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Labour market search and monetary shocks

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1 INTRODUCTION

In recent years, dynamic stochastic general equilibrium (DSGE) models of monetary economies have focused on the role of nominal rigidities in affecting the economy's adjustment to monetary policy and non-policy disturbances. While these rigidities appear important for understanding the impact nominal shocks have on such real variables as output and employment, models with only nominal rigidities have been unable to match the responses to monetary disturbances that have been estimated in the data. Typically, empirical studies have concluded that monetary shocks generate large and persistent real responses that display a hump shape. After a positive money shock, for example, output rises over several quarters and then declines. Christiano, Eichenbaum and Evans (1999) document this effect and provide an extensive discussion of the empirical evidence on the effects of monetary shocks. Sims (1992) finds large, hump-shaped responses of real output to monetary shocks in several OECD countries. Inflation also displays a hump-shaped response, although inflation is usually found to respond more slowly than output to monetary shocks.

The 'stylised facts' emphasised by Christiano, Eichenbaum and Evans, by Sims, and by others are illustrated in figure 9.1, which shows estimated impulse responses of output and inflation following a shock to the growth rate of money. These responses were obtained from a three-variable VAR (output, inflation, and money growth) estimated using US quarterly data for 1965–2001. Output is real GDP, inflation is measured by the Consumer Price Index, and M2 is the aggregate used to measure money. The real persistence and inflation inertia seen in figure 9.1 has been hard for

I would like to thank Ryota Kojima for research assistance, and the referees, seminar participants at UC Davis and UCSB and participants in the Brown Bag workshop at UCSC for helpful comments on this research.

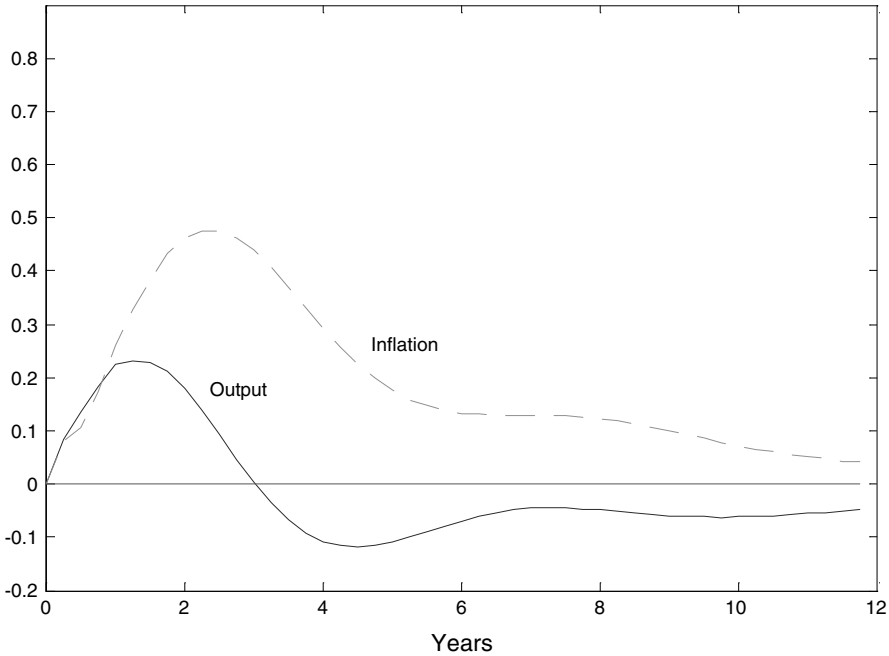

Figure 9.1 Impulse responses to a money growth rate shock (estimated VAR)

models based on nominal rigidities to match.[1] As Dotsey and King (2001) have expressed it, 'modern optimizing sticky price models have displayed a chronic inability to generate large and persistent real responses to monetary shock'.

In order to capture at least some of the real persistence seen in empirical studies, models based on nominal rigidity generally must assume a high degree of price stickiness. For example, it is common to assume that individual prices remain fixed on average for as much as nine months. Micro data on individual prices, however, suggests that prices typically change more frequently than this. Consequently, a number of researchers have recently argued that simply adding nominal rigidities to an otherwise standard DSGE model is not sufficient to match the persistence observed in the data. Instead, the real side of the economy must also be modified to capture additional factors affecting both aggregate production and aggregate spending decisions. Christiano, Eichenbaum and Evans (2001), in particular, have argued that models with nominal rigidities are capable of

[1] Chari, Kehoe and McGratten (2000) find little persistence in their model of staggered price adjustment, and Nelson (1998) demonstrates that several optimising models of price stickiness are unable to match the time series properties of inflation.

matching macro evidence but only if the real side of the model is properly specified.

Despite the recent interest on the real side of monetary models, almost all existing DSGE models of monetary economies continue to assume that labour can be costlessly and instantaneously reallocated across firms. Yet the complex process through which workers seeking jobs and firms with open vacancies are matched is likely to be important in influencing the way economic disturbances are propagated over time. The work of Mortensen and Pissarides (1994, 1999) and Pissarides (2000) has emphasised the costly and time-consuming process of matching workers seeking employment with firms seeking to fill job vacancies. How quickly unemployment returns to its steady-state level after an adverse shock, whether it is a real or nominal shock, is likely to be influenced by how efficiently the labour market is able to generate new matches between firms and unemployed workers.

In this chapter, the dynamic implications of labour market search and sticky prices are analysed. The model that is developed is the first to combine a labour market structure based on a Mortensen–Pissarides aggregate matching function with an optimising model of price rigidity. The introduction of price stickiness allows the interactions between labour market rigidities and nominal price rigidities to be investigated.

Section 2 reviews some of the related literature that has focused on the labour market and other real aspects of the economy that affect the economy's response to nominal shocks. Section 3 develops the basic model. In section 4, the dynamic adjustment of the economy to nominal money growth rate shocks is examined under flexible prices and under sticky prices. Conclusions and some suggestions for further research are discussed in section 5.

2 RELATED LITERATURE

This chapter brings together two previously unrelated strands of the literature – models such as those of Andolfatto (1996), Cooley and Quadrini (1999, 2000) and Merz (1995) that study the implications of matching models of the labour market for DSGE models of the business cycle, and models such as those of Chari, Kehoe and McGratten (2000), Goodfriend and King (1997), Rotemberg and Woodford (1997) and Yun (1996), among others, that introduce price stickiness in DSGE models of monopolistic competition. Cole and Rogerson (1999) argue that aggregate labour market matching models based on the work of Mortensen and Pissarides (1994, 1999) can replicate important aspects of business cycles, but only

if the models are calibrated to imply average durations of unemployment spells that are much longer than data on actual duration suggest. Similarly, models of price stickiness are typically calibrated to imply individual prices are fixed for durations that are also much longer than suggested by evidence on individual prices (Bils and Klenow 2002). By incorporating both a labour market matching model and price stickiness within a single model, one can investigate whether the interactions of these two allow the model to match important business cycle facts with more plausible calibrations of unemployment duration and price rigidity. In fact, it has been the failure of models based solely on nominal price stickiness to account for output persistence and inertia in inflation that has led some researchers to explore the role of real factors that might interact with nominal rigidities to account for the dynamic responses seen in the data.

In an early contribution, Ball and Romer (1990) argued that real rigidities act to amplify the effects of nominal rigidities. More recently, Dotsey and King (2001) argue that 'real flexibilities' – variable capital utilisation and produced inputs – are critical for generating the persistence displayed in the data. These real flexibilities permit output to vary with relatively small effects on marginal cost, reducing the elasticity of marginal costs with respect to output. A nominal shock to aggregate demand and output has only a small impact on marginal cost, and therefore on inflation, when this elasticity is small.[2]

Similarly, Christiano, Eichenbaum and Evans (2001) (hereafter, CEE) emphasise that the interaction of real and nominal rigidities seems to be critical in matching empirical evidence. CEE allow for habit persistence in consumption, variable capital utilisation and investment adjustment costs (real rigidities) as well as both price and nominal wage stickiness. They conclude that nominal wage rigidity (as opposed to price rigidity) is critical in matching aggregate data for the USA. However, King and Goodfriend (2001: 4) take the position that nominal wage rigidity may not be important for business cycle phenomena, arguing that 'The labour market is characterised by long-term relationships where there is opportunity and reason for firms and workers to neutralise the allocative effects of temporarily sticky nominal wages'.[3]

The real rigidities introduced by long-term relationships between firms and workers has, to date, not been incorporated into models with nominal rigidities. This neglect is perhaps surprising. Jeanne (1998) combined a

[2] See Burnside and Eichenbaum (1996) for an earlier analysis of variable capital utilisation in a RBC model.

[3] The issue of nominal price versus nominal wage rigidity is analysed in detail in chapter 4 by Canzoneri, Cumby and Diba in this volume.

Calvo-type model of price stickiness with an ad hoc specification of the equilibrium real wage. This allowed him to exogenously vary the response of the real wage to output movements. He showed that an increase in labour market real wage rigidity reduced the degree of price stickiness that was needed to match the response of output to a monetary shock. This work suggested that the specification of the labour market might play an important role in explaining the persistent output response to nominal shocks, but his model provided no underlying theory to explain the source of the labour market rigidity.

