# Labor Market Search and Monetary Shocks: Appendix*

Carl E. Walsh

This draft: Feb. 2003

## 1 Introduction

This appendix provides further details on the model and the linearized system used for the simulations.

## 2 Model specification

### 2.1 The labor market and wholesale production

Wholesale production takes place in competitive output markets. Production is carried out by matched pairs consisting of a worker and a firm. At the beginning of the period, there are $N_t$ matched workers and firms; $U_t = 1 - N_t$ workers are unmatched. If a worker is part of an existing match at the start of period $t$, she travels to her place of employment. At that point, there is an exogenous probability $0 \leq \rho^x < 1$ that the match is terminated.For the $(1-\rho^x)N_t$ matches that survive, the worker and firm jointly observe the current realization of productivity and decide whether to continue the match.

If the realization of productivity is low enough, it will be unprofitable for the match to continue. If the match does continue, production occurs. The output of a matched worker/firm pair $i$ in period $t$ that does produce is

$$y_{it} = a_{it} z_t \tag{1}$$

where $a_{it}$ is a match-specific productivity disturbance with mean 1 and $z_t$ is a common, aggregate productivity disturbance, also with mean 1.

#### 2.1.1 Endogenous job destruction

Because of the cash-in-advance constraint, proceeds from output produced in period $t$ are only available for consumption in period $t+1$. Thus, the time $t$

*Elements of Dynamic Macroeconomic Analysis*, S. Altuĝ, J. Chadha, and C. Nolan (eds), Cambridge University Press, 2003.

value of the revenues obtained from production in period $t$ is

$$\beta \mathrm{E}_t \left( \frac{u'_{t+1}}{u'_t} \right) \left[ \frac{P^w_t a_{it} z_t}{P_{t+1}} \right] = \left( \frac{\delta_t}{\mu_t} \right) a_{it} z_t,$$

where $\delta_t$ is the discount rate given by

$$\delta_t = \beta \mathrm{E}_t \left( \frac{P_t}{P_{t+1}} \frac{u'_{t+1}}{u'_t} \right),$$

$P^w_t$ is the wholesale price, $P_t$ is the retail price index, and

$$\mu_t = \frac{P_t}{P^w_t}$$

is the retail mark up.

The expected value of a match that produces in period $t$ is

$$\left( \frac{\delta_t}{\mu_t} \right) a_{it} z_t - A + g_{it};$$

$g_{it}$ is equal to the expected present value at time $t$ of a match that continues into period $t+1$ and $A$ is the worker's disutility of effort.

Assume that the share of the surplus from a match received by each participant is fixed. The worker and firm will maintain a match as long as it yields a positive expected surplus in present value terms. Thus, a matched firm/worker pair will maintain their match as long as $(\delta_t/\mu_t) a_{it} z_t - A + g_{it}$ exceeds the match's opportunity cost, $w^u_t$.

The match's opportunity costs is equal to the value of home consumption an unmatched worker can produce plus the present value of future worker opportunities if unmatched in period $t$. Hence, a match will be continued as long as the realization of the firm specific productivity shock is greater than the value $\tilde{a}_{it}$ defined by

$$\left( \frac{\delta_t}{\mu_t} \right) \tilde{a}_{it} z_t - A + g_{it} = w^u_t.$$

This critical value $\tilde{a}_{it}$ can be expressed using equation the representative household's Euler condition and the definition of $\delta_t$ as

$$\tilde{a}_t = \frac{\mu_t R_t \left( w^u_t + A - g_{it} \right)}{z_t} = \frac{\mu_t R_t (A - q_t)}{z_t}$$

where

$$q_t \equiv g_{it} - w^u_t.$$

Let $\rho^n_t$ be the fraction of matches that endogenously decide to separate. Let $F$ denote the cumulative distribution function of the match specific productivity shock $a$. Then the endogenous separation rate is the probability that $a_t \leq \tilde{a}_{it}$.

$$\rho^n_{it} = \Pr \left[ a_t \leq \tilde{a}_{it} \right] = \mathrm{F} \left[ \frac{\mu_t R_t \left( A - q_t \right)}{z_t} \right] \tag{2}$$

The aggregate separation rate $\rho_t$ is equal to

$$\rho_t = \rho^x + (1-\rho^x)\rho_t^n \tag{3}$$

Define

$$s_{t+1} = \delta_{t+1}\left(\frac{a_{it+1}z_{t+1}}{\mu_{t+1}}\right) - A + q_{it+1} \tag{4}$$

as the joint surplus to a worker-firm pair who are matched at the start of $t+1$ and do not separate. Note that everything is in terms of the present value as of the beginning of period $t+1$.

