© Oxford University Press 2000 *Oxford Economic Papers* 52 (2000), 249–271 **249**

Market discipline and monetary policy

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The effects of forward looking expectations of future inflation on equilibrium inflation and interest rates are examined within an imperfect information framework. Expectations of future inflation affect equilibrium in a manner similar to an increase in the central bank's weight on future social welfare, making it more likely an opportunistic central bank will actually deliver on its announced inflation targets, and output expansions can arise even if the central banker is revealed to be a low inflation type. The model also illustrates the channels through which inflation scares raise current real interest rates.

1. Introduction

In the standard models of discretionary monetary policy, the central bank is tempted to engineer a real economic expansion. If private agents are locked into nominal wage contracts prior to observing the current stance of policy, the central bank can generate a surprise inflation that produces a temporary output expansion. The wage and price adjustment models of Taylor (1979), Rotemberg (1982), and Calvo (1983), however, imply that current inflation will depend on expectations of future inflation. A policy action that signals higher future inflation acts to increase current inflation, and this raises the cost of any attempt to engineer an output expansion. With most major central banks using a short-term interest rate to implement monetary policy, policy changes are immediately and widely noted in the press. Expectations about future inflation can respond immediately to any change in policy, affecting both current and future equilibria. This response has the potential to discipline an opportunistic central bank, helping to support a low inflation equilibrium. The disciplinary channel emphasized in this paper adds to a growing list of channels that suggest a more limited role for any inflationary bias under discretionary monetary policy.[1]

Expectations of future inflation will be affected by current policy, however, only if current policy provides informaton about future inflation. To generate such an informational role, I assume the central bank 'type' is unknown by the public; thus,

[1] Other channels through which the inflation bias may be reduced include reputation (Barro and Gordon, 1983b; Backus and Driffill, 1985; Canzoneri, 1985), voting in democracies (Minford, 1995), the appointment process (Rogoff, 1985; O'Flaherty, 1990; Waller, 1992; Walsh, 1995b; Waller and Walsh, 1996), policy lags (Goodhart and Huang, 1998), and inflation contracts or targets (Persson and Tabellini, 1993; Walsh, 1995a; Svensson, 1997; Herrendorf, 1998).

observing current policy may affect the public's priors concerning the identity of the central bank. This approach follows the work on preference uncertainty by Backus and Driffill (1985), Vickers (1986), Cukierman and Liviatan (1991), and Ball (1995). The importance of preference uncertainty has been stressed recently by Briault *et al.* (1997), Schaling (1997), and Beetsma and Jensen (1998). The form of uncertainty emphasized in this paper is particularly relevant when the head of a central bank changes or a new institution is launched (such as the European Central Bank) whose commitment to an announced promise of low inflation may be doubted, or at least untested. Uncertainty about future policy is shown in influence the decision problem faced by current policy makers; skepticism about a future central banker's commitment to low inflation forces the current central banker to raise real interest rates.

Perhaps closest in spirit to the present paper are the recent papers by Goodhart and Huang (1998) and Herrendorf (1998). In both, observations on current policy can also discipline the central bank, but the mechanisms are quite different. Goodhart and Huang incorporate real output persistence, overlapping multi-period nominal wage contracts, and a lag between policy actions and their effects on the economy. These last two factors imply that current policy actions have a muted impact on the economy, since some nominal wages will be renegotiated before actual inflation can affect real output. While this does not change the basic nature of the inflation bias under discretion, it does reduce the absolute magnitude of the bias. This channel also operates in the present model, but the emphasis is on a second channel that is absent in the Goodhart and Huang model. Wage and price setting behavior based on forward looking expectations alters the nature of the equilibrium, and, for some parameter values, even implies lower than expected inflation may generate an economic expansion. Thus, output expansions can occur when a central banker is revealed to be strong on inflation. Goodhart and Huang also emphasize the role of uncertainty arising from underlying economic disturbances, and announcements do not play a role in their analysis. In contrast, I focus on the public's uncertainty about the central bank's committment to its announced inflation targets. As Briault *et al.* (1997) stress, uncertainty about the central bank's preferences can be important in influencing inflation expectations.

Herrendorf (1998) considers chisel-proof trigger strategies (al-Nowaihi and Levine, 1994) in a model with a monopoly union and imperfect information about the central bank's actual choice of policy. He shows that publicly announced inflation targets alleviate this informational problem by making deviations from the low inflation equilibrium apparent. In contrast, I assume the central bank's policy setting (for example, the value of the short-term interest rate used to implement policy) is directly observable; thus the public has perfect information about any change in monetary policy. Instead, imperfect information about the type of central banker is emphasized, and, because the public is assumed to be atomistic, equilibrium is Bayesian rather than based on trigger strategies.

In the next section, the basic model is presented. Section 3 characterizes equilibria in the model. Forward looking expectations are shown to affect the nature of the equilibria in a manner similar to the central bank's discount factor. Thus, the discipline provided by forward looking expectations forces the central bank to behave as if it were more patient. Section 4 examines the model's implications for long-term (two period) real and nominal interest rates. The model illustrates what Goodfriend (1993) has characterized as 'the inflation scare problem'. A rise in expected future inflation (caused in this model by an exogenous change in the public's prior beliefs about the next central banker) forces the current central banker to raise real interest rates. Conclusions are contained in Section 5.

2. The model

The basic model is a variant of one developed by Cukierman and Liviatan (1991). Their model is modified in two ways. First, forward looking expectations are incorporated. Second, the framework is extended beyond the two-period version employed by Cukierman and Liviatan. In this section, these aspects of the model structure are discussed.

2.1 Inflation and output

The literature that has developed from the original Barro–Gordon (1983a) model has been based on a simplified version of an aggregate supply relationship that takes the form

$$y_t = a(\pi_t - \mathrm{E}_{t-1}\pi_t) + \varepsilon_t \tag{1}$$

where y is the deviation of output around the economy's equilibrium output in the absence of price surprises and supply shocks, π_t is the rate of inflation, $\mathrm{E}_{t-1}\pi_t$ is the expected period t inflation rate based on information available at $t-1$, and ε_t is a supply or productivity disturbance. This specification is normally rationalized on the basis of one-period nominal wage contracting models of the type developed by Fischer (1977) and Taylor (1979). With nominal wages fixed prior to the start of the period, price fluctuations alter the *ex-post* real wage, employment, and output.[2] A critical aspect of (1) is the presence of $\mathrm{E}_{t-1}\pi_t$; the relevant expectational variable is past expectations of current inflation. With this formulation it is natural to assume the central bank treats $\mathrm{E}_{t-1}\pi_t$ as given.

In models in which only a fraction of all nominal wages are set each period and remain fixed for several periods, workers and firms will be concerned with the path of inflation in future periods. Roberts (1995) shows that the popular adjustment models of Taylor (1979), Rotemberg (1982), and Calvo (1983) all lead to a

[2] This type of relationship can also be derived from Lucas (1972), in which case the parameter a reflects the outcome of a signal extraction problem due to the inability of individual economic decision makers to distinguish contemporaneously between general inflation and relative price changes. Lockwood (1997), Svensson (1997), and Goodhart and Huang (1998) introduce persistence by including an effect of y_{t-1} on y_t.

common expectations augmented Phillips Curve in which current inflation depends upon market expectations of both current and future inflation. King and Watson (1996), Yun (1996), and Bernanke and Woodford (1997) among others, have employed inflation equations in which current inflation depends on $E_t\pi_{t+1}$ rather than $E_{t-1}\pi_t$.