Andolfatto (1996) and Merz (1995) have shown that a RBC model that incorporates a Mortensen–Pissarides aggregate matching function (Mortensen and Pissarides 1994, 1999; Pissarides 2000) to represent the search process in the labour market is able to provide a better match with evidence on employment and wages than do models based on a traditional Walrasian labour market. Cole and Rogerson (1999) note, however, that one must assume that workers face a small probability each period of finding a match if aggregate matching models are to capture important labour market behaviour. Den Haan, Ramey and Watson (2000) show how a search model of the labour market can serve to amplify and propagate productivity shocks. These models capture the long-term nature of employment relationships that King and Goodfriend emphasise alter the allocative implications of observed rigidity in nominal wages.

The only examples to date of monetary models with a matching model of the labour force are due to Cooley and Quadrini (1999, 2000). They introduced money into a DSGE model with a matching model of the labour market and show that monetary shocks have highly persistent impacts on inflation and the real economy. However, they assume that prices are completely flexible, and persistent real effects arise, in part, because they assume that nominal portfolios adjust slowly over time. This portfolio rigidity generates a liquidity effect (a fall in the nominal interest rate after an increase in money growth).[4] By assuming portfolio readjustments take several periods to occur, the real impact of money shocks is propagated over time in the Cooley–Quadrini model. Thus, their dynamics reflect both the specification of the labour market and the assumption of sticky portfolio adjustment.

While the work by Cooley and Quadrini has helped to highlight the role of the matching process in a monetary economy, their assumption of flexible prices meant that they were unable to study the possible interactions

[4] This form of limited participation model (Fuerst 1992) assumes that households cannot immediately reallocate their bond and money holdings after a monetary shock.

between the dynamic adjustment of employment through the matching process and the dynamic adjustment of inflation when prices are sticky.

3 THE MODEL ECONOMY

To study the interaction of labour search and price stickiness, I employ a model that distinguishes between wholesale and retail sectors. Goods are produced in the wholesale sector and then sold by retail firms to households. The production of wholesale goods requires that a firm and a worker be matched. Unmatched workers and firms with vacancies are matched through a process characterised by an aggregate matching function. Because some workers and firms will be matched, while others will not be, distributional issues arise. To avoid these issues, I follow Andolfatto (1996), den Haan, Ramey, and Watson (2000) and Merz (1995) in assuming that households pool consumption, both market-purchased consumption and home consumption produced by workers who do not have an employment match in period t. The output produced by worker–firm matches is sold in a competitive goods market to retail firms. These firms costlessly transform the wholesale good into retail goods that are sold to households in markets characterised by monopolistic competition. Prices at the retail level are sticky, with only a fraction of retail firms optimally adjusting their price each period. This separation between wholesale firms participating in the labour market matching process and retail firms with sticky prices follows the approach of Bernanke, Gertler and Gilchrist (1999) in their study of credit market imperfections. This separation simplifies the structure of the model.

Money is introduced through a cash-in-advance constraint.[5] Income generated in period t is not available for consumption until period $t + 1$, so the nominal interest rate affects the present discounted value of current production. This generates a channel through which nominal interest rate changes affect output and employment. An increase in the nominal interest rate reduces the present value of production and leads to an increase in job destruction, a fall in employment and a decline in output in the wholesale sector.

The role played by the labour market specification can be highlighted by considering the effects of an unanticipated increase in money growth in period t that persists for periods t and $t + 1$, after which money growth returns to its steady-state value. When prices are flexible, inflation rises immediately and remains above its steady-state value during periods t and

[5] For a general discussion of cash-in-advance models, see Walsh (2003, chapter 3).

$t + 1$. In period t, expected inflation also rises, causing the nominal rate of interest to rise. In period $t + 1$, the nominal rate returns to its steady-state value. In a model with an inelastic labour supply and no capital but with a standard Walrasian labour market, this money growth shock has no impact on employment or output. If labour supply is elastic, however, the rise in the nominal interest rate causes a substitution towards leisure (a 'cash' good), and employment and output fall, but only for one period.[6] In contrast, in the present model, the rise in the nominal interest rate leads to a rise in job destruction; employment and output fall. The economy enters period $t + 1$ with fewer employment matches. This propagates the output and employment decline into period $t + 1$. The matching process causes the return to the steady state to be spread over several periods. Thus, the labour market dynamics contribute to the persistence displayed by the economy in response to a money growth rate shock.

The adverse output effects of a rise in the nominal interest rate also have implications for inflation. The decline in wholesale output raises wholesale prices relative to retail prices. The fall in the retail price markup acts to raise retail price inflation. A positive impact on inflation of an increase in the nominal interest rate is variously called the 'cost channel of monetary policy' or the 'Wright Patman' effect, after the late, popularist Texas Congressman. Barth and Ramey (2001) have used industry-level data to examine this cost channel. They argue that the cost channel can account for the price puzzle – the finding in empirical VARs that inflation initially rises after an interest rate increase. Christiano, Eichenbaum and Evans (2001) also incorporate a cost channel into their model, but they do not provide information on its contribution to matching the dynamic responses of output and inflation to a monetary shock.

In the remainder of this section, the details of the model are developed.

3.1 Households

The representative household consists of a worker and a shopper. Each of these actors engages in different activities during the period before reuniting at the end of each period (Lucas and Stokey 1983, 1987). There are assumed to be a continuum of such households on the interval [0, 1]. Shoppers carry cash balances to the goods market to purchase market consumption goods; these purchases are subject to a cash-in-advance constraint. For simplicity, households are assumed to supply their unit of labour inelasticity.

[6] The effect can be spread over several periods if capital is introduced into the model. See, for example, Walsh (2003, chapter 3).

Households are also the owners of all firms in the economy. Households maximise the expected present discounted value of utility:

$$E_t \sum_{i=0}^{\infty} \beta^i \left[u(C_{t+i}) + (1 - \chi_t)h - \chi_t A\right] \tag{9.1}$$

where C_t is a composite consumption good consisting of the differentiated products produced by monopolistically competitive retail firms. There are a continuum of such firms of measure 1. C_t is defined as

$$C_t = \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \qquad \theta > 1. \tag{9.2}$$

The variable χ_t is an indicator variable, equal to 1 if the household's worker is employed and 0 otherwise. The disutility of work is A, and h is the utility of home production when unemployed.

Households maximise expected utility subject to two constraints. First, they face a cash-in-advance constraint that takes the form

$$P_t C_t \leq M_{t-1}^h + T_t - B_t \tag{9.3}$$

where M_t^h (B_t) is the household's nominal holdings of money (one-period nominal bonds), P_t is the retail price index and T_t is a lump-sum transfer received from the government. In the aggregate, this transfer is equal to $M_t - M_{t-1} = (G_t - 1)M_{t-1}$ where M (without the superscript h) is the aggregate nominal money stock. Note that current income is unavailable for purchasing current market consumption. This timing assumes that financial asset markets open before the goods market. Bonds purchased at the start of period t, B_t, pay a gross nominal interest rate of R_t. These interest payments are received when the asset market reopens in period $t + 1$. Thus, the budget constraint households face can be written as

$$M_t^h = P_t Y_t^l + D_t + R_t B_t + M_{t-1}^h + T_t - B_t - P_t C_t, \tag{9.4}$$

where Y_t^l is the household's real labour income and D_t is their share of aggregate profits from wholesale and retail firms.

Given prices p_{jt} for the final goods, this preference specification implies that the household's demand for good j is

$$c_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{-\theta} C_t, \tag{9.5}$$

where the aggregate retail price index P_t is defined as

$$P_t = \left[\int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}}. \tag{9.6}$$

The following two conditions, obtained from the household's first-order conditions and the cash-in-advance constraint, must hold in equilibrium:

$$\frac{u'_t}{P_t} = \beta R_t E_t \left(\frac{u'_{t+1}}{P_{t+1}} \right) \tag{9.7}$$

$$C_t = \frac{M_t}{P_t}, \tag{9.8}$$

where u' denotes the marginal utility of consumption. It will be convenient to define the one-period discount factor

$$\delta_t \equiv \beta E_t \left(\frac{P_t}{P_{t+1}} \frac{u'_{t+1}}{u'_t} \right), \tag{9.9}$$

so that (9.7) can be written as $\delta_t R_t = 1$.

3.2 The labour and goods markets

The production side of the model, and the labour market specification, is similar to that used by den Haan, Ramey and Watson (2000). Their focus is on the role of aggregate productivity shocks, and because their model does not incorporate money, they do not study the role of price stickiness. In order to simplify the non-monetary aspects of the model, I ignore the capital stock dynamics that den Haan, Ramey and Watson include. Production takes place in the wholesale sector, where firms and workers are paired through a matching process.

The wholesale sector

At the beginning of the period, there are N_t matched workers and firms; $U_t = 1 - N_t$ workers are unmatched. If a worker is part of an existing match at the start of period t, she travels to her place of employment. At that point, there is an exogenous probability $0 \leq \rho^x < 1$ that the match is terminated. For the $(1 - \rho^x)N_t$ surviving matches, the worker and firm jointly observe the current realisation of productivity and decide whether to continue the match. If the realisation of productivity is low enough, it will be unprofitable for the match to continue. If the match does continue,

production occurs. The output of a matched worker/firm pair i in period t that does produce is

$$y_{it} = a_{it} z_t \tag{9.10}$$

where a_{it} is a serially uncorrelated, match-specific productivity disturbance and z_t is a common, aggregate productivity disturbance. The means of both productivity disturbances are equal to 1, and both are bounded below by zero. Wholesale firms sell their output in a competitive market at the price P_t^w.