Let $\eta$ denote the share of this surplus received by the worker; the firm receives $1-\eta$ of the joint surplus.

If an unmatched worker in period $t$ succeeds in making a match that produces in period $t+1$, she receives

$$\eta s_{t+1} + w_{t+1}^u$$

The probability of this occurring is $k_t^w(1-\rho_{t+1})$, where $k_t^w$ is the period $t$ probability an unmatched worker finds a job.

Therefore, the expected discounted value to an unmatched worker in the labor matching market is

$$w_t^u = h + \beta \mathrm{E}_t\left(\frac{u'_{t+1}}{u'_t}\right)\left[k_t^w(1-\rho^x)\int_{\tilde{a}_{t+1}}^{\bar{a}} \eta s_{t+1} f(a)da + w_{t+1}^u\right] \tag{5}$$

since an unmatched worker is assumed to produce home consumption goods $h$ while unmatched.

If an unmatched firm posts a vacancy and succeeds in making a match that produces in period $t+1$, it receives

$$(1-\eta)s_{t+1} - \gamma$$

where $\gamma$ is the cost of posting a vacancy. Otherwise (i.e., if no match is made or if the match separates before production), the firm receives nothing. If $k_t^f$ is the probability a vacancy is filled, free entry ensures that firms will post vacancies until

$$\beta \mathrm{E}_t\left(\frac{u'_{t+1}}{u'_t}\right)\left[k_t^f(1-\rho^x)\int_{\tilde{a}_{t+1}}^{\bar{a}} (1-\eta)s_{t+1} f(a)da\right] - \gamma = 0 \tag{6}$$

Define

$$\mathrm{E}_t X_{t+1} \equiv \beta \mathrm{E}_t\left(\frac{u'_{t+1}}{u'_t}\right)\left[\int_{\tilde{a}_{t+1}}^{\bar{a}} s_{t+1} f(a)da\right].$$

Then the job posting condition becomes

$$(1-\eta)(1-\rho^x)k_t^f \mathrm{E}_t X_{t+1} = \gamma \tag{7}$$

For a worker and firm who are already matched, the discounted value of an existing match is

$$g_{it} = \beta \mathrm{E}_t \left( \frac{u'_{t+1}}{u'_t} \right) \left[ (1-\rho^x) \int_{\tilde{a}_{t+1}}^{\bar{a}} s_{t+1} f(a) da + w^u_{t+1} \right]. \tag{8}$$

Hence,

$$\begin{aligned} q_t &= g_t - w^u_t = \beta \mathrm{E}_t \left( \frac{u'_{t+1}}{u'_t} \right) \left[ (1-\rho^x) \int_{\tilde{a}_{t+1}}^{\bar{a}} s_{t+1} f(a) da + w^u_{t+1} \right] \\ &\quad - \beta \mathrm{E}_t \left( \frac{u'_{t+1}}{u'_t} \right) \left[ \eta k^w_t (1-\rho^x) \int_{\tilde{a}_{t+1}}^{\bar{a}} s_{t+1} f(a) da + w^u_{t+1} \right] - h \\ &= (1-\rho^x)(1-\eta k^w_t) \beta \mathrm{E}_t \left( \frac{u'_{t+1}}{u'_t} \right) \left[ \int_{\tilde{a}_{t+1}}^{\bar{a}} s_{t+1} f(a) da \right] - h \end{aligned} \tag{9}$$

or

$$q_t = (1-\rho^x)(1-\eta k^w_t) \mathrm{E}_t X_{t+1} - h.$$

Note that using (7), this becomes

$$\begin{aligned} q_t &= \frac{\gamma(1-\rho^x)(1-\eta k^w_t)}{(1-\eta)(1-\rho^x) k^f_t} - h \\ &= \left[ \frac{\gamma(1-\eta k^w_t)}{(1-\eta) k^f_t} \right] - h. \end{aligned}$$

### 2.1.2 Matching

$\rho_t N_t$ matches dissolve prior to engaging in production during period $t$. If the worker is not part of an existing match, or if her current match ends, she travels to the labor matching market. Unmatched firms, or firms whose matches terminated, may choose to enter the labor matching market and post vacancies. Based on a matching function, some fraction of workers and firms in the labor market establish new matches. These, plus the worker-firm matches that produced during the period, constitute the stock of matches that enter period $t+1$. Thus, a total of

$$u_t \equiv U_t + \rho_t N_t = 1 - (1-\rho_t) N_t \tag{10}$$

workers will not produce market goods during the period and will be searching for a new match.