To investigate the implications of assuming that wage and price decisions may be based on expectations of future inflation, it is useful to employ a model that allows for both one-period wage contracts (as in the standard model) and two-period wage contracts. Assume that a fraction γ of all workers and firms negotiate one-period nominal wage contracts at the start of each period. In the standard model, $\gamma = 1$. The remaining $1 - \gamma$ fraction negotiate two-period nominal wage contracts, with one half adjusting nominal wages each period. If employment is based on realized ex-post real wages, the aggregate production function is Cobb–Douglas with labor coefficient a_L, and the future is discounted at the rate $\rho (0 < \rho < 1)$, then it is shown in Appendix 1 that output will be given by[3]

$$y_t = a[\pi_t - (1 - k)E_{t-1}\pi_t - \rho k E_t \pi_{t+1}] \tag{2}$$

where $k = (1 - \gamma)/[(1 + \gamma)(1 + \rho)]$ and $a = a_L(1 + \gamma)/(2(1 - a_L))$. The parameter k then captures the relative importance of forward looking expectations. If $\gamma = 1$, all wage contracts are set for one period at the start of each period, $k = 0$ and the standard model (1) is obtained. If $\gamma = 0$, all contracts last for two period and $k \approx 0.5$.[4] In this case, expectations of future inflation and past expectations of current inflation will be equally important.

Equation (2) illustrates the two distinct channels through which the nature of the wage setting process affects the aggregate supply relationship. First, for given expectations, the impact of inflation on output, given by a, is increasing in γ. If only half of all firms are locked into nominal wages ($\gamma = 0$), the output effects of inflation are smaller than if all firms are locked in ($\gamma = 1$).[5] A smaller γ reduces the incentive

[3] Because stabilization policy issues will not be the focus of this paper, (2) has been written without a disturbance term. This setup is similar to that described by Goodhart and Huang (1998). However, they do not provide an explicit derivation of their aggregate supply equation (eq. 2, p. 383) and instead assume that two period wage contracts depend only on the inflation rate expected during the first period of the contract.

[4] With $\gamma = 0$, $k = 1/(1 + \rho)$. If for example, $\rho = 0.9$, then $k = 0.53$. Roberts (1995) obtains $\pi_t = \rho E_t \pi_{t+1} + \alpha y_t + \varepsilon_t$ for the Taylor model with two-period wage contracts (see his eq. 8, p. 979). However, in his derivation, $\varepsilon_t = -\rho(\pi_t - E_{t-1}\pi_t)$ so that his inflation equation can also be written as

$$\pi_t = \frac{\rho}{1 + \rho}(E_{t-1}\pi_t + E_t\pi_{t+1}) + \frac{\alpha}{1 + \rho} y_t$$

which is equivalent to eq. (2) of the text when all contracts last two peiods ($\gamma = 0$) and $\alpha = 2(1 - a_L)(1 + \rho)/a_L$. Because he assumes high frequency data, he sets $\rho = 1$. McCallum (1994) criticizes price adjustment models of the form $\pi_t = E\pi_{t+1} + \alpha y_t$ as violating monetary neutrality; output can be maintained above its natural rate (i.e. y can be kept positive) by a constantly declining rate of inflation. Fuhrer (1994) argues that historical evidence does not allow one to speculate whether one should automatically rule out models that produce such ‘second degree’ non-neutrality results.

[5] The impact of inflation is $(\frac{1}{2})$ $(a_L/1 - a_L)$ if $\gamma = 0$ and $(a_L/1 - a_L)$ if $\gamma = 1$.

to inflate and lowers the inflationary bias under discretion; this is the channel Goodhart and Huang analyze. But changes in γ have a second effect; they alter the relative importance of $E_{t-1}\pi_t$ and $E_t\pi_{t+1}$. This channel will turn out to have effects that alter the nature of the model's equilibrium.[6]

2.2 The policy framework

Following Cukierman and Liviatan (1991), assume central banks come in two types; one type, denoted type S, always delivers on its commitments, while the other, denoted type W, pursues an opportunistic policy. Type is private information, initially known only to the actual central bank. The central banker is assumed to hold office for two periods, with a new central banker chosen (by nature) in odd periods. The probability a newly chosen central banker is of type S is q. This probability is known by the public, as are k and ρ, and this information is used by private agents in forming their expectations. Depending on the nature of the equilibrium during the central banker's first period in office, the private sector may, in period 2, update its estimate of the probability that a type S is in office. Assuming new central bankers are drawn from a stationary distribution, the probability that a type S is chosen in the first period of each new term is always equal to q.

Most central banks employ interest rate oriented operating procedures, with the underlying transmission mechanism running from interest rates to real aggregate spending (via an IS relationship) to inflation (via an expectations augmented Phillips Curve). In Section 4, interest rates, a term structure relationship, and a simple IS relationship will be introduced to study the interest rate implications of the model. It simplifies the exposition, however, to initially follow the standard approach in the literature and treat inflation as if it were the central bank's choice variable. Thus, for the remainder of this section and Section 3, policy is assumed to involve the direct setting of the inflation rate.

The underlying game between the public and the central bank evolves in a number of steps. To begin, if t is odd, nature chooses a new central banker who holds office for period t and $t+1$; these will be referred to as periods 1 and 2. The central banker then announces a path for inflation over the two periods (π_t^a, π_{t+1}^a). Since a type W will always make the same announcements as would a type S, announcements do not need to be indexed by type. Following these announcements, the public forms expectations of period t inflation, denoted by $E_{t-1}\pi_t$ and all one-period nominal wage contracts are set. The central bank then choses π_t, and, based on this new information, the public forms expectations of inflation for period $t+1$. Period t output is then realized.

Actual policy is implemented by setting inflation in each period to maximize the expected value of an objective function given by

[6] As will be clear below, these two channels are distinct; the value of a affects the level of inflation but does not enter into the conditions that determine whether a separating, pooling, or mixed strategy equilibrium arises.

$$U^i = [y_t - \tfrac{1}{2}\beta\pi_t^2] + \rho[y_{t+1} - \tfrac{1}{2}\beta\pi_{t+1}^2] \qquad (3)$$

for $i = W, S$, where $0 < \rho < 1$ is the central bank's discount factor (the same for both types) and β is the weight placed on inflation objectives (also the same for both types). Since type S can commit, it is assumed that a type S always delivers an inflation rate equal to the rate announced.[7]

3. Equilibria

We consider perfect Bayesian equilibria. The public's expectations formed in period t are optimal (in the sense of minimizing mean squared forecast errors) given the central bank's actions in setting period t policy, and the public updates its assessment of the probability the central bank is of type S using Bayes rule. The central bank recognizes that its choice of π_t will affect $E_t\pi_{t+1}$. That is, in setting first period policy, the central bank takes into account how the public will revise its expectations of period $t+1$ inflation once π_t is set. We consider stationary equilibria in which subsequent central banks behave identically (although different types may behave differently). This means that the expected outcomes in periods $t, t+2, t+4, \ldots$ will be the same. Equilibria may be pooling (with both types behaving similarly during the first period in office), separating (in which they behave differently so that the public is able to identify who is in office), or involve a mixed strategy by type W (in which the type W mimics a type S with some probability).

Because the model reduces to that of Cukieman and Liviatan (1991) when $k = 0$, the details of the derivations of equilibrium outcomes can be kept to a minimum and can be found in Appendix 2.