Firms seeking workers must incur a cost of posting a vacancy, and workers seeking jobs must engage in a search process that takes time. As a consequence, existing matches may earn an economic surplus, and both the firm and the worker will wish to maintain a match with a positive expected surplus. The expected surplus an existing match generates depends, in part, on the value of the current output the match produces. Because of the cash-in-advance constraint, proceeds from output produced in period t are available for consumption only in period $t + 1$. Thus, the time-t value of the revenues obtained from production in period t is $\delta_t P_t^w a_{it} z_t / P_t$, where δ_t is the discount rate given by (9.9). In addition, there is a continuation value of being part of an existing match that survives into period $t + 1$. Therefore, the expected value of a match that produces in period t is

$$\delta_t \left(\frac{a_{it} z_t}{\mu_t} \right) - A + g_{it},$$

where $\mu_t = P_t / P_t^w$ is the markup of retail over wholesale prices, and g_{it} is the expected present value of a match that continues into period $t + 1$.

To simplify, assume that the share of the surplus from a match received by each participant is fixed.[7] The surplus is the difference between $(\delta_t a_{it} z_t / \mu_t) - A + g_{it}$ and the alternative opportunities available to the firm and the worker. If the firm has no alternative opportunities, the match's opportunity cost, w_t^u, is equal to the value of home consumption an unmatched worker can produce plus the present value of future worker opportunities if unmatched in period t. Define

$$q_t \equiv g_t - w_t^u$$

as the expected excess value of a match that continues into period $t + 1$. Since all matches are identical, the subscript i has been suppressed. A match continues as long as $(\delta_t a_{it} z_t / \mu_t) - A + q_t \geq 0$. Matches endogenously

[7] The assumption of fixed shares is common and would arise under risk neutrality in a Nash bargaining solution.

separate if the match-specific productivity shock is less than $\tilde{a}_t$, where this critical value is defined using the definition of δ_t in (9.9) as

$$\tilde{a}_t = \frac{\mu_t R_t (A - q_t)}{z_t}. \tag{9.11}$$

If $A - q_t < 0$, then matches would never endogenously end since the support of a is strictly positive. When $A - q_t > 0$, matches do endogenously break up. In this case, a higher realisation of the aggregate productivity shock z_t will, *ceteris paribus*, lower $\tilde{a}_t$, making it more likely that existing matches produce. A higher z_t realisation directly increases the production of all matched worker/firms (see (9.10)). It also leads more matches to produce because fewer endogenously separate (see (9.11)). Thus, the role of z_t in affecting $\tilde{a}_t$ tends to amplify the impact of the aggregate productivity shock on output, an effect emphasised by den Haan, Ramey and Watson (2000). A decrease in the nominal interest rate also leads more matches to produce. Because income earned in period t is available for the household to consume only in period $t + 1$, a rise in the value of future income (as would be caused by a fall in R_t) makes current production more valuable and decreases the probability that a match will be dissolved.[8] A rise in the markup of retail over wholesale prices reduces the profitability of wholesale production and increases $\tilde{a}_t$. These results are only partial equilibrium effects, since changes in aggregate productivity or the nominal interest rate also affect w_t^u, the present discounted value of unemployment, and g_t, the present discounted value of a match.

Let ρ_t^n be the aggregate fraction of matches that endogenously separate, and let F denote the cumulative distribution function of the match specific productivity shock. Then the probability match i endogenously separates is $F(\tilde{a}_t)$ and, because all matches are identical, the aggregate endogenous separation rate is the probability that $a_t \leq \tilde{a}_t$:

$$\rho_t^n = \Pr\left[a_t \leq \tilde{a}_t\right] = F(\tilde{a}_t). \tag{9.12}$$

The aggregate total separation rate ρ_t is equal to

$$\rho_t = \rho^x + (1 - \rho^x)\rho_t^n \tag{9.13}$$

while the survival rate, $\varphi_t \equiv (1 - \rho_t) = (1 - \rho^x)\left[1 - F(\tilde{a}_t)\right]$, is decreasing in $\tilde{a}$.

[8] The non-neutrality that this creates is similar to other cash-in-advance models that, for example, require firms to pay wages prior to the receipt of revenues from production as in Carlstrom and Fuerst (1995), Christiano, Eichenbaum, and Evans (2001), or Cooley and Quadrini (1999). For evidence on the existence of such a 'cost channel' of nominal interest rates, see Barth and Ramey (2001).

Define the joint surplus of a worker/firm pair who are matched at the start of $t+1$ and do not separate as

$$s_{it+1} = \delta_{t+1}\left(\frac{a_{it+1}z_{t+1}}{\mu_{t+1}}\right) - A + q_{t+1}. \tag{9.14}$$

Note that this is expressed in terms of the present value as of the beginning of period $t+1$. Let η denote the share of this surplus received by the worker; the firm receives $1-\eta$ of the joint surplus. If an unmatched worker in period t succeeds in making a match that produces in period $t+1$, she receives her opportunity utility w^u_{t+1} plus the fraction η of the joint surplus, or $\eta s_{it+1} + w^u_{t+1}$. The probability of this occurring is $k^w_t(1-\rho_{t+1})$, where k^w_t is the period t probability an unmatched worker finds a job and $1-\rho_{t+1}$ is the probability that the match actually produces in period $t+1$. With probability $1-k^w_t(1-\rho_{t+1})$ the worker either fails to make a match or makes a match that fails to survive to produce in $t+1$. In either case, the worker is unmatched in $t+1$ and receives w^u_{t+1}. Therefore, the expected discounted value to an unmatched worker in the labour matching market is[9]

$$w^u_t = h + \beta E_t\left(\frac{u'_{t+1}}{u'_t}\right)\left[k^w_t(1-\rho^x)\int_{\tilde{a}_{t+1}}^{\bar{a}} \eta s_{it+1} f(a_i)da_i + w^u_{t+1}\right] \tag{9.15}$$

since an unmatched worker is assumed to enjoy utility h while unmatched.

For a worker and firm who are already matched, the joint discounted value of an existing match is

$$g_t = \beta E_t\left(\frac{u'_{t+1}}{u'_t}\right)\left[(1-\rho^x)\int_{\tilde{a}_{t+1}}^{\bar{a}} s_{it+1} f(a_i)da_i + w^u_{t+1}\right]. \tag{9.16}$$

Hence, subtracting (9.15) from (9.16),

$$\begin{aligned} q_t \equiv g_t - w^u_t &= (1-\rho^x)\left(1-\eta k^w_t\right) \\ &\quad \times \beta E_t\left(\frac{u'_{t+1}}{u'_t}\right)\left[\int_{\tilde{a}_{t+1}}^{\bar{a}} s_{t+1} f(a_i)da_i\right] - h. \end{aligned} \tag{9.17}$$

Unmatched firms, or firms whose matches are terminated, may choose to enter the labour matching market and post vacancies. Posting a vacancy

[9] The expected surplus is

$$k^w_t(1-\rho_{t+1})\int_{\tilde{a}_{t+1}}^{\bar{a}} \eta s_{t+1}\frac{f(a)}{1-F(\tilde{a}_{t+1})}da = k^w_t(1-\rho^x)\int_{\tilde{a}_{t+1}}^{\bar{a}} \eta s_{t+1} f(a)da.$$

costs γ per period. If an unmatched firm does post a vacancy and succeeds in making a match that produces in period $t+1$, it receives $(1-\eta)s_{t+1} - \gamma$. Otherwise (i.e. if no match is made or if the match separates before production), the firm receives nothing. If k_t^f is the probability that a vacancy is filled, free entry ensures that firms post vacancies until

$$\beta E_t\left(\frac{u'_{t+1}}{u'_t}\right)\left[k_t^f(1-\rho^x)\int_{\tilde{a}_{t+1}}^{\tilde{a}}(1-\eta)s_{t+1}f(a)da\right] - \gamma = 0. \quad (9.18)$$

Combining (9.17) and (9.18)

$$q_t = \frac{\gamma\left(1-\eta k_t^w\right)}{(1-\eta)k_t^f} - h. \quad (9.19)$$

Increases in either k^w or k^f reduce the value of continuing an existing match by making it easier to find a new match.

A total of $\rho_t N_t$ matches dissolve prior to engaging in production during period t. If the worker is not part of an existing match, or if her current match ends, she travels to the labour matching market. Thus, a total of

$$u_t \equiv U_t + \rho_t N_t = 1 - (1-\rho_t)N_t \quad (9.20)$$

workers will not produce market goods during the period and will be searching for a new match.

Based on an aggregate matching function, a fraction of workers and firms in the labour market establish new matches. These, plus the worker/firm matches that produced during the period, constitute the stock of matches that enter period $t+1$. The number of matches is equal to $m(u_t, V_t)$ where V_t is the number of posted vacancies and $m(.)$ is the aggregate matching function. The probability an unemployed worker makes a match, k_t^w, is equal to

$$k_t^w = \frac{m(u_t, V_t)}{u_t}. \quad (9.21)$$

Similarly, the probability that a firm with a posted vacancy finds a match, k_t^f, is

$$k_t^f = \frac{m(u_t, V_t)}{V_t}. \quad (9.22)$$

The total number of matches evolves according to

$$N_{t+1} = (1-\rho_t)N_t + m(u_t, V_t). \quad (9.23)$$

The aggregate output of the wholesale sector is obtained by aggregating over all matches that actually produce:

$$Q_t = (1-\rho_t)N_t z_t \left[\int_{\tilde{a}_t}^{\infty} a_t \left(\frac{f(a)}{1-F(\tilde{a}_t)}\right) da\right]. \tag{9.24}$$

The retail sector

Firms in the retail sector purchase output from wholesale producers at the price P_t^w and sell directly to households. For simplicity, assume that retail firms have no other inputs. Given the structure of demand facing each retail firm (see (9.5)), all retail firms would charge the same price in a flexible price equilibrium. This price would be a constant markup over wholesale prices, with the markup equal to $\theta/(\theta-1)$.