The number of matches is equal to $m(u_t, V_t)$ where $V_t$ is the number of posted vacancies and $m(\quad)$ is the aggregate matching function.

The probability an unemployed worker makes a match, $k^w_t$, is equal to

$$k^w_t = \frac{m(u_t, V_t)}{u_t} \tag{11}$$

The probability a firm with a posted vacancy finds a match, $k_t^f$, is

$$k_t^f = \frac{m(u_t, V_t)}{V_t} \tag{12}$$

The total number of matches evolves according to

$$N_{t+1} = (1-\rho_t)N_t + m(u_t, V_t) \tag{13}$$

#### 2.1.3 Output of the wholesale sector

$(1-\rho_t)N_t$ is the number of producing matches in period $t$. Average output per match is

$$\mathrm{E}\left(a_{it}z_t \mid a_{it} > \tilde{a}\right) = \int_{\tilde{a}} a_{it}z_t \frac{dF(a)}{1-F(\tilde{a})}$$

Wholesale sector output is

$$Q_t = z_t \left[\int_{\tilde{a}} a_{it} \frac{dF(a)}{1-F(\tilde{a})}\right](1-\rho_t)N_t.$$

### 2.2 The retail sector

Retail firms are monopolistically competitive. Firm $j$ faces a demand curve

$$c_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{-\theta} C_t$$

Nominal profits:

$$p_{jt}c_t(j) - P_t^w c_t(j)$$

Real profits:

$$\left[\left(\frac{p_{jt+i}}{P_{t+i}}\right) - \frac{1}{\mu_t}\right] c_t(j)$$

Real marginal cost

$$mc_t = \frac{1}{\mu_t}$$

The firm's decision problem then involves picking $p_{jt}$ to maximize

$$\begin{aligned} & \mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i}\left[\left(\frac{p_{jt+i}}{P_{t+i}}\right) - mc_{t+i}\right]\left(\frac{p_{jt}}{P_{t+i}}\right)^{-\theta} C_{t+i} \\ = \quad & \mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i}\left[\left(\frac{p_{jt+i}}{P_{t+i}}\right)^{1-\theta} - mc_{t+i}\left(\frac{p_{jt}}{P_{t+i}}\right)^{-\theta}\right] C_{t+i} \end{aligned}$$

where the discount factor $\Delta_{i,t+i}$ is given by $\beta^i (C_{t+i}/C_t)^{-\frac{1}{\sigma}}$ and $\omega$ is the probability of *not* adjusting (i.e., $1-q$).

The first order condition is

$$\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[ (1-\theta) \left( \frac{1}{p_{jt}} \right) \left( \frac{p_{jt}}{P_{t+i}} \right)^{1-\theta} + \theta mc_{t+i} \left( \frac{1}{p_{jt}} \right) \left( \frac{p_{jt}}{P_{t+i}} \right)^{-\theta} \right] C_{t+i} = 0$$

or

$$\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[ (1-\theta) \left( \frac{1}{P_{t+i}} \right) + \theta mc_{t+i} \left( \frac{1}{p_{jt}} \right) \right] \left( \frac{p_{jt}}{P_{t+i}} \right)^{-\theta} C_{t+i} = 0.$$

This can be rewritten as

$$\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[ (1-\theta) \left( \frac{1}{P_{t+i}} \right) + \theta mc_{t+i} \left( \frac{1}{p_{jt}} \right) \right] \left[ \left( \frac{p_{jt}}{P_t} \right) \left( \frac{P_t}{P_{t+i}} \right) \right]^{-\theta} C_{t+i} = 0.$$

Dividing by $p_{jt}/P_t$ and rearranging,

$$\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[ \left( \frac{1}{P_{t+i}} \right) \left( \frac{P_t}{P_{t+i}} \right)^{-\theta} C_{t+i} \right] = \frac{\theta}{\theta-1} E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[ mc_{t+i} \left( \frac{1}{p_{jt}} \right) \left( \frac{P_t}{P_{t+i}} \right)^{-\theta} C_{t+i} \right].$$