3.1 Separating equilibrium

In a separating equilibrium, S and W set different inflation rates in period t; this reveals their identity, and the public's expectations about second period inflation reflect their knowledge of the central bank's true type. In such an equilibrium, a type W inflates at the one-shot discretionary rate in both periods. This inflation rate is given by

$$\pi^d \equiv \frac{a}{\beta}$$

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[7] To rationalize the different behavior of the two types, one can view the true underlying utility functions as equal to

$$U^i = [y_t - \tfrac{1}{2}\beta\pi_t^2] + \rho[y_{t+1} - \tfrac{1}{2}\beta\pi_{t+1}^2] - \theta^i[\pi_t - \pi_t^a]^2 - \rho\theta^i[\pi_{t+1} - \pi_{t+1}^a]^2$$

For type W, $\theta^W = 0$; type W suffers no loss if actual inflation differs from announced inflation. For type S, $\theta^S \to \infty$ so that this type always sets actual inflation to its announced value. Vickers (1986) analyzes a signalling model in which types differ with respect to the parameter β. Backus and Driffill (1986) and Ball (1995) study models in which one type always delivers a zero rate of inflation; announcements play no role in these models since the type S behaves non-strategically.

Table 1 Separating equilibrium

	Type *S*	Type *W*
π_1	$[1-(1-k)q]\pi^d$	π^d
y_1	$a\pi^d[k-(1-k)q+(1-k)^2q^2] \lesseqgtr 0$	$a\pi^d[k(1-\rho)+(1-k)^2q^2] > 0$
π_2	0	π^d
y_2	$-\rho ak\pi^d[1-(1-k)q^2] < 0$	$a\pi^d k[1-\rho(1-(1-k)q^2)] > 0$

which, from the definition of a, is decreasing in the fraction of two-period contracts. This is the role of staggered contracts analyzed by Goodhart and Huang (1998).

In contrast to a type W, a type S announces inflation targets at the start of period t and then adjusts policy to ensure actual inflation equals the announced targets. Immediately after announcements are made, the public's expectation of period t inflation, given that the equilibrium is separating, is

$$\mathrm{E}_{t-1}\pi_t = q\pi_t^a + (1-q)\pi^d$$

since with probability q the central bank is actually a type S, and so inflation will be at the announced rate π_t^a, and with probability $1-q$, the central bank is a type W and we have seen that a type W will inflate at the rate π^d. Thus, a type S can influence expectations about inflation through its announced target.

The optimal policy for a type S consists of announcements (π_t^a, π_{t+1}^a) and actual inflation (π_t, π_{t+1}) that maximize its two-period objective function, subject to $\pi_i = \pi_i^a$ for $i = t, t+1$, and the aggregate supply relationships linking output, inflation, and expectations. The first order conditions for this problem yield (see Appendix 2)

$$\pi_t^a = [1-(1-k)q]\pi^d < \pi^d$$

and

$$\pi_{t+1}^a = 0$$

So in the first period in office a type S sets policy to achieve an inflation rate below the one-shot discretionary level π^d (in a separating equilibrium). In the second period, a type S delivers an inflation rate of zero.[8]

Actual outcomes for inflation and output are summarized in Table 1.

If $k = 0$, all wages are set at the start of the period and for only one period, and we have the special case considered by Cukierman and Liviatan. And, as can be easily checked, a recession occurs in period t when $k = 0$ if the central bank is actually of type S. However, if $k > 0$, and multi-period wage contracts are present, the output predictions of the model are more complex. What matters for output is actual inflation relative to a weighted sum of expected current inflation and

[8] While positive inflation in period $t+1$ would generate higher output, the effect of π_{t+1}^a on y_{t+1} is $ak > 0$ (from eq. (20) when $\pi_{t+1}^a = \pi_{t+1}$); the value of this gain, when discounted at the rate ρ, is just offset by the lower period t output caused by a higher announced inflation rate for $t+1$ (see eq. (19)).

expected future inflation (see eq. (2)). Because the public assigns some probability to the central bank being a type S, expected inflation in the first period of office ($E_{t-1}\pi_t$) is less than π^d, so if the central bank is a type W, actual inflation in period t exceeds the weighted sum of expected current inflation and expected future inflation. Thus, output expands. In the following period, inflation remains at the level π^d, and this is expected, but the public anticipates lower future inflation due to the possibility a type S will take over in $t+2$ (as long as $q > 0$). Thus, actual inflation in $t+1$ remains above the relevant weighted sum, and output is again greater than zero. The output gap is positive in both periods under a type W.[9]

It does not follow that a recession occurs under a type S. In fact, an expansion occurs in period t under a type S for all q if $k \geq 0.25$.[10] As soon as type S reveals itself by its policy choice in period t, agents' expectation of period $t+1$ inflation falls as they know inflation will be zero under the type S. This decline in $E_t\pi_{t+1}$ is expansionary in period t as long as $k > 0$. Thus, if future expectations are important in current wage contract settlements, having a type S in office can produce both lower inflation and an economic expansion in the first period of a separating equilibrium.[11]

For the separating outcome to be an equilibrium, the type W central banker must have no incentive to derivate from it. Appendix 2 shows that a type W will derivate from the separating equilibrium if and only if[12]

$$\psi \equiv \frac{\rho}{(1-k)^2} > \tfrac{1}{2}q^2 \tag{4}$$

If a large weight is placed on the future (ρ large), then a type W will find it advantageous to deviate from the separating equilibrium and mimic the behavior of a type S in the first period in office. This allows the type W to create a larger surprise inflation and output expansion in $t+1$. Revealing itself to be a type W in period t causes expected future inflation to rise, and when $k > 0$ this reduces the output level consistent with maintaining period t inflation equal to π^d. If this channel is important (i.e., if k is large), the central bank will not wish to separate. As (4) shows, ρ and k affect the profitability of a deviation through ψ which is increasing in both ρ and k; a rise in the importance of forward looking wage contracts (a fall in γ) plays essentially the same role as an increase in the discount

[9] It is easy to show, however, that $y_1 > y_2$; output expands more in the first period of a type W's term in office.

[10] If $\rho = .9$, $k > 0.25$ whenever the fraction of two-period contracts, $1-\gamma$, exceeds .36. If $k = 0$, as in the standard case, a recession occurs upon the appointment of a type S for all values of q. For $0 < k \leq .25$, expansions can occur for extreme values of q. Notice that these conditions do not depend on a, again emphasizing the two distinct channels through which k affects inflation outcomes. The effects operating through a affect only the size of the output effect, not its sign.

[11] This result arises for the same reason Ball (1994) finds that credible disinflations may generate expansions.

[12] When $\gamma = 1$, k is equal to zero and this condition becomes $\rho > \frac{1}{2}q^2$; this is the condition derived by Cukierman and Liviatan (1991).

Table 2 Pooling equilibrium

	Type *S*	Type *W*
π_1	$k\pi^d$	$k\pi^d$
y_1	$a\pi^d k[k - \rho(1 - q^2)]$	$a\pi^d k[k - \rho(1 - q^2)]$
π_2	$(1 - q)\pi^d$	π^d
y_2	$a\pi^d[k(1 - \rho k) + (1 - k)q^2 - q]$	$a\pi^d[k(1 - \rho k) + (1 - k)q^2] > 0$

factor in affecting the incentive to derivate from a separating equilibrium. A larger k, just like a larger ρ, makes it less likely a type W would wish to separate and makes it more likely that a type W will behave like a type S. Importantly, if all wage contracts are two period contracts ($\gamma = 0$), then $k = 1/(1 + \rho)$ and $\psi = (1 + \rho)^2/\rho > 1$; separating in period 1 is never an equilibrium strategy for the type W. Finally, note that as claimed earlier, the condition that must be satisfied to support a separating equilibrium does not depend on a. Thus, as Goodhart and Huang showed, the presence of two period staggered contract setting can lower the one-shot discretionary equilibrium inflation rate, but such contracts also ensure that the type W may never actually chose to separate and inflate at this rate, and will not do so if all contracts are for two periods ($\gamma = 0$).

3.2 Pooling equilibrium

If eq. (4) holds, a type W would find it profitable to deviate from the separating equilibrium. In this case, there may be pooling and/or mixed strategy equilibria. In a pooling equilibrium, both types inflate at the same rate during the first period, and, when forming expectations about second period inflation, the public remains uncertain about the central bank's type.