Rather than assume flexible prices, I assume prices at the retail level are sticky. To model this price stickiness, I adopt the approach due originally to Calvo (1983) and now widely used in macroeconomics (see, for example, Christiano, Eichenbaum and Evans 2001; Erceg, Henderson and Levin 2000; or Woodford 1999, 2000). The Calvo model is based on the assumption that each period a randomly chosen fraction of all firms are allowed to adjust their price.

Let the probability a firm adjusts its price each period be given by $1-\omega$. If firm j sets its price at time t, it will do so to maximise expected profits, subject to the demand curve it faces. The price of retail firm j is p_{jt}. Let p_t^* be the price chosen by all firms who set prices in period t; all retail firms setting prices in period t will choose the same price. Each retail firm's nominal marginal cost is just P_t^w. Real marginal cost is $P_t^w/P_t = \mu_t^{-1}$. Using (9.5), the firm's decision problem when it adjusts its price involves picking p_t^* to maximise

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+1} \left[\left(\frac{p_t^*}{P_{t+i}}\right)^{1-\theta} - \mu_{t+i}^{-1}\left(\frac{p_t^*}{P_{t+i}}\right)^{-\theta}\right] C_{t+i}$$

where the i-period discount factor $\Delta_{i,t+1}$ is given by $\beta^i(u'_{t+i}/u'_{t+1})$. The first-order condition implies[10]

$$\left(\frac{p_t^*}{P_t}\right) = \left(\frac{\theta}{\theta-1}\right) \frac{E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+1}\left[\mu_{t+i}^{-1}\left(\frac{P_{t+i}}{P_t}\right)^{\theta} C_{t+i}\right]}{E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+1}\left[\left(\frac{P_{t+i}}{P_t}\right)^{\theta-1} C_{t+i}\right]}. \tag{9.25}$$

[10] See, for example, Sbordone (2002) for a more complete derivation.

The aggregate retail price index is

$$P_t^{1-\theta} = (1-\omega)(p_t^*)^{1-\theta} + \omega P_{t-1}^{1-\theta}. \tag{9.26}$$

Equations (9.25) and (9.26) jointly determine p_t^* and P_t.

3.3 The monetary authority

To close the model, it is necessary to specify the behaviour of the monetary authority. Most central banks implement monetary policy by controlling a short-term nominal rate of interest. Thus, one way to analyse the impact of monetary policy is to specify a rule for setting the nominal interest rate. For example, Taylor rules, in which the nominal rate responds to output and inflation, are commonly used to describe monetary policy. However, one cannot simply specify arbitrary, exogenous (stationary) rules for the nominal interest rate and compare the adjustments to policy shocks in a flexible-price version of the model with a sticky-price version. The reason is that the nominal rate must react endogenously to inflation in the sticky-price model to ensure the existence of a unique, stationary, rational expectations equilibrium (Svensson and Woodford 1999). The model may have multiple stationary equilibria if the nominal interest rate is a function solely of exogenous disturbances. It is common in recent sticky-price models to require that the policy rule satisfy the 'Taylor Principle' under which the nominal interest rate responds more than one-for-one to changes in either actual or expected inflation, yet such rules would seem less relevant when prices are perfectly flexible. Thus, to provide the most transparent comparisons between the flexible-price and sticky-price versions of the model, I assume that monetary policy can be represented by an exogenous process for the growth rate of the money supply.[11]

Specifically, let Θ_t denote the growth rate of the nominal money supply. It is assumed that

$$\Theta_t = (1-\rho_m)\,\bar{\Theta} + \rho_m \Theta_{t-1} + \phi_t \bar{\Theta} \tag{9.27}$$

where ϕ_t is a serially uncorrelated, mean zero stochastic process. The steady-state gross inflation rate is equal to the average growth rate of money $\bar{\Theta}$. A similar stochastic process for the growth rate of the nominal money supply is employed by Cooley and Quadrini (1999) and Christiano, Eichenbaum and Evans (2001).

[11] Cooley and Quadrini (1999), Christiano, Eichenbaum, and Evans (2001) and Dostey and King (2001) are other examples of papers using money growth rate rules to evaluate the effects of monetary shocks.

3.4 Equilibrium and the steady state

The final equilibrium condition in the model requires that consumption equal aggregate household income which, in turn, is equal to production net of vacancy posting costs:

$$C_t = Y_t = Q_t - \gamma V_t = z_t(1-\rho^x)N_t \int_{\tilde{a}_t}^{\infty} a_{it} f(a)da - \gamma V_t. \quad (9.28)$$

An equilibrium in the model consists of an initial value of the nominal money stock M_t, the initial number of matchers N_t, and sequences for $\tilde{a}_t$, ρ_t^n, ρ_t, q_t, u_t, V_t, k_t^w, k_t^f, N_{t+1}, Y_t, C_t, P_t, p_t^*, μ_t and R_t that satisfy equations (9.4), (9.7), (9.8), (9.11)–(9.13), (9.17), (9.19)–(9.26) and (9.28), and the Central Bank's policy rule governing the evolution of the nominal money stock M_t given by (9.27). If prices are flexible, the model consists of (9.4), (9.7), (9.8), (9.11)–(9.13), (9.17), (9.19)–(9.24), (9.27), $p_t^* = P_t$ and $\mu_t = \theta/(\theta - 1)$.

The steady state is the same for both the flexible-price and sticky-price versions of the model. In a zero-inflation steady state, (9.7) implies $R = \bar{\Theta}/\beta$. Using this in (9.11), $q = A - (\beta/\bar{\Theta}\mu)\tilde{a}$. Equation (9.17) can then be written as

$$\left(\frac{\beta}{\mu}\right)\tilde{a} + (1-\rho^x)(1-\eta k^w)\beta\left(\frac{\beta}{\mu}\right)\left[\int_{\tilde{a}}^{\bar{a}} (a-\tilde{a})\, f(a_i)da_i\right] = (A+h)\,\bar{\Theta}. \quad (9.29)$$

Rearranging this condition yields the following steady-state condition for $\tilde{a}$:

$$[1-(1-\rho^x)(1-\eta k^w)\beta\,(1-F(\tilde{a}))]\tilde{a} = G(\tilde{a}) \quad (9.30)$$

where $G(\tilde{a}) \equiv \mu(A+h)\bar{\Theta}/\beta - (1-\rho^x)(1-\eta k^w)\beta[\int_{\tilde{a}}^{\bar{a}} af(a_i)da_i]$. Note that this condition also depends on the endogenous k^w. Both the sides of (9.30) are continuous and increasing in $\tilde{a}$. The support of a is $(0, \bar{a}]$. For a given k^w, $G(\bar{a}) = \mu(A+h)\bar{\Theta}/\beta = \mu(A+h)R$, while the left-hand side of (9.30) is equal to $\bar{a}$. Evaluated at zero, the left-hand side of (9.30) is zero and the right-hand side is equal to $G(0) = \mu(A+h)R - (1-\rho^x)(1-\eta k^w)\beta$ as the expected value of a is equal to 1. A unique solution $\tilde{a}(k^w)$ as a function of k^w exists as long as $G(0) > 0$ and $G(\bar{a}) < \bar{a}$, or

$$(1-\rho^x)(1-\eta k^w)\beta < \mu(A+h)R < \bar{a}.$$

Assume A (the disutility of work) and $\bar{a}$ are such that this holds. Then $\rho = \rho^x + (1 - \rho^x)F[\tilde{a}(k^w)]$ and the steady-state values of N, u, V, k^f, k^w and C are given by the solution to

$$u = 1 - (1 - \rho)N$$

$$\rho N = m(u, V)$$

$$k^f = \frac{m(u, V)}{V}$$

$$k^w = \frac{m(u, V)}{u}$$

$$\frac{\gamma(1 - \eta k^w)}{(1 - \eta)k^f} = A + h - \left(\frac{\beta}{\bar{\Theta}\mu}\right)\tilde{a}(k^w)$$

$$C = (1 - \rho)N\left(\frac{1}{1 - F[\tilde{a}(k^w)]}\right)\left[\int_{\tilde{a}(k^w)}^{\infty} af(a)da\right] - \gamma V.$$

The steady-state markup is equal to $\theta/(\theta - 1)$, while real money balances are equal to consumption.