Multiplying and dividing the left side by $P_t$ and then multiplying both sides by $p_{jt}$,

$$\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[ \left( \frac{p_{jt}}{P_t} \right) \left( \frac{P_t}{P_{t+i}} \right) \left( \frac{P_t}{P_{t+i}} \right)^{-\theta} C_{t+i} \right] = \frac{\theta}{\theta-1} E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[ mc_{t+i} \left( \frac{P_t}{P_{t+i}} \right)^{-\theta} C_{t+i} \right]$$

or

$$\left( \frac{p_{jt}}{P_t} \right) = \left( \frac{\theta}{\theta-1} \right) \frac{\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[ mc_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\theta} C_{t+i} \right]}{\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[ \left( \frac{P_{t+i}}{P_t} \right)^{\theta-1} C_{t+i} \right]}.$$

Since all firms adjusting in period $t$ set the same price, let $p_t^*$ be the optimally set price at time $t$. Then,

$$\left( \frac{p_t^*}{P_t} \right) = \left( \frac{\theta}{\theta-1} \right) \frac{\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[ mc_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\theta} C_{t+i} \right]}{\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[ \left( \frac{P_{t+i}}{P_t} \right)^{\theta-1} C_{t+i} \right]} \tag{14}$$

The aggregate price index is

$$P_t^{1-\theta} = (1-\omega)(p_t^*)^{1-\theta} + \omega P_{t-1}^{1-\theta} \tag{15}$$

Price adjustment (linear approximation around zero-inflation steady-state):

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} - \kappa \hat{\mu}_t$$

where $\hat{\mu}_t$ is the deviation of real marginal cost around flexible price level. For details, see Sbordone (2002).

# 3 Zero inflation steady-state

The steady-state is the same for both the flexible price and sticky price versions of the model.

In a zero-inflation steady-state, the household Euler condition implies $R = 1/\beta$. Using this in (**??**), $q = A - (\beta/\mu)\,\tilde{a}$. Using this result and the definition of the surplus $s$, equation (9) becomes, in the steady-state,

$$\left(\frac{\beta}{\mu}\right)\tilde{a} + \alpha\left(\frac{\beta}{\mu}\right)\left[\int_{\tilde{a}}^{\bar{a}} (a-\tilde{a})\,f(a_i)da_i\right] = A + h$$

where $\alpha \equiv (1-\rho^x)(1-\eta k^w)\beta < 1$.

The steady-state values of $\tilde{a}$, $\rho$, $N$, $u$, $V$, $k^f$, $k^w$, and $\mu$ are given by the solution to

$$\left(\frac{\beta}{\mu}\right)\tilde{a} + (1-\rho^x)(1-\eta k^w)\beta\left(\frac{\beta}{\mu}\right)\left[\int_{\tilde{a}}^{\bar{a}} (a-\tilde{a})\,f(a_i)da_i\right] = A + h$$

$$\rho = \rho^x + (1-\rho^x)\mathrm{F}(\tilde{a})$$

$$u = 1 - (1-\rho)N$$

$$\rho N = m(u, V).$$

$$k^f = \frac{m(u,V)}{V}.$$

$$k^w = \frac{m(u,V)}{u}.$$

$$\frac{\gamma(1-\eta k^w)}{(1-\eta)k^f} = A + h - (\beta/\mu)\,\tilde{a}$$

$$\mu = \frac{\theta}{\theta - 1}$$

# 4 Simulations

The model of the previous section is expressed in terms of percentage deviations and linearized around the steady state. The basic approach is described in Uhlig (1999), and the model solution and its properties are obtained using the "toolkit" of programs written by Harald Uhlig.[1]

The utility function for the composite consumption good is assumed to be of isoelastic form:

$$u(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma}; \qquad \sigma > 0,$$

where $\sigma$ is the coefficient of relative risk aversion. The matching function is taken to be

$$m(u_t, V_t) = \mu u_t^a V_t^\xi, \qquad 0 < a < 1,\ 0 < \xi < 1. \tag{16}$$

[1] Uhlig's programs are available at http://cwis.kub.nl/~few5/center/STAFF/uhlig/toolkit.dir/toolkit.htm.