In the second period of a pooling equilibrium, type W sets inflation equal to π^d as this is the last period in office and reputation has no further value. Since policy in period t does not provide any information about the central bank's type, the public expects inflation during period $t + 1$ to equal the announced level with probability q and the rate π^d with probability $1 - q$; thus, $E_t\pi_{t+1} = q\pi_2^a + (1 - q)\pi^d$. From the decision problem faced by the type S in a pooling equilibrium, one can show that $\pi_t^S = \pi_1^a = k\pi^d$ and $\pi_{t+1}^S = \pi_2^a = (1 - q)\pi^d$. In contrast to the second period of a separating equilibrium, inflation is positive even under a type S. The outcomes in a pooling equilibrium are summarized in Table 2.

As in Cukierman and Liviatan (1991), the type S is able to inflate at a lower rate in the pooling equilibrium than in a separating equilibrium ($k\pi^d < [1 - (1 - k)q]\pi^d$), but in contrast to their results, inflation remains positive even with pooling. This effect arises from the role of expected future inflation. Since both $E_{t-1}\pi_t$ and $E_t\pi_{t+1}$ matter for period t inflation, the announcement π_1^a has a smaller effect on period t inflation (it is weighted by $1 - k$). Thus, even though the

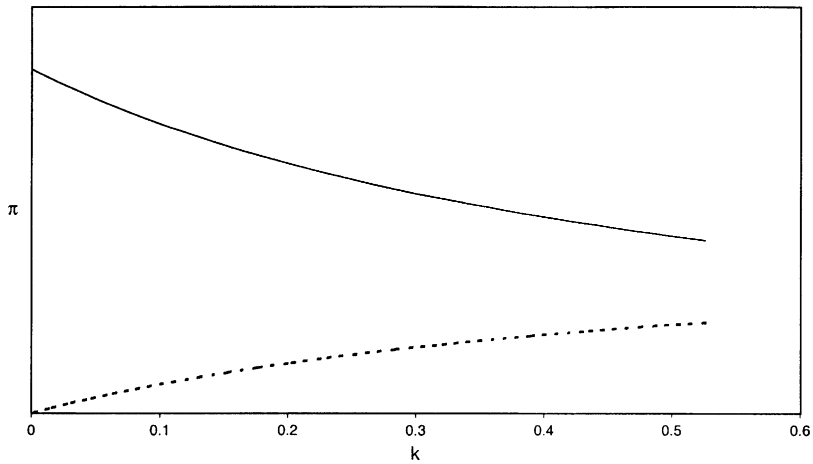

Fig. 1. One-shot discretionary inflation rate (solid) versus first period pooling inflation rate (dashed)

announcement is believed (since both types inflate at the announced rate in a pooling equilibrium), the type S is unable to fully affect the relevant combination of $E_{t-1}\pi_t$ and $E_t\pi_{t+1}$ as the public continues to expect a positive inflation rate in period $t+1$. Inflating at a zero rate in period t would cause a recession. In contrast, if future inflation does not matter (i.e., $k=0$), only $\pi_t - E_{t-1}\pi_t$ is relevant for output; announcing and delivering zero inflation in period t does not lead to a recession.

If $k>0$, the inflation rate in the pooling equilibrium is positive and increasing in k. Both k and π^d depend on the parameter γ; and a decrease in γ (more two-period contracts) raises k and decreases π^d. When k reaches its maximum value (when $\gamma=0$), $k\pi^d \approx \frac{1}{2}\pi^d$ so that inflation in the pooling equilibrium is considerably less than would occur in a one-shot discretionary equilibrium. Both π^d and $k\pi^d$ are shown as functions of k in Figure 1.[13]

Again, the implications for output when $k>0$ differ from those that are standard in the Barro–Gordon framework, with the first period involving either an output expansion (if $k-\rho(1-q^2)>0$) or recession (if $k-\rho(1-q^2)<0$). To understand the implications for output in period t, consider a case in which q is large; the public believes it likely that the new central banker is a type S. Expected future inflation, equal to $(1-q^2)\pi^d$, is therefore low, and this is expansionary in period t. If future expectations are important enough (as measured by k) and expected future inflation is low enough so that $k-\rho(1-q^2)>0$, output will be postive in period t. Thus, a large k or a high probability of a type S can produce an

[13] The figure is drawn for $\rho=0.9$. As the fraction of two-period contracts rises, π^d falls (this is the Goodhart–Huang effect), but in a pooling equilibrium, actual inflation is even lower than π^d. If $\gamma=0$ and all contracts last two periods, then inflation is approximately equal to $\frac{1}{2}\pi^d$ when $\rho=0.9$.

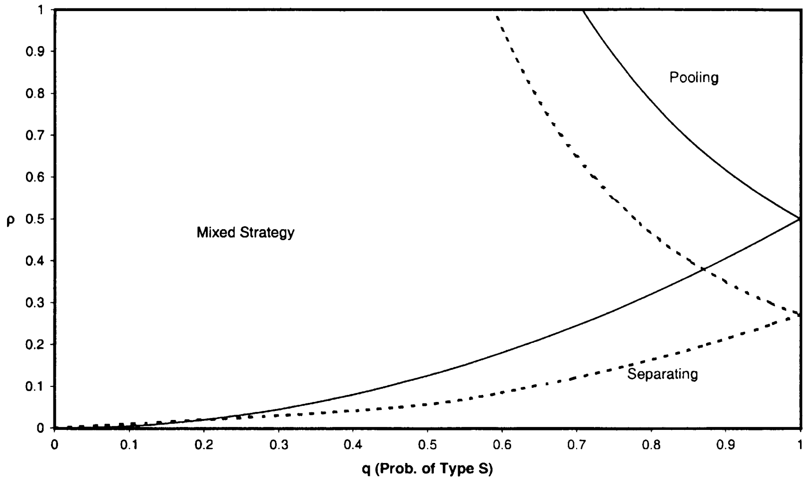

Fig. 2. Equilibria for $\gamma = 1$ (solid) and $\gamma = 0.5$ (dashed)

expansion in period t even though first period inflation is anticipated correctly. A high q lowers expected second period inflation which is expansionary in period t. It is more expansionary, the larger is k.

A deviation from the pooling equilibrium means that the type W inflates in period t at a rate that differs from π_1^a. The optimal deviation is to inflate at the one-shot discretionary rate π^d, and, since the public now knows a type W is in office, expected inflation for period $t + 1$ jumps to π^d. Appendix 2 shows that type W will deviate from the pooling equilibrium if and only if

$$\psi < \tfrac{1}{2}q^{-2} \tag{5}$$

Since $\psi = \rho/(1 - k)^2$, deviation from the pooling equilibrium is more likely if k is small (less weight on forward looking expectations), ρ is small (less weight placed on period 2 outcomes by the central bank), or q is small (high probability of a type W).

Figure 2 illustrates the combinations of ρ and q consistent with pooling and separating for $\gamma = 1$ $(k = 0$, the Cukierman and Liviatan case) and $\gamma = 0.5(k = 0.33)$. Two conclusions are immediately apparent. First, the set of discount factors and initial probabilities of type S consistent with pooling increases as γ falls. Thus, the greater the prevalence of multi-period wage contracts, the more likely it is that pooling results and even a type W inflates at a rate designed to mimic the type S. Second, the set of discount factors and initial probabilities of type S consistent with a separating equilibrium decreases as γ falls. If multi-period wage contracts are important, it is unlikely even an impatient type W would find it advantageous to inflate at the one-shot discretionary rate.

A large region in the figure is labeled as mixed strategy equilibria. We have not yet demonstrated this, a point to which we now turn.