4 SIMULATIONS

The recent literature in monetary economics has used simulations extensively to study the dynamic properties of stochastic general equilibrium models. To cite just a few recent examples, Dotsey and King (2001), Dotsey, King and Wolman (1999), Fuhrer (2000), Jensen (2002), McCallum and Nelson (1999) and Walsh (2002). In keeping with that literature, the model of section 3 is expressed in terms of percentage deviations and linearised around the steady state. The basic approach is described in Uhlig (1999), and the model solution and its properties are obtained using the 'toolkit' of programs written by Harald Uhlig.[12]

In solving the model, functional forms of the utility function and the aggregate matching function need to be specified. The utility function for the composite consumption good is assumed to be of isoelastic form:

$$u(C_t) = \frac{C_t^{1-\sigma}}{1 - \sigma}; \quad \sigma > 0,$$

[12] Uhlig's programs are available at http://cwis.kub.nl/~few5/center/STAFF/uhlig/toolkit.dir/toolkit.htm.

where σ is the coefficient of relative risk aversion. The matching function is taken to be

$$m(u_t, V_t) = \mu u_t^a V_t^{\xi}, \quad 0 < a < 1, 0 < \xi < 1. \tag{9.31}$$

A Cobb–Douglas specification of the matching function is common, and is the form used by Cooley and Quadrini (1999). With constant returns to scale, $a + \xi = 1$.[13] Equation (9.31) does allow for the matching function to display increasing or decreasing returns to scale if $a + \xi \neq 1$.[14]

Let $\hat{z}_t$ denote the log deviation from steady state of the aggregate productivity disturbance. As is standard in the literature, $\hat{z}_t$ is assumed to follow an $AR(1)$ process with innovation ε_t:

$$\hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon_t. \tag{9.32}$$

4.1 The linearised model

Given the assumed functional forms, the model is linearised around the steady state.[15] Let $\hat{x}_t$ denote the percentage deviation of a variable X_t around its steady-state value. The linearised model consists of (9.32) for the aggregate productivity disturbance and the following thirteen equations:

- The policy rule for nominal money growth, (9.27):

$$\hat{\Theta}_t = \rho_m \hat{\Theta}_{t-1} + \hat{\phi}_t; \tag{9.33}$$

[13] Blanchard and Diamond (1989, 1990) provide evidence that the aggregate matching function displays constant returns to scale.

[14] den Haan, Ramey and Watson (2000) assume that the matching function displays constant returns to scale and is of the form

$$m(u_t, V_t) = \frac{u_t V_t}{\left[u_t^s + V_t^s\right]^{1/s}}.$$

As den Haan, Ramey and Watson note, this functional form ensures the probabilities k^w and k^f are bounded between 0 and 1. This specification and the Cobb–Douglas form lead to similar equilibrium conditions when the model is linearised around the steady state. For example, the specification of den Haan, Ramey and Watson implies $k_t^w = V/[u_t^s + V_t^s]^{1/s}$ which is approximated by

$$\hat{k}_t^w = [1 - (k^w)^s]\hat{V}_t - (k^f)^s \hat{u}_t = [1 - (k^w)^s](\hat{V}_t - \hat{u}_t)$$

where $\hat{x}$ denotes the percentage deviation around the steady state. With constant returns to scale, (9.31) implies $k_t^w = \mu u_t^{a-1} V_t^{1-a}$ which leads to

$$\hat{k}_t^w = (1 - a)(\hat{V}_t - \hat{u}_t)$$

When $a = (k^w)^s$, the two specifications produce identical dynamic simulations. In the calibration used by Den Haan, Ramey and Watson $s = 1.27$ and $k^w = 0.45$, implying that $(k^w)^s = 0.3627$. Cooley and Quadrini set $a = 0.4$ in their base calibration, implying the two specifications are essentially identical.

[15] Details are in an appendix available at http://econ.ucsc.edu/~walshc/.

- The cash-in-advance constraint (in first-difference form), (9.8):

$$\hat{\Theta}_t = \hat{y}_t - \hat{y}_{t-1} + \hat{\pi}_t; \tag{9.34}$$

- The evolution of the number of matches, (9.23):

$$\hat{n}_{t+1} = \varphi\hat{\varphi}_t + \varphi\hat{n}_t + \left(\frac{vk^f}{N}\right)\hat{v}_t + \left(\frac{vk^f}{N}\right)\hat{k}_t^f; \tag{9.35}$$

- The endogenous job destruction margin, (9.11):

$$\hat{a}_t = \hat{r}_t + \hat{\mu}_t - \left(\frac{\mu Rq}{\tilde{a}}\right)\hat{q}_t - \hat{z}_t; \tag{9.36}$$

- The survival rate $\varphi_t = 1 - \rho_t$, using (9.12):

$$\hat{\varphi}_t = -\left(\frac{\rho^n}{1-\rho^n}\right) e_{F,a}\hat{a}_t; \tag{9.37}$$

- The number of unemployed job seekers, (9.20):

$$\hat{u}_t = -\left(\frac{\varphi N}{u}\right)\hat{n}_t - \left(\frac{\varphi N}{u}\right)\hat{\varphi}_t; \tag{9.38}$$

- The probability a vacancy is filled, (9.22):

$$\hat{k}_t^f = a\hat{u}_t - (1-\xi)\hat{v}_t; \tag{9.39}$$

- The equality of firms filling vacancies and workers finding matches:

$$\hat{v}_t + \hat{k}_t^f = \hat{u}_t + \hat{k}_t^w; \tag{9.40}$$

- The job-posting condition, (9.19):

$$\hat{k}_t^f = -\left(\frac{\eta k^w}{1-\eta k^w}\right)\hat{k}_t^w - \left(\frac{q}{q+h}\right)\hat{q}_t; \tag{9.41}$$

- The output equation (9.28):

$$\hat{y}_t = \left(\frac{Q}{Y}\right)(e_{H,a}\hat{a}_t + \hat{n}_t + z_t) - \left(\frac{\gamma V}{Y}\right)\hat{v}_t; \tag{9.42}$$

- The Euler condition from the household's optimisation problem (9.7):

$$0 = E_t\hat{y}_{t+1} - \hat{y}_t - \left(\frac{1}{\sigma}\right)\hat{r}_t + \left(\frac{1}{\sigma}\right)E_t\hat{\pi}_{t+1}; \tag{9.43}$$

- The inflation equation from the retail firms' pricing decisions, obtained from (9.25) and (9.26):

$$0 = \beta E_t \hat{\pi}_{t+1} - \hat{\pi}_t - \kappa \hat{\mu}_t \tag{9.44}$$

- The present value condition for matches, (9.17):

$$\begin{aligned}\hat{q}_t &= AB(e_{H,a} E_t \hat{a}_{t+1} - E_t \hat{\mu}_{t+1} - E_t \hat{r}_{t+1} + E_t z_{t+1}) \\ &\quad + \left[\frac{(1-\eta k^w)\beta\varphi(q-A)}{q}\right] E_t \hat{\varphi}_{t+1} - \left(\frac{q+h}{q}\right)(\hat{r}_t - E_t \hat{\pi}_{t+1}) \\ &\quad - \left(\frac{\eta k^w}{1-\eta k^w}\right)\left(\frac{q+h}{q}\right)\hat{k}_t^w + (1-\eta k^w)\beta\varphi E_t \hat{q}_{t+1}. \end{aligned} \tag{9.45}$$

In these conditions, $e_{F,a}$ is the elasticity of the cumulative density function of a, $H(\tilde{a}) \equiv E_t(a|a \geq \tilde{a})$, $e_{H,a}$ is the elasticity of $H(\tilde{a})$ with respect to $\tilde{a}$, evaluated at the steady-state and

$$AB = \frac{(1-\eta k^w)\beta H(\tilde{a})}{\mu R q}.$$

Note that, while the distribution of the idiosyncratic shock a appears in the form of the function $H(\tilde{a})$ and the elasticities $e_{F,a}$ and $e_{H,a}$, the actual realisations of the a_{it} shocks average out across matches, so they do not appear in the equilibrium conditions.

In the flexible-price version of the model, the markup μ is constant, equal to $\theta/(\theta-1)$, and (9.44) is dropped.

Let $x_t = (\hat{r}_t, \hat{n}_t, \hat{\varphi}_t, \hat{y}_t, \hat{a}_t, \hat{v}_t, \hat{k}_t^f, \hat{k}_t^w, \hat{q}, \hat{u}_t, \hat{\mu}_t, \hat{\pi}_t)'$ be the vector of endogenous variables, and let $\psi_t = (\hat{\Theta}_t, z_t)'$ be the vector of exogenous aggregate disturbances. Equations (9.34)–(9.45) can be written as

$$AE_t x_{t+1} + Bx_t + CE_t \psi_{t+1} + D\psi_t = 0 \tag{9.46}$$

where

$$\psi_{t+1} = N\psi_t + \chi_{t+1},$$

and $\chi_t = (\phi_t, \varepsilon_t)'$ is the innovation vector.

If an equilibrium solution to this system of equations exists, it takes the form of stable laws of motion given by

$$x_t = Px_{t-1} + Q\psi_t.$$

Uhlig (1999) provides a complete discussion of the methods used to solve systems such as (9.46).

Table 9.1 *Calibrated parameters*

Preferences							
			β	σ	θ	h	
			0.989	2	11	0	
Labour market							
ρ^s	ρ^x	α	ξ	κ^f	κ^w	N	η
0.10	0.068	0.4	0.6	0.7	0.6	0.94	0.5
Price rigidity							
				ω			
				0.67			
Policy							
		ρ_m		σ_ϕ		$\bar{\Theta}$	
		0.73		0.00624		1.01	
Productivity							
		σ_ε		ρ_z		σ_a	
		0.0033		0.95		0.15	

4.2 Calibration

The model is characterised by five sets of parameters – those describing household preferences, those describing the aggregate matching function, those characterising the degree of price rigidity at the retail level, those specifying the behaviour of the growth rate of money and those characterising the stochastic distribution of the exogenous shocks. Parameter values are chosen to be largely consistent with those shown to match US data in non-monetary models. The baseline parameters values are shown in table 9.1 and discussed in this section.

Preferences

Household preferences are characterised by the parameters β, σ, θ, h and A. The discount rate and the coefficient of relative risk aversion appear in standard DSGE models. Choosing the time period to correspond to a calendar quarter, β is set equal to 0.989, implying a steady-state real annual return of 4.5 per cent. A value $\sigma = 2$ is chosen for the coefficient of relative risk aversion, implying greater risk aversion than log utility. The parameter θ determines the elasticity of demand for the differentiated retail goods. This elasticity in turn determines the markup μ. This markup is set equal to 1.1, corresponding to a value of $\theta = 11$. The value of home production while unemployed, h, is set equal to zero. Finally, A is determined by

the steady-state condition (9.11) once q is found from the labour market calibrations.