## 4.1 The linearized model

Let $\hat{z}_t$ denote the percentage deviation of a variable $Z_t$ around its steady-state value. The linearized model consists of the following equations:

- The policy rule for nominal money growth equation:

$$\Theta_t = (1-\rho_m)\bar{\Theta} + \rho_m \Theta_{t-1} + \phi_t \bar{\Theta} \tag{17}$$

In the steady-state,

$$\bar{\Theta} = (1-\rho_m)\bar{\Theta} + \rho_m \bar{\Theta}.$$

Divide (17) by $\bar{\Theta}$:

$$\frac{\Theta_t}{\bar{\Theta}} = (1-\rho_m) + \rho_m \frac{\Theta_{t-1}}{\bar{\Theta}} + \phi_t$$

$$\frac{\Theta_t}{\bar{\Theta}} - 1 = \rho_m \left( \frac{\Theta_{t-1}}{\bar{\Theta}} - 1 \right) + \phi_t$$

$$\hat{\Theta}_t = \rho_m \hat{\Theta}_{t-1} + \phi_t; \tag{18}$$

- The cash-in-advance constraint, $M_t/P_t = Y_t$, becomes

$$\hat{m}_t - \hat{p}_t = \hat{y}_t.$$

In first difference form,

$$\hat{\Theta}_t = \hat{y}_t - \hat{y}_{t-1} + \hat{\pi}_t; \tag{19}$$

- The evolution of the number of matches:

$$\begin{aligned} N_{t+1} &= (1-\rho_t)N_t + m(u_t, V_t) \\ &= (1-\rho_t)N_t + \mu u_t^a V_t^\xi \end{aligned}$$

$$\begin{aligned} \bar{N}(1+\hat{n}_{t+1}) &= \left[1 - \bar{\rho}(1+\hat{\rho}_t)\right] \bar{N}(1+\hat{n}_t) + \mu \bar{u}^a (1 + a\hat{u}_t) \bar{V}^\xi (1 + \xi \hat{v}_t) \\ &\approx \bar{N}(1+\hat{n}_t) - \bar{\rho}\bar{N}(1 + \hat{\rho}_t + \hat{n}_t) + \mu \bar{u}^a \bar{V}^\xi (1 + a\hat{u}_t + \xi \hat{v}_t) \end{aligned}$$

But

$$\bar{N} = (1-\bar{\rho})\bar{N} + \mu \bar{u}^a \bar{V}^\xi$$

so

$$\bar{N}\hat{n}_{t+1} \approx \bar{N}\hat{n}_t - \bar{\rho}\bar{N}(\hat{\rho}_t + \hat{n}_t) + \mu \bar{u}^a \bar{V}^\xi (a\hat{u}_t + \xi \hat{v}_t)$$

$$\hat{n}_{t+1} \approx (1-\bar{\rho})\hat{n}_t - \bar{\rho}\hat{\rho}_t + \left( \frac{\mu \bar{u}^a \bar{V}^\xi}{\bar{N}} \right) (a\hat{u}_t + \xi \hat{v}_t)$$

Since $\varphi_t = 1 - \rho_t$,

$$\bar{\varphi}(1+\hat{\varphi}_t) = 1 - \bar{\rho}(1+\hat{\rho}_t) = 1 - \bar{\rho} - \bar{\rho}\hat{\rho}_t$$

$$\varphi\hat{\varphi}_t = -\bar{\rho}\hat{\rho}_t$$

and $m = k^f v$, so

$$\left(\frac{\mu \bar{u}^a \bar{V}^\xi}{\bar{N}}\right)(a\hat{u}_t + \xi\hat{v}_t) = \left(\frac{\bar{m}}{\bar{N}}\right)\hat{m}_t = \left(\frac{\bar{v}\bar{k}^f}{\bar{N}}\right)\left(\hat{v}_t + \hat{k}_t^f\right).$$

Hence,

$$\hat{n}_{t+1} = \bar{\varphi}\hat{n}_t + \varphi\hat{\varphi}_t + \left(\frac{vk^f}{N}\right)\hat{v}_t + \left(\frac{vk^f}{N}\right)\hat{k}_t^f; \tag{20}$$

- The endogenous job destruction margin:

$$\tilde{a}_t = \frac{\mu_t R_t (A - q_t)}{z_t}$$

can be approximated by first writing

$$\begin{aligned}
\tilde{a}(1+\hat{a}_t) &= \frac{\mu_t R_t A}{z_t} - \frac{\mu_t R_t q_t}{z_t} \\
&= \mu R A\left(1 + \hat{\mu}_t + \hat{r}_t - \hat{z}_t\right) - \mu R q\left(1 + \hat{\mu}_t + \hat{r}_t + \hat{q}_t - \hat{z}_t\right) \\
&= \mu R(A-q)\left(1 + \hat{\mu}_t + \hat{r}_t - \hat{z}_t\right) - \mu R q \hat{q}_t
\end{aligned}$$