3.3 Mixed strategies

Neither pooling nor separating is supportable if

$$\tfrac{1}{2}q^2 < \psi < \tfrac{1}{2}q^{-2}$$

However, there exist mixed strategy equilibria in this case. Consider the following strategy for type W in period t

$$\pi_t^W = \begin{cases} \pi_1^a \text{ with probability } P \\ \pi^d \text{ with probability } 1 - P \end{cases}$$

In period $t+1$, type W always inflates at the rate π^d.

If $\pi_t^W = \pi^d$ (an event that occurs with probability $1 - P$), private agents know the central bank is a type W and $\mathrm{E}_t\pi_{t+1} = \pi^d$. If $\pi_t^W = \pi_1^a$ (an event that occurs with probability P), then private agents are still unable to determine the central bank's identity, but with $P < 1$, observing π_1^a is more likely to have occurred if the true type is actually S. Therefore, if the central bank delivers on its announced inflation rate, agents will update their prior probabilities using Bayes rule. If q_1 is the initial probability of a type S and q_2 is the revised probability

$$q_2 = \frac{q_1}{q_1 + P(1 - q_1)} \tag{6}$$

The type W central banker must be indifferent between picking the same inflation rate as type S with probability P and picking the discretionary rate π^d with probability $1 - P$. Appendix 2 shows that this requires the probability P to satisfy a quadratic, only one solution of which satisfies the condition $0 \leq P \leq 1$. This solution is given by

$$P = \frac{-q_1}{1 - q_1} + \frac{1}{1 - q_1}\sqrt{q_1\sqrt{2\psi}} \tag{7}$$

As occurred in the case of separating and pooling equilibria, the central bank's discount factor ρ and the weight on future expectations k affect P solely through the parameter ψ. Increases in either ρ or k (decreases in γ) increase the probability that the type W will mimic the type S in period 1.

Figure 3 shows the optimal mimicking probability P as a function of q_1 for $\gamma = 1$, $\gamma = 0.5$ and $\gamma = 0$. P is decreasing in γ; fewer one-period contracts and greater weight on future inflation increase the likelihood that a type W will mimic the behavior of a type S during period 1. With expected future inflation playing a larger role, the cost to a type W of revealing itself rises, so a type W is more likely to mimic the behavior of a type S. As the figure also shows, the optimal probability of mimicking is increasing in the initial probability that a type S will be selected. If the public attaches a high prior probability to the central bank being a type S, expected inflation will be low and a type W has a larger incentive to mimic the behavior of a type S.

3.4 Implications

The model has several interesting implications. The dependence of the conditions for determining whether the equilibrium is separating, pooling or mixed strategy

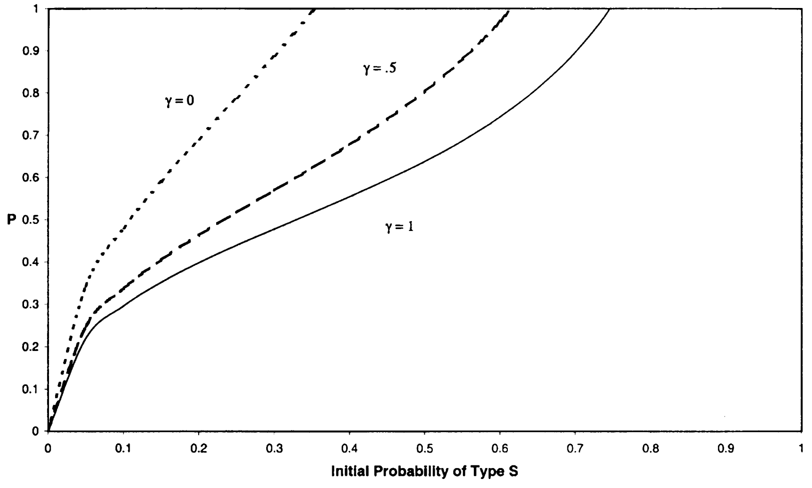

Fig. 3. Probability of mimicking $(\rho = 0.9)$

on the parameter $\psi = \rho/((1-k)^2)$ implies that forward looking market expectations play much the same role as the central bank's discount factor. Even a type *W* central bank who places little weight on future outcomes (i.e. has a low ρ) may find it advantageous to mimic the type *S* central bank if *k* is large. In that sense, the market forces the type *W* to be patient. Because revealing oneself as a type *W* has a cost in the form of higher expected future inflation, the public's response to policy serves to discipline deviations from the policies of a type *S*. Thus, type *W* central banks are unlikely to separate, at least not with certainty. Even moderate values of *k* make the separating equilibrium unlikely and imply a quite high probability that the type *W* will mimic in a mixed strategy equilibrium. If forward looking expectations are important, then, market discipline may serve to constrain the behavior of the central bank, forcing the central bank to 'do the right thing' as argued by McCallum (1995, 1997).[14]

A second implication is that the standard prediction that a recession occurs under a type *S* is not robust when expectations respond to current instrument settings and expected future inflation affects current inflation. If period 1 policy causes the public to reduce the inflation rate they expect in period 2, this can offset the otherwise contractionary impact of a low inflation policy in period 1. The effect of forward looking expectations is to make more immediate the credibility benefits that are gained from having a type *S* setting policy. If this effect is strong enough, output can actually rise when the public perceives that the central banker is a type *S*.

[14] This role of forward looking expectations is distinct from the role of staggered wage contracts in reducing the one-shot discretionary inflation rate.

4. Interest rates and the 'inflation scare problem'

In the previous section, inflation was treated as the direct policy instrument of the central bank. This approach is common, but central banks typically operate using a short-term nominal interest rate as their instrument of monetary policy. Because of the simple structure of the model, the equilibrium values of output and inflation could be determined without specifically deriving the implied values for interest rates. It is useful to introduce interest rates explicitly into the analysis, however, to examine how they respond to information about the central bank type.

The one-period nominal interest rate, denoted by i_t, is now assumed to be the central bank's policy instrument. Output is assumed to be related through an IS relationship to a longer term real rate of interest, with variations in the policy rate affecting output through the term structure of interest rates. To maintain an analytically simple framework, the long rate is taken to be a two-period expected real rate of interest. In this case

$$y_t = -bR_t \tag{8}$$

where R is defined by[15]

$$R_t = \tfrac{1}{2}[i_t - \mathrm{E}_t\pi_{t+1} + \mathrm{E}_t i_{t+1} - \mathrm{E}_t\pi_{t+2}] \tag{9}$$

The corresponding two-period nominal rate of interest is defined by

$$I_t = \tfrac{1}{2}[i_t + \mathrm{E}_t i_{t+1}] = R_t + \tfrac{1}{2}[\mathrm{E}_t\pi_{t+1} + \mathrm{E}_t\pi_{t+2}] \tag{10}$$

Through its choice of i_t, and the public's expectations about inflation and future policy, the central bank influences aggregate spending through the IS relationship. Movements in output, together with expectations, then affect inflation through an expectations augmented Phillips Curve, which, in the present model, is obtained by rewriting eq. (2) in the form

$$\pi_t = (1-k)\mathrm{E}_{t-1}\pi_t + \rho k\mathrm{E}_t\pi_{t+1} + a^{-1}y_t$$

Marvin Goodfriend (1993) has provided an interpretation of Federal Reserve policy in terms of what he describes as the 'inflation scare problem'. According to Goodfriend, the Fed was, at several times, faced with increases in long-term interest rates that reflected increases in expected future inflation. To counteract this rise in expected inflation, and maintain actual inflation at targeted levels, the Fed was forced to raise short-term real rates. In the context of Europe, reduced confidence that the European Central Bank will maintain low inflation could generate a similar inflation scare problem. This phenomena can be illustrated in the present model by considering the impact on interest rates of an exogenous shift in the public's assessment of future monetary policy. This could occur if the public's priors over the likely type of a future central banker were to change. An inflation scare