Matching and the labour market

Den Haan, Ramey and Watson (2000) set the steady-state separation rate ρ^s equal to 0.1. This is based on Hall's conclusion that 'around 8 or 10 percent of workers separate from their employer each quarter' (Hall 1995: 235) and the Davis, Haltiwanger and Schuh (1996) finding of about an 11 per cent quarterly separation rate. This is higher than the 0.07 value adopted by Merz (1995), but lower than the 0.15 used by Andolfatto (1996). Given a value of 10 per cent for ρ^s, den Haan, Ramey and Watson use evidence on permanent job destruction to calibrate the exogenous separation probability ρ^x as 0.068. I use this value for the baseline simulations. These values for ρ^x and ρ^s imply an endogenous separation probability ρ^n of 0.0343. From this value, and the assumed distribution function for the match-specific productivity shock, the steady-state value of the cut-off productivity realization $\tilde{a}$ can be derived. I assume $\tilde{a}$ is log-normally distributed with standard deviation 0.15; this is somewhat higher than the value used by den Haan, Ramey and Watson (they set this standard deviation equal to 0.1).

For the Cobb–Douglas matching function (9.31), I follow Cooley and Quadrini and set $\alpha = 0.4$ and $\xi = 0.6$ based on the estimates of Blanchard and Diamond (1989). Both Cooley and Quadrini (1999) and den Haan, Ramey and Watson fix $k^f = 0.7$. Cooley and Quadrini (1999) cite Cole and Rogerson (1996) to set the average duration of unemployment at 1.67 quarters, which implies $k^w = 0.6$. I set $N = 0.94$, implying a steady-state unemployment rate of 0.06 and a value of 0.154 for u, the steady-state number of workers searching each period. The steady-state value of V is 0.132.[16]

I follow den Haan, Ramey and Watson and set the share of the match surplus that the worker receives, η, equal to 0.5. Finally, the steady-state value of a match q, is obtained from (9.19). Finally, (9.17) is used to calibrate γ.

Price rigidity

The degree of nominal rigidity is determined by ω, the fraction of firms each period that do not adjust their price. Empirical estimates of

[16] Den Haan, Ramey and Watson (2000) chose a value of 1.27 for the parameter s that appears in their matching function (see n. 9). This, together with the steady-state value of u implies a value of 0.0993 for V and 0.4515 for k^w. This last value is close to the value of 0.45 used by den Haan, Ramey and Watson and is significantly lower than the 0.6 employed by Cooley and Quadrini.

forward-looking price-setting models of the type employed here suggest prices are fixed on the order of nine months. This would imply a value of 0.67 for ω (Sbordone 2002) and this is taken as the baseline value. However, evidence based on BLS data on price changes suggests a median time between changes of six months (Bils and Klenow 2002); this would imply a value of 0.5 for ω. In the simulations, various values for ω ranging from $\omega = 0$ (price flexibility) to $\omega = 0.9$ are used to explore the impact of nominal rigidity on the dynamic response of the economy.

Policy

The process for the growth rate of M is calibrated by estimating an $AR(1)$ process for US M2 using quarterly data over the period 1965:3–2001:4. This yielded $\rho_m = 0.73$ and the standard deviation of ϕ is set to 0.00624.[17]

Shocks

In addition to the money growth rate shock, there are two other exogenous disturbances in the model: the match-specific shock and the aggregate productivity shock. The specification of the distribution of the match-specific shock has already been discussed. The log aggregate productivity shock was given in (9.32). Standard calibrations for the productivity disturbance process in the RBC literature are $\rho_z = 0.95$ and $\sigma_\varepsilon = 0.007$ (see Cooley and Prescott 1995). This calibration is, however, based on models in which the productivity disturbance is the sole source of fluctuations. In the present model there are nominal money growth shocks in addition to the aggregate productivity shocks. Thus, a smaller value of σ_ε is appropriate for matching output fluctuations. Cooley and Quadrini (1999) choose σ_ε so the model's prediction for the standard deviation of output matches the standard deviation of US real GDP. For their baseline parameter values, this implies $\sigma_\varepsilon = 0.0033$ when prices are assumed to be flexible. I employ this as the baseline value for σ_ε, together with $\rho_z = 0.95$.

4.3 Results

Flexible prices

Column (1) of table 9.2 presents standard deviations based on US data for 1959:1–1996:4; column (2) expresses these relative to the standard deviation of output. These values are taken from den Haan, Ramey and Watson (2000) and Cooley and Quadrini (1999). Columns (3) and (4) report the

[17] Cooley and Quadrini (1999) set the standard deviation of ϕ to 0.00623 and $\rho_m = 0.49$. Christiano, Eichenbaum and Evans (2001) set $\rho_m = 0.5$.

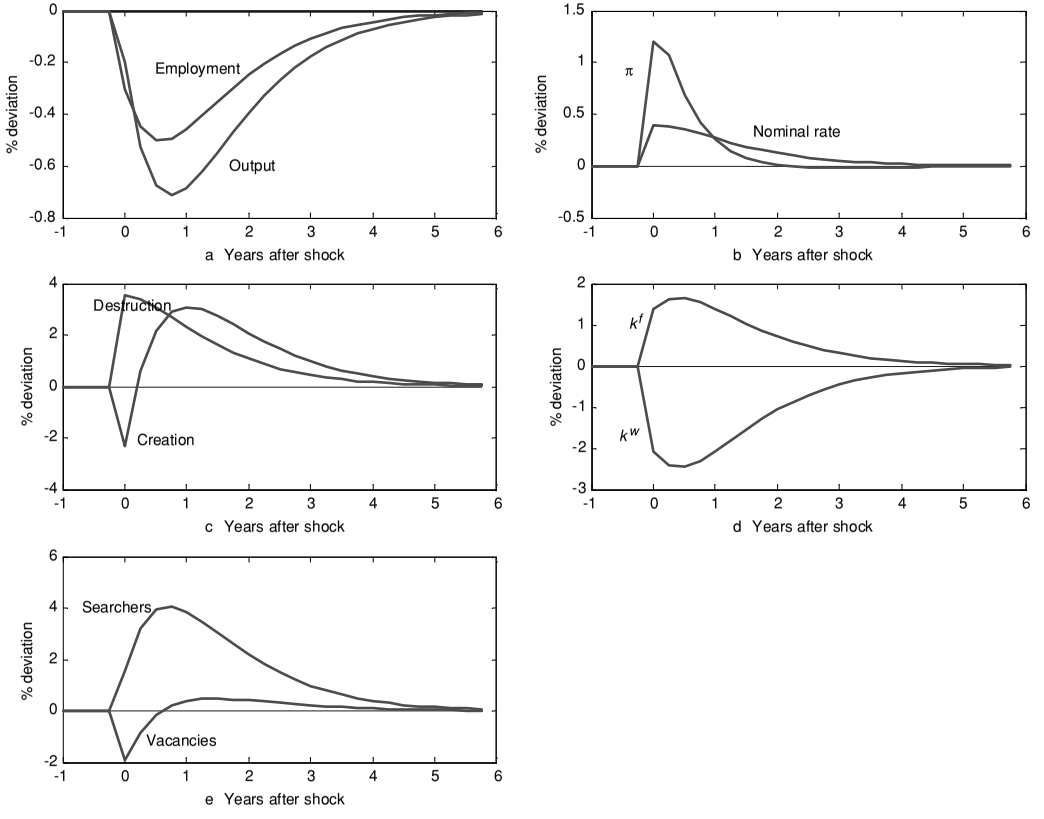

Figure 9.2 Effects of a money growth shock with flexible prices

Table 9.2 *Business cycle properties*

	US		Model outcomes				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
			$\omega = 0$	$\omega = 0$	$\omega = 0.33$	$\omega = 0.67$	$\omega = 0.9$
Variable	σ_i	σ_i/σ_y	σ_i	σ_i/σ_y	σ_i/σ_y	σ_i/σ_y	σ_i/σ_y
Output	1.60	1.00	1.18	1.00	1.00	1.00	1.00
Employment	0.99	0.62	0.66	0.56	0.53	0.57	0.78
Job creation rate	4.62	2.89	4.77	4.05	3.93	4.93	4.26
Job destruction rate	6.81	4.26	4.05	3.44	3.34	5.97	7.10
Inflation	0.56	0.35	1.05	0.89	0.96	0.84	0.14
Nominal interest rate	1.27	0.79	0.41	0.35	0.48	0.87	0.62

results from the flexible-price model. The model based on productivity and money growth rate shocks implies less output variability than is observed in the data. The model implies slightly less employment volatility relative to the standard deviation of output than found in the data (0.56 compared to 0.62), and it also reverses the relative volatility of job creation and destruction. In the data, destruction exhibits more volatility, while in the flexible price model job creation has a larger standard deviation. Greater volatility in job creation than destruction is also a property of the real model studied by den Haan, Ramey and Watson (2000). Perhaps not surprisingly, the flexible price model implies greater inflation volatility than is evident in the US data.