But $\tilde{a} = \mu R(A-q)$,

$$\tilde{a}\hat{a}_t = \mu R(A-q)\left(\hat{\mu}_t + \hat{r}_t - \hat{z}_t\right) - \mu R q \hat{q}_t$$

or

$$\hat{a}_t = \hat{r}_t + \hat{\mu}_t - \hat{z}_t - \left(\frac{\mu R q}{\tilde{a}}\right)\hat{q}_t; \tag{21}$$

- The survival rate $\varphi_t = 1 - \rho_t$ using equation (2):

$$\varphi_t = (1-\rho^x)(1-\rho_t^n)$$

$$\bar{\varphi}(1+\hat{\varphi}_t) = (1-\rho^x) - (1-\rho^x)\bar{\rho}^n(1+\hat{\rho}_t^n)$$

$$\bar{\varphi}\hat{\varphi}_t = -(1-\rho^x)\bar{\rho}^n\hat{\rho}_t^n$$

$$\begin{aligned}
\hat{\varphi}_t &= -\left[\frac{(1-\rho^x)\bar{\rho}^n}{\bar{\varphi}}\right]\hat{\rho}_t^n \\
&= -\left[\frac{(1-\rho^x)\bar{\rho}^n}{(1-\rho^x)(1-\bar{\rho}^n)}\right]\hat{\rho}_t^n
\end{aligned}$$

or

$$\hat{\varphi}_t = -\left(\frac{\rho^n}{1-\rho^n}\right)\hat{\rho}_t^n.$$

Since $\rho_t^n = \mathrm{F}(\tilde{a})$,

$$\hat{\rho}_t^n = \left[\frac{\partial \mathrm{F}(\tilde{a}_t)}{\partial \tilde{a}_t}\frac{\tilde{a}_t}{\mathrm{F}(\tilde{a}_t)}\right]\hat{a}_t = e_{F,a}\hat{a}_t,$$

so

$$\hat{\varphi}_t = -\left(\frac{\rho^n}{1-\rho^n}\right)e_{F,a}\hat{a}_t; \tag{22}$$

- The number of unemployed job seekers equation (10):

$$\begin{aligned} u_t &\equiv \bar{u}(1+\hat{u}_t) = 1 - [1-\bar{\rho}(1+\hat{\rho}_t)]\,\bar{N}(1+\hat{n}_t) \\ &= 1 - \bar{N}(1+\hat{n}_t) + \bar{\rho}\bar{N}(1+\hat{\rho}_t+\hat{n}_t) \end{aligned}$$

$$\bar{u}\hat{u}_t = -\bar{N}(\hat{n}_t) + \bar{\rho}\bar{N}(\hat{\rho}_t+\hat{n}_t)$$

Therefore,

$$\begin{aligned} \hat{u}_t &= -\left(\frac{1-\bar{\rho}}{\bar{u}}\right)\bar{N}\hat{n}_t + \frac{\bar{\rho}\bar{N}}{\bar{u}}\hat{\rho}_t \\ &= -\left(\frac{\varphi\bar{N}}{\bar{u}}\right)\hat{n}_t - \frac{\bar{\rho}\bar{N}}{\bar{u}}\frac{\varphi}{\bar{\rho}}\hat{\varphi}_t \\ &= -\left(\frac{\varphi\bar{N}}{\bar{u}}\right)\hat{n}_t - \left(\frac{\varphi N}{u}\right)\hat{\varphi}_t; \end{aligned}$$

- The probability a vacancy is filled equation becomes

$$\left(\frac{\mu\bar{u}^a\bar{V}^\xi}{\bar{N}}\right)(a\hat{u}_t+\xi\hat{v}_t) = \left(\frac{\bar{m}}{\bar{N}}\right)\hat{m}_t = \left(\frac{\bar{v}\bar{k}^f}{\bar{N}}\right)\left(\hat{v}_t+\hat{k}_t^f\right),$$

where this was derived above. Hence,

$$\hat{k}_t^f = a\hat{u}_t - (1-\xi)\hat{v}_t; \tag{23}$$

- The equality of firms filling vacancies and workers finding matches:

$$u_t k_t^w = V_t k_t^f$$

or

$$\hat{v}_t + \hat{k}_t^f = \hat{u}_t + \hat