[15] Recall that y is an output gap measure, so R is interpreted as the deviation from the real rate consistent with a zero output gap. Kerr and King (1996), McCallum and Nelson (1997), and Walsh (1998) develop IS relationships in which $\mathrm{E}_t y_{t+1}$ appears, thus generating another channel through which expectations affect the current equilibrium.

problem may arise if there is a rise in the perceived probability that the next central banker will be a type W; this corresponds to a fall in q_1.[16]

Suppose a new central banker is appointed in period t. The real interest rate during the second period of the central banker's term can be written as

$$R_{t+1} = -\frac{a}{b}[\pi_{t+1} - (1-k)\mathrm{E}_t\pi_{t+1} - \rho k \mathrm{E}_{t+1}\pi_{t+2}]$$

where $\mathrm{E}_{t+1}\pi_{t+2}$ denotes the expectation, formed during the second period of the current central banker's term of office, of inflation during the first period of the next central banker's term. The impact on R_{t+1} of a change in q_1 is given by

$$\frac{\partial R_{t+1}}{\partial q_1} = \left(\frac{\rho a k}{b}\right)\left(\frac{\partial \mathrm{E}_{t+1}\pi_{t+2}}{\partial q_1}\right) \tag{11}$$

The exact expression for $(\partial \mathrm{E}_{t+1}\pi_{t+2})/\partial q_1$ will depend on the nature of the equilibrium. In a mixed strategy equilibrium, $\mathrm{E}_{t+1}\pi_{t+2} = [q_1 + (1-q_1)P]\pi_1^a + (1-q_1)(1-P)\pi^d$, so

$$\frac{\partial \mathrm{E}_{t+1}\pi_{t+2}}{\partial q_1} = (1-P)(\pi_1^a - \pi^d) + (q_1 + (1-q_1)P)\frac{\partial \pi_1^a}{\partial q_1} + (1-q_1)(\pi_1^a - \pi^d)\frac{\partial P}{\partial q_1} \tag{12}$$

All three terms in (12) are non-positive, contributing to higher expected inflation if q_1 falls. The first term reflects the effect of a change in q_1 on the probability that the inflation rate π_1^a rather than π^d will occur in the following period. Since $\pi_1^a < \pi^d$, a fall in q_1 makes π_1^a less likely and so increases expected future inflation. This can be thought of as the direct channel; since a type S inflates at a lower rate than a type W, an increased probability of a type W will raise expected inflation. Indirect effects operate as well since a change in q_1 will affect the inflation rate a type S would pick and the probability a type W will mimic a type S. The second term in (12) arises because a change in the public's assessment that a type S is in office affects the inflation rate a type S finds it optimal to announce for the first period in office. Announcements have less of an effect on expectations when q_1 is small, and eq. (23) shows that π_1^a decreases with q_1. Finally, the third term in (12) reflects the impact of q_1 on the optimal probability of mimicking. From (7), this probability is increasing in q_1. A fall in q_1 therefore decreases the probability a type W will inflate at the rate π_1^a rather than at the rate π^d; this raises expected future inflation. All three channels operate to increase expected future inflation if q_1 decreases; thus, from (11), a fall in the probability a type S will be appointed (a reduction in q_1) forces the current central banker (regardless of type) to raise the real rate of interest.

The effect of a fall in q_1 on the two-period nominal interest rate I_{t+1} depends on the effect on the two-period real rate and on any change in expected future inflation

[16] In an alternative interpretation, Kerr and King (1996) suggest the inflation scare problem might arise if the rational expectations solution is of the backward looking form that allows for sunspot equilibria.

$$\frac{\partial I_{t+1}}{\partial q_1} = \frac{\partial R_{t+1}}{\partial q_1} + \frac{1}{2}\left(\frac{\partial E_{t+1}\pi_{t+2}}{\partial q_1} + \frac{\partial \mathrm{E}_{t+1}\pi_{t+3}}{\partial q_1}\right)$$

The nominal rate moves more than the real rate since a change in q_1 causes expected future inflation to change in the same direction as the current real rate; a fall in q_1 increases R_{t+1} and expected future inflation. $(\partial \mathrm{E}_{t+1}\pi_{t+2})/(\partial q_1)$ has already been derived in equation (12); $(\partial \mathrm{E}_{t+1}\pi_{t+3})/(\partial q_1)$ equals the effect of a change in q_1 on the time $t+1$ expectation of period $t+3$ inflation. Period $t+3$ represents the second period in office of the central banker appointed at time $t+2$. $\mathrm{E}_{t+1}\pi_{t+3}$ increases with a fall in the prior probability future policy will be conducted by a type S central banker, with the same three channels discussed above operatng. Since a type W is likely to inflate at a higher rate, an increased probability of a type W directly raises expected inflation. A fall in q_1 also makes it less likely a type W will attempt to mimic a type S, and this further raises expected inflation. Finally, a fall in q_1 affects the inflation rate that a type S will pick, so that the inflation expected even if a type S is appointed rises. These direct and indirect effects raise the inflation premium incorporated into the two-period nominal rate. They are reinforced by the effect of a change in q_1 on the expected two-period real interest rate. Thus, inflation scares raise nominal interest rates by increasing both expected real returns and expected future inflation.

5. Conclusions

The specific model analyzed here focused on the presence of forward looking expectations in the aggregate supply relationship due to the presence of two-period nominal wage contracts. This provided a convenient means of developing a comparison with the standard specfication in terms of the single parameter k. Expectations of future inflation responded to current policy because today's policy decisions potentially reveal information about the central bank, causing private agents to change their expectations concerning future inflation. If the parameter k is related to the relative proportions of one-period and two-period wage contracts, an increase in the importance of multi-period contracts both reduces the one-shot discretionary inflation rate (as Goodhart and Huang showed) and makes it more likely a weak central bank will mimic a strong central bank.

The implications for output movements were found to be much richer than in the standard model. Economic expansions and low inflation may co-exist if the central bank is revealed to be committed to fulfilling its announced inflation targets. Consistent with Goodfriend's notion of inflation scares, a rise in expected future inflation forces the central bank to raise current real interest rates. Thus, uncertainty over future policy, such as may occur while Europe waits to see how the ECB will conduct policy, can force policy makers to alter their current policy stance.

Several extensions of the model would be worth exploring. First, the framework could be modified to incorporate a role for stabilization and exogenous non-policy disturbances. A second extension would involve generalizing the treatment of the term of office. Waller and Walsh (1996) have shown how the relationship between the central banker's term of office and the weight placed on inflation objectives (Rogoff's degree of conservatism) can interact to affect fluctuations in real economic activity. The assumption of a fixed, nonrenewable two-period term of office simplified the present analysis, but it would be interesting to pursue the role reappointment might play in affecting the conduct of policy.[17]

Acknowledgements

I would like to thank three anonymous referees and seminar participants at the Bank of Italy, a CEPR Conference at the European University Institute, and the Federal Reserve Board for helpful comments, and Alessandra Cassar for research assistance. This research was partially supported by the National Science Foundation.

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Appendix 1

Derivation of eq. (2)

Suppose a fraction γ of all firms and workers set wages for one period at the start of each period; prices then adjust during each period. Call these worker-firms type *A*. The remaining fraction $1-\gamma$ of all firms and workers set nominal wages for two periods, and, following the standard formulation, assume that one half of these firms adjust each period. Such worker-firms will be called type *B*.