Figure 9.2 illustrates the impact of a money growth rate shock when prices are flexible. Nominal money growth rate fluctuations have very small real effects in a basic CIA model with a neoclassical specification of the labour market and flexible prices (for example, see Walsh 2003, chapter 3). In those models, higher inflation taxes consumption and leads households to reduce their labour supply. In the present model, labour supply is completely inelastic, so this channel is absent. Instead, fluctuations in the nominal interest rate alter the job destruction margin and have real effects even when prices are flexible. Figure 9.2 shows that a 1 percentage point money growth rate shock (which raises the nominal interest rate – see panel b) causes a fall in employment and output (panel a). For a given level of employment at the beginning of each period, fewer worker/firm matches actually remain together to produce when the nominal interest rate increases. The contraction in production at the wholesale level increases wholesale prices. With a fixed markup of retail over wholesale

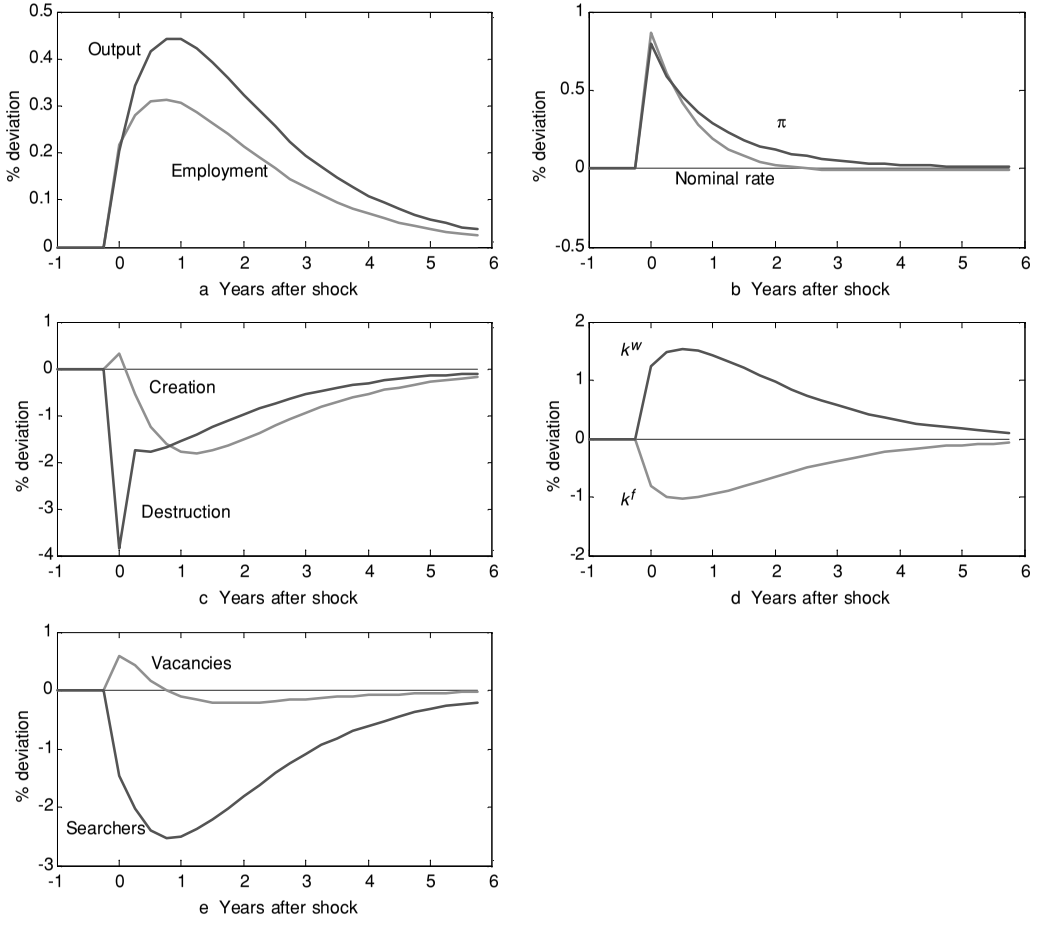

Figure 9.3 Effects of a money growth rate shock with sticky prices ($\omega = 0.67$)

prices, inflation spikes as the retail price level jumps (panel b). Job creation and job destruction initially move in opposite directions, as the number of endogenous separations rise and fewer jobs are created (panel c). However, job creation rebounds after one period. The job-finding probability falls for workers and rises for firms (panel d), reflecting the rise in the number of searching workers relative to vacancies (panel e).

The results in figure 9.2 are qualitatively similar to those reported by Cooley and Quadrini (1999) for a contractionary money shock. The key difference is the presence of a liquidity effect in the Cooley–Quadrini model and the lack of one in the present model. Thus, a *negative* money shock raises the nominal interest rate in their model while a *positive* money shock does so in the present model.

Sticky prices

When prices are sticky, monetary disturbances have traditional demand effects as well as the supply-side effects. Columns (5)–(7) of table 9.2 show the impact of increasing price rigidity on the relative variability of key variables. As expected, the relative standard deviation of inflation falls as the degree of price stickiness increases. In other sticky-price models, $\omega = 0.67$ is a common parameterisation. It implies that prices are fixed, on average, for nine months. This degree of price stickiness does provide a better match between the model's predictions for the standard deviation of inflation relative to that of output. When $\omega = 0.67$, the standard deviation of job destruction is increased relative to that of job creation, moving these statistics closer to the values found in US data.

Figure 9.3 shows how important price stickiness is in affecting the model's predictions for the response to a money supply disturbance. In contrast to figure 9.2, a positive money shock now increases real output and employment. There is a jump in job creation and a significant drop in job destruction. As a consequence, the number of workers searching for matches falls and then gradually returns to the steady state. In this CIA model, there is no liquidity effect – the nominal interest rate rises immediately in reaction to the positive shock to money growth. As expected, the inflation impact of the money growth shock is smaller when prices are sticky. Two effects operate on inflation. First, the expansion in output induced by the rise in demand increases wholesale prices and leads to a rise in retail inflation. Second, the associated rise in the nominal interest rate (reflecting the rise in expected inflation) reduces wholesale output and also contributes to an increase in wholesale prices and retail inflation. If the cost channel is eliminated,

Table 9.3 *Effects of parameter variation*

							Response of output					
							Multipliers			Lags		
	ω	k^f	k^w	ρ_m	η	Cost Channel	Impact	Peak	Total	Peak	Median	Mean
(1)	0.67	0.70	0.60	0.73	0.5	Yes	0.20	0.44	5.49	3	7	8.37
(2)	0.50	0.95	0.95	0	0.5	No	0.39	0.39	1.03	0	1	1.60
(3)	0.50	0.95	0.95	0	0.5	Yes	0.43	0.43	1.43	0	1	2.31
(4)	0.67	0.95	0.95	0	0.5	Yes	0.63	0.63	2.83	0	2	3.45
(5)	0.67	0.70	0.60	0	0.5	Yes	0.68	0.68	4.55	0	4	5.60
(6)	0.67	0.70	0.60	0.73	0.5	Yes	0.20	0.44	5.49	3	7	8.37
(7)	0.67	0.70	0.60	0.73	0.1	Yes	0.27	0.75	13.05	5	13	15.20

the standard deviation of inflation relative to output falls from 0.84 to 0.54, a 35 per cent decline.

For the baseline parameter values used for figure 9.3, the immediate impact effect on output of a 1 percentage point rise in the money growth rate is 0.20. This increases to a peak effect of 0.44, with this peak occurring three periods after the initial shock. The total multiplier is 5.49 and the mean lag is 8.37 periods. There are several elements of the model that account for the dynamic pattern displayed by this output response. First, the degree of nominal price stickiness influences the model dynamics. Second, the money growth rate process itself is serially correlated, and this accounts for some of the persistence in the output effects seen in figure 9.3. Third, the nominal interest rate affects job separation and creation, and fourth, the labour market matching process affects the evolution of employment after a shock.

Insight into how the dynamic response of output is affected by the various aspects of the model specification is provided by table 9.3. Each row of that table represents a different set of parameter values. Row (1) reports results for the baseline parameter values. Row (2) has a lower degree of price rigidity than in the baseline parameter set ($\omega = 0.50$ versus 0.67). In addition, ρ_m is set equal to 0 so that the growth rate of money is serially uncorrelated, and the direct cost channel of nominal interest rates is eliminated. Finally, both k^f and k^w are increased to 0.95 to capture a matching process in which workers and firms are able to find new matches much more quickly. As a result of these changes, the impact effect of a money growth shock on output is actually increased to 0.39, but this impact effect is also the peak effect as output starts declining in the period immediately following the shock. The response no longer displays the typical hump shape seen in

estimated VARs. The total multiplier falls dramatically to 1.03 from 5.49 for the baseline parameter set (row 1), and the mean lag falls to 1.60 periods. Row (3) of table 9.3 adds back in the supply channel of the nominal interest rate. This increases the impact multiplier on output and the total impact, but it has only minor effects on the dynamic pattern of output's response. The maximum effect still occurs in the period of the shock, with output declining thereafter.

Row (4) increases the degree of price rigidity by raising ω from 0.5 to 0.67. This, like the supply channel, increases the impact of a money shock but has little impact on the shape of the response. The maximum effect occurs immediately, although the mean and median lags are both increased.

Row (5) returns k^f and k^w to their baseline values, so row (5) serves to illustrate the impact of more sluggish labour market adjustment. While the impact effect of a money shock is not significantly changed, the median lag is doubled to four periods, while the mean lag rises from 3.45 to 5.60 periods. Labour market search stretches out the response, adding to the overall persistence due to a monetary shock, but it does not induce the hump-shape response seen in VARs.

