOSS FUNCTION (33) WITH  $\lambda = 0.0521$  ( $\pi$  AND  $\theta$  SCALED IN PERCENTAGE POINT DEVIATIONS FROM STEADY STATE; UNEMPLOYMENT SCALED AS PERCENTAGE POINTS OF TOTAL LABOR FORCE)

under the US calibration. Second, the effect of the interest rate on inflation is much stronger under the EU calibration, and this improves the inflation-unemployment trade-off. When matches persist with greater probability, as is the case under the EU calibration, the expected savings in future vacancy costs from having an existing match is larger, and changes in the interest rate that affect the present value of these savings have a bigger impact on the effective cost of labor. A positive bargaining shock increases inflation and increases unemployment, but reducing the interest rate to boost employment through the standard demand channel also acts to significantly reduce inflation through its direct effect on labor costs, improving the inflation-unemployment trade-off.

Results for alternative loss functions with the EU calibration are given in Table 5. Regardless of the objective function, both inflation and the unemployment gap are significantly less volatile when the labor market is less flexible. Comparing the last column of Table 5 with the corresponding column of Table 2 shows that the relative decline in inflation volatility is greater, however. As under the US calibration, the last row of Table 5 shows that a policy that places a large weight on unemployment volatility in the loss function leads to a significant deterioration in welfare. Outcomes when policy minimizes the standard loss function (33) with  $\lambda = 0.0521$  are shown in Figure 8. In this case, the inflation response increases by nearly an order of magnitude under both calibrations (compare scales in Figures 7 and 8), while the change in the behavior of labor market variables is much more modest. Under discretion, the adoption of the inefficient policy leads to drastically different outcomes for the EU and US calibrations. The costs of bargaining shocks is still

relatively small in absolute value for the EU case, resulting in a loss of 0.06 percent of steady-state consumption (Table 5, row 4), while it rises to 0.48 percent of steady-state consumption for the US calibration (Table 3, row 4).

#### IV. Conclusions

A growing literature has incorporated labor market frictions into models with nominal rigidities. When these frictions are modelled using the search and matching approach of Mortensen and Pissarides (1994), optimal policy has generally been studied using ad-hoc loss functions of the form commonly employed in models that lack labor market frictions. We derive an explicit second-order approximation to the welfare of the representative agent in the presence of search and matching frictions. The resulting model-consistent loss function includes an additional quadratic term involving labor market tightness that is missing from the standard loss function. This extra term captures the welfare loss due to search frictions when labor market tightness deviates from its efficient level. Thus, all the cost from search inefficiency can be summarized in a single term in the welfare function. This term can also be expressed solely in terms of what we have labeled the unemployment gap, the gap between unemployment and the efficient level of unemployment.

Just as inflation creates an inefficiency in the way market goods are combined to generate market consumption, volatility in the labor market tightness gap generates inefficiency in the way market and home consumption are combined to generate overall utility. While we focus on search frictions, this result suggests that any distortion in the allocation of time among alternative uses that generate utility would lead to a labor market term in the loss function. In that sense, the basic results are not special to search frictions, though of course the exact form of the loss would depend on the nature of the distortion.

The trade-off between stabilizing the terms in the welfare function results from random deviations from efficient surplus sharing across firms and workers. Our approach does not take a stand on the sources of these fluctuations and does not assume any endogenous constraint affecting wage adjustment, exactly as in the standard NK model. By preserving the assumption of real wage flexibility, our results are directly comparable to the models with staggered price adjustment and Walrasian labor markets with flexible wages that have been prominent in monetary policy analysis. However, our framework allows for a rich characterization of the labor market in terms of exogenous separation rates, labor bargaining power, matching technology productivity and efficiency, and job posting costs. We find, for example, that if the labor market becomes less flexible, as is generally assumed to be the case for European labor markets, employment dynamics are more muted in response to random shocks and optimal policy calls for greater inflation stability.

Ignoring the role of labor market frictions in setting the objectives of policy can lead to large losses. In particular, a policy designed to minimize inflation volatility and unemployment gap volatility can lead to a significant reduction in welfare if a moderately large weight is placed on unemployment objectives. If the central bank ignores the impact of labor market frictions in both the objective function and the transmission mechanism and implements the optimal targeting rule derived from a

standard NK model, outcomes are inferior to those obtained under strict inflation targeting.

In standard NK models, cost shocks generate a wedge between the marginal rate of substitution between leisure and consumption and the marginal product of labor, producing a welfare cost that calls for deviating from price stability. With long-term employment relationships and efficient bargaining, temporary fluctuations in bargaining shares do not generate large welfare costs since workers and firms are concerned with the present value of the match surplus. This is why price stability turns out to remain close to the optimal policy.

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