Suppose that all wage contracts set the nominal wage to achieve a target equilibrium (log) real wage ω. By choice of normalization, set $\omega = 0$. Prices are free to adjust, and this means that price movements affect the real wage and, via labor demand, employment. Type *A* worker and firms set the log nominal wage at the beginning of the period equal to

$$w_t^A = \omega + \mathrm{E}_{t-1}p_t = \mathrm{E}_{t-1}p_t$$

The actual log real wage in period t will be given by

$$w_t^A - p_t = -(p_t - \mathrm{E}_{t-1}p_t)$$

If the aggregate production function is Cobb–Douglas, with labor's share equal to a_L (so $Y_t = L_t^{a_L}$), labor demand and employment will, in log terms, be given by

$$n_t = \left(\frac{1}{1-a_L}\right)(p_t - \mathrm{E}_{t-1}p_t)$$

and output by

$$y_t^A = \left(\frac{a_L}{1-a_L}\right)(p_t - \mathrm{E}_{t-1}p_t) = \bar{a}(\pi_t - \mathrm{E}_{t-1}\pi_t) \tag{13}$$

where $\bar{a} = (a_L/1-a_L)$. This is eq. (1) of the text.

Now turn to type *B* contracts. Let ω_t^B denote the nominal wage for two-period contracts negotiated in period t. The average expected discounted real wage over the life of the contract is $\frac{1}{2}[w_t^B - p_t + \rho(w_t^B - \mathrm{E}_t p_{t+1})]$, where $0<\rho<1$ is the discount factor. As with the type *A* contracts, assume the contract nominal wage is set to achieve an average expected discounted real wage of ω:

$$\tfrac{1}{2}[w_t^B - p_t + \rho(w_t^B - \mathrm{E}_t p_{t+1})] = \omega = 0$$

Rearranging

$$w_t^B = \frac{p_t + \rho E_t p_{t+1}}{1+\rho} \tag{14}$$

Employment will depend on the actual real wage. For type B firms setting wages in period t, the real wage is

$$\begin{aligned} w_t^B - p_t &= \frac{p_t + \rho E_t p_{t+1}}{1+\rho} - p_t \\ &= \frac{\rho E_t \pi_{t+1}}{1+\rho} \end{aligned}$$

For type B firms who set nominal wages in period $t-1$ (and these are still in effect since contracts run for two periods), the real wage faced in period t is

$$\begin{aligned} w_{t-1}^B - p_t &= \frac{p_{t-1} + \rho E_{t-1} p_t}{1+\rho} - p_t \\ &= -\left(\pi_t - \frac{\rho E_{t-1}\pi_t}{1+\rho}\right) \end{aligned}$$

Thus, employment in sector B will be

$$n_t = -\frac{1}{2}\left(\frac{1}{1-a_L}\right)\left[\left(\frac{\rho E_t \pi_{t+1}}{1+\rho}\right) - \left(\pi_t - \frac{\rho E_{t-1}\pi_t}{1+\rho}\right)\right]$$

and output is

$$y_t^B = \tfrac{1}{2} a[\pi_t - \bar{\rho} E_t \pi_{t+1} - \bar{\rho} E_{t-1}\pi_t] \tag{15}$$

where $\bar{\rho} \equiv (\rho/1+\rho)$.

Combining (13) and (15), aggregate output will equal the weighted average $\gamma y_t^A + (1-\gamma) y_t^B$, or

$$y_t = \bar{a}\{\gamma(\pi_t - E_{t-1}\pi_t) + \tfrac{1}{2}(1-\gamma)[\pi_t - \bar{\rho} E_t \pi_{t+1} - \bar{\rho} E_{t-1}\pi_t]\}$$

Collecting terms,

$$\begin{aligned} y_t &= \bar{a}(\gamma + \tfrac{1}{2}(1-\gamma))\left[\pi_t - \left(\frac{\gamma + \frac{1}{2}\bar{\rho}(1-\gamma)}{\gamma + \frac{1}{2}(1-\gamma)}\right) E_{t-1}\pi_t - \bar{\rho}\left(\frac{\frac{1}{2}(1-\gamma)}{\gamma + \frac{1}{2}(1-\gamma)}\right) E_t \pi_{t+1}\right] \\ &= a[\pi_t - (1-k) E_{t-1}\pi_t - \rho k E_t \pi_{t+1}] \end{aligned} \tag{16}$$

where

$$a = \frac{\bar{a}(1+\gamma)}{2} = \left(\frac{1+\gamma}{2}\right)\left(\frac{a_L}{1-a_L}\right)$$

and

$$k \equiv \left(\frac{1-\gamma}{1+\gamma}\right)\left(\frac{1}{1+\rho}\right)$$

Equation (16) is eq. (2) of the text.

Appendix 2

Derivation of equilibria

1. *Separating equilibrium*

To characterize a separating equilibrium, we begin with the decision problem of a type W in period $t+1$, the last period of the two-period term of office. A type W's objective is to choose inflation to maximize $y_{t+1} - \frac{1}{2}\beta\pi_{t+1}^2$ subject to

$$y_{t+1} = a[\pi_{t+1} - (1-k) E_t \pi_{t+1} - \rho k E_{t+1}\pi_{t+2}] \tag{17}$$

taking $E_t\pi_{t+1}$ and $E_{t+1}\pi_{t+2}$ as given. Private agents have already committed to $E_t\pi_{t+1}$ (it was formed in period t), while nothing the central banker does in period $t+1$ will influence $E_{t+1}\pi_{t+2}$ since π_{t+2} will be determined by a new central banker. The first order condition implies

$$\pi_{t+1}^W = \frac{a}{\beta} \equiv \pi^d$$

Since a type W's identity has (by definition) been revealed in period t of a separating equilibrium, $E_t\pi_{t+1} = \pi^d$. Using (17), it is straightforward to show that the type W's utility in period $t+1$ is

$$U_2^W = ak[\pi^d - \rho E_{t+1}\pi_{t+2}] - \tfrac{1}{2}a\pi^d \tag{18}$$

Higher expected future inflation lowers the central bank's utility by reducing the output level consistent with achieving a current inflation rate of π^d.

In the first period of a separating equilibrium, the problem faced by a type W is to pick π_t to maximize $U_1^W + \rho U_2^W$, subject to (2), where U_2^W is given by (18) and $E_t\pi_{t+1} = \pi^d$ since type is revealed in a separating equilibrium. The optimal choice of period t inflation is $\pi_t^W = \pi^d$. So in a separating equilibrium, the type W central bank inflates at the one-shot discretionary rate π^d each period.

A type S announces inflation targets at the start of perod t and then adjusts policy to ensure actual inflation equals the announced targets. Immediately after announcements are made, $E_{t-1}\pi_t = q\pi_t^a + (1-q)\pi^d$. The optimal policy for a type S consists of announcements (π_t^a, π_{t+1}^a) and actual inflation (π_t, π_{t+1}) that maximize

$$[y_t - \tfrac{1}{2}, \beta\pi_t^2] + \rho[y_{t+1} - \tfrac{1}{2}\beta\pi_{t+1}^2]$$

subject to $\pi_i = \pi_i^a$ for $i = t, t+1$,

$$y_t = a[\pi_t - (1-k)[q\pi_t^a + (1-q)\pi^d] - \rho k\pi_{t+1}^a] \tag{19}$$

and

$$y_{t+1} = a[\pi_{t+1} - (1-k)\pi_{t+1}^a - \rho k E_{t+1}\pi_{t+2}] \tag{20}$$

Note that $E_t\pi_{t+1} = \pi_{t+1}^a$ since a type S's actions in period t reveal its type to the public. The first order condtions for this problem yield

$$\pi_t^a = [1 - (1-k)q]\pi^d < \pi^d$$

and

$$\pi_{t+1}^a = 0$$

For the separating outcome to be an equilibrium, the type W central banker must have no incentive to deviate from it. Using the results in Table 1, utility for the type W in the separating equilibrium is equal to

$$\begin{aligned} U_{sep}^W &= a[\pi^d - (1-k)E_{t-1}\pi_t - \rho k\pi^d] + \rho ak[\pi^d - \rho E_{t+1}\pi_{t+2}] - \tfrac{1}{2}(1+\rho)a\pi^d \\ &= a[\pi^d - (1-k)E_{t-1}\pi_t] - \rho^2 ak E_{t+1}\pi_{t+2} - \tfrac{1}{2}(1+\rho)a\pi^d \end{aligned}$$