Row (5) differs from the baseline parameter set only in setting $\rho_m = 0$. If ρ_m is increased to its baseline value of 0.73, the impact is quite dramatic. To facilitate the comparison, row (6) repeats row (1), the outcomes for the baseline parameters. Comparing rows (5) and (6) shows that serially correlated money shocks reduce the impact effect of a money growth shock on output. While the impact effect is smaller, the peak effect is now both larger and delayed, occurring three periods after the shock. The total effect also rises, and the mean lag increases to over seven periods. Thus, the persistence in the money supply growth process appears to have an important effect on the response to a monetary shock, and it is serial correlation in the money shock that generates the hump-shaped response of output.

Rows (1)–(6) employed a value of 0.5 for the share parameter η. This is the value used by den Haan, Ramey and Watson (2000) and is common in other applications of the Mortensen–Pissarides framework. In contrast, Cooley and Quadrini (1999) set η equal to 0.01 and 0.1 in their alternative model economies. Row (7) of table 9.3 sets η equal to 0.1. This has a major impact on the model's response to a money growth rate shock. The impact effect rises from 0.20 in row (1) to 0.27 in row (7), the peak effect increases from 0.44 to 0.75, and the total impact rises from 5.49 to 13.05. The peak impact now occurs five periods after the shock, and the mean lag rises to over 10 periods.

The results reported in table 9.3 help to identify the key parameters affecting both the magnitude and the persistence of the response of output.

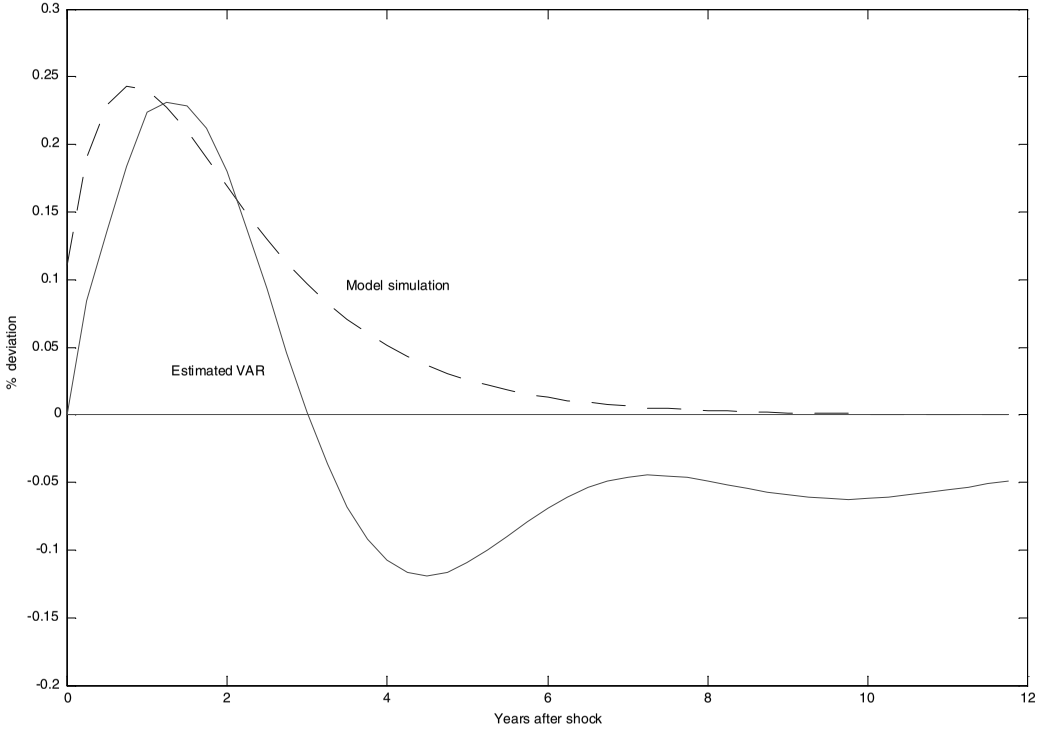

Figure 9.4 Output responses: VAR and model simulations

The magnitude of the total impact depends importantly on the values of ω, ρ_m, k_t^w, k_t^f and η. The persistence displayed by output in response to a nominal money growth shock, as measured by the median lag, is affected by the degree of price rigidity, the speed of labour market adjustment and the degree of persistence in the money growth process itself. The role of ω, the degree of price rigidity, is not surprising. Nominal rigidities have long been viewed as the key explanation for sizeable real effects of nominal disturbances. The ease with which employment matches are formed, as reflected in the values of k^f and k^w, also have a major effect on the size of the output effect of a money growth shock. The values of ω, k_t^w, and k_t^f affect mainly the total impact multiplier but not the shape of the output response. The hump-shaped response of output is determined by the degree of serial correlation in the money growth rate (ρ_m) and η (the labour share parameter).

The role of η is unexpected, in part because this share parameter plays no role in traditional dynamic general equilibrium models. Cooley and Quadrini (1999) find that the volatility of output and employment increases with η in their flexible-price model. This is no longer the case when prices are sticky, and increasing the share of the surplus going to the firm (a reduction in η) increases overall volatility.[18] As η goes to zero, (9.19) implies that the present value of a match becomes less sensitive to k^w and more sensitive to k^f. As a consequence, a contractionary monetary policy shock that reduces employment and increases the number of searching workers increases k^f and reduces the continuation value of a match. Because firms are able to find new matches more easily, the value of continuing in an existing match falls. This raises the probability of endogenous separations and, as a result, the economy takes longer to return to its steady-state level of employment. Thus, a decline in η leads to greater persistence in the real effects of a monetary policy shock.

Table 9.3 serves to separate the influence on output dynamics of various aspects of the model. To assess the 'match' between the model and the data, it is useful to compare impulse responses directly. Figure 9.4 shows the response of output to a money growth shock estimated from US data and the response obtained from the model simulations.[19] Under the baseline parameter values, the peak impact of a money shock on output is too large (0.44 – see row (1) of table 9.3 – versus 0.23 from the estimated VAR). A better match is obtained with $\omega = 0.62$, implying slightly more price

[18] For example, decreasing η from 0.5 to 0.1 increases the standard deviation of output from 0.73 to 1.64. Decreasing it further to 0.01 raises the standard deviation to 2.15.

[19] The estimated response for output is the same as in figure 9.1.

flexibility than with the baseline parameter values, and this value of ω is used to generate figure 9.4. The model captures the basic hump-shaped response, but there are several aspects of the estimated impulse response that are not captured by the model. The estimated output dynamics are much more complex than those exhibited by the model. The model implies output reaches a peak and then declines smoothly to the steady state; the estimate VAR shows output reaching its peak and then falling below the steady state. The impulse response function from the estimated VAR also shows output peaking five periods after the shock versus three periods in the case of the model simulation.[20]

5 CONCLUSIONS

Dynamic stochastic general equilibrium (DSGE) models are well suited for studying the interactions of real and nominal stickiness, and this chapter has examined the role of the labour market matching function and price stickiness in affecting the way the economy responds to money growth shocks. The model incorporated both an aggregate labour market matching function and price stickiness by incorporating a wholesale production sector, in which matched firms and workers produce output, and a retail sector characterised by monopolistic competition and sticky prices. Money growth shocks led to employment and output responses that were hump-shaped, just as the empirical evidence suggests. Replacing the Walrasian labour market with a simple model of labour market search appears to be a promising avenue to pursue in understanding the dynamic adjustment of the economy to monetary policy shocks.

In the sticky-price version of the model, monetary shocks had both demand and supply effects. Increases in money growth lead to increases in consumption and the nominal interest rate (via the expected inflation channel). The rise in the nominal rate reduced production among wholesale firms by altering the job destruction margin. By pushing up wholesale prices relative to retail prices, positive nominal interest rate movements also affected retail price inflation. This supply, or cost channel, effect reinforced the inflationary impact of a money growth rate increase.

As in traditional models, the response of output to monetary shocks depended on the degree of nominal price stickiness. It also depended on the degree of persistence displayed by the money process itself. Perhaps more

[20] However, it should be noted that the VAR impulse responses are obtain using a Choleski decomposition with money ordered last. This means that the VAR restricts money growth shocks to have no impact on output until at least one period after the shock.

interestingly, the dynamic behaviour of the model economy was sensitive to the parameter that determined how a match surplus was divided between the worker and the firm. An interesting direction for future research will be to explore further the implications of this share parameter for economic dynamics. Hosios (1990) has shown that the relationship between the share parameter and the elasticity of the vacancy matching probability with respect to labour market tightness (measured by the ratio of vacancies to searchers) is critical for determining the efficiency of the steady-state unemployment rate (Pissarides 2000, chapter 8). While Friedman argued that the optimal rate of inflation is the rate that produces a zero nominal interest rate, Cooley and Quadrini (1999) argue that the optimal level of the nominal interest rate may be positive if steady-state unemployment is inefficiently low.[21] The positive nominal rate increases job destruction and raises the average unemployment rate.

Monetary policy has been represented by a process for the growth rate of the nominal money supply. It is common in much of the recent monetary literature to represent policy by a rule for the nominal interest rate. An extension of the present model would be to replace the money growth rate rule with a nominal interest rate rule. A policy shock that raised the nominal interest rate would reduce output through a traditional demand channel and through its effect on job destruction. These channels, however, would have countervailing effects on inflation. The negative impact on wholesale output would raise wholesale prices relative to retail prices and lead to an increase in retail price inflation. The demand-side reduction in consumption would put downward pressure on inflation. These opposing effects could account for the small initial net impact on inflation of a nominal interest rate shock that is observed in the data.

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[21] This occurs in the model of this chapter when $\eta < \alpha$, as in the calibration of Cooley and Quadrini (1999).

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