A deviation would involve setting the inflation rate in period t equal to $[1-(1-k)q]\pi^d$ rather than π^d. Since the public, observing this rate of inflation, would infer that the true type is S, expected second period inflation will be zero, not π^d as would be the case if W had separated. Utility for the type W under a deviation would be

$$\begin{aligned} U_{sep}^{dev} = a[(1-(1-k)q)\pi^d - (1-k)E_{t-1}\pi_t] - \tfrac{1}{2}a\pi^d[1((1-k)q]^2 \\ + \rho a[\pi^d - \rho k E_{t+1}\pi_{t+2}] - \tfrac{1}{2}\rho a\pi^d \end{aligned}$$

since in period 2 type W will inflate at the rate π^d while the public will expect zero inflation.

The type W will deviate from the separating equilibrium if and only if $U^{dev}_{sep} > U^{W}_{sep}$. But using the expressions for U^{dev}_{sep} and U^{W}_{sep}, this condition holds if and only if $\rho > \frac{1}{2}(1-k)^2 q^2$, which can be rewritten as condition (4) of the text.

2. *Pooling equilibrium*

In a pooling equilibrium, both types will inflate at the announced rate during the first period in office. Hence, if a new central banker is appointed in period t, $E_{t-1}\pi_t = \pi^a_t$. In period $t+1$, the public is still uncertain as to the true type holding office, so $\mathrm{E}_t\pi_{t+1} = q\pi^a_{t+1} + (1-q)\pi^d$ since with probability $1-q$ a type W is in office and will inflate at the rate π^d. The optimal policy for a type S consists of announcements (π^a_t, π^a_{t+1}) and actual inflation (π_t, π_{t+1}) that maximize

$$[y_t - \tfrac{1}{2}\beta\pi^2_t] + \rho[y_{t+1} - \tfrac{1}{2}\beta\pi^2_{t+1}]$$

subject to $\pi_i = \pi^a_i$ for $i = t, t+1$

$$\begin{aligned} y_t &= a[\pi_t - (1-k)\pi^a_t - \rho k(q\pi^a_{t+1} + (1-q)\pi^d)] \\ &= a[k\pi^a_t - \rho k(q\pi^a_{t+1} + (1-q)\pi^d)] \end{aligned}$$

and

$$\begin{aligned} y_{t+1} &= a[\pi_{t+1} - (1-k)(q\pi^a_{t+1} + (1-q)\pi^d) - \rho k\mathrm{E}_{t+1}\pi_{t+2}] \\ &= a[(1-(1-k)q)\pi^a_{t+1} - (1-k)(1-q)\pi^d - \rho k\mathrm{E}_{t+1}\pi_{t+2}] \end{aligned}$$

The first order conditions imply

$$\pi^a_t = \frac{ak}{\beta} = k\pi^d$$

and

$$\pi^a_{t+1} = \frac{a(1-q)}{\beta} = (1-q)\pi^d < \pi^d$$

A deviation from the pooling equilibrium means that the type W inflates in period t at a rate that differs from π^a_1, thereby revealing her identity. The optimal deviation is $\pi^{dev} = \pi^d$. Expected inflation for period $t+1$ then jumps to π^d. But in period t, $\mathrm{E}_{t-1}\pi_t = k\pi^d < \pi^d$ since the public expected the pooling outcome. Output under a deviation by type W will be $a\pi^d[1-(1-k)k - \rho k]$ in period 1 and $a\pi^d k(1-\rho k)$ in period 2. If the type W pools, utility for the type W will be

$$\begin{aligned} U^W_{pool} &= a\pi^d[k - \rho(1-q^2)] - \tfrac{1}{2}ak^2\pi^d \\ &\quad + \rho a\pi^d[1-(1-k)(1-q^2) - \rho k^2] - \tfrac{1}{2}\rho a\pi^d \end{aligned} \tag{21}$$

The utility of type W from a deviation is

$$U^{dev}_{pool} = a\pi^d[1 - k + k^2(1-\rho^2)] - \tfrac{1}{2}a\pi^d(1+\rho) \tag{22}$$

Type W will deviate from the pooling equilibrium if and only if $U^{dev}_{pool} > U^W_{pool}$, or, comparing (21) and (22), if and only if $\psi < \frac{1}{2}q^{-2}$, which is condition (5) of the text.

3. Mixed strategy equilibrium

Consider the strategy of type S. At the start of period 1, the central bank must announce a path for inflation $\{\pi^a_1\pi^a_2\}$ that a type S is committed to delivering. The decision problem faced by the type S upon coming into office is to maximize $(y_t - \frac{1}{2}\beta\pi^2_t) + \rho(y_{t+1} - \frac{1}{2}\beta\pi^2_{t+1})$ subject to $\pi_t = \pi^a_1$ and $\pi_{t+1} = \pi^a_2$, together with

$$\pi_t = (1-k)\{q\pi_1^a + (1-q)[P\pi_1^a + (1-P)\pi^d]\} - \rho k[q_2\pi_2^a + (1-q_2)\pi^d] + ay_t$$

and

$$\pi_{t+1} = (1-k)[q_2\pi_2^a + (1-q_2)\pi^d] - \rho k \mathrm{E}_{t+1}\pi_{t+2} + ay_{t+1}$$

The solution to this decision problem yields

$$\pi_t^S = \pi_1^a = [1-(1-k)(q_1+(1-q_1)P)]\pi^d \tag{23}$$

and

$$\pi_{t+1}^S = \pi_2^a = (1-q_2)\pi^d \tag{24}$$

With type S announcing an inflation rate of (23) in period t, the type W central banker must be indifferent between picking the same inflation rate as type S with probability P and picking the discretionary rate π^d with probability $1-P$. Type W's utility from picking π_t^S, and therefore mimicking so that $\mathrm{E}_t\pi_{t+1} = q_2\pi_2^a + (1-q_2)\pi^d = (1-q_2^2)\pi^d$, is given by

$$U^W(\pi_1^a) = a[\pi_1^a - (1-k)(q_1\pi_1^a + (1-q_1)[P\pi_1^a + (1-P)\pi^d]) - \rho k(1-q_2^2)\pi^d] - \tfrac{1}{2}\beta(\pi_1^a)^2 + \rho a[\pi^d - (1-k)(1-q_2^2)\pi^d - \rho k \mathrm{E}_{t+1}\pi_{t+2}] - \tfrac{1}{2}\rho a\pi^d$$

Type W's utility from inflating at the rate π^d in period 1 (which reveals W's type) is equal to

$$U^W(\pi^d) = a[\pi^d - (1-k)(q_1\pi_1^a + (1-q_1)[P\pi_1^a + (1-P)\pi^d]) - \rho k\pi^d] - \tfrac{1}{2}\beta\pi^{d2}(1+\rho) + \rho a\pi^d k(1-\rho \mathrm{E}_{t+1}\pi_{t+2})$$

Setting these two equal implies that $U^W(\pi_1^a) = U^W(\pi^d)$ if and only if

$$\frac{q_1\sqrt{2\psi}}{q_1 + P(1-q_1)} = q_1 + P(1-q_1)$$

This yields a quadratic in P, only one solution of which satisfies the condition $0 \le P \le 1$. This solution is given by eq. (7) of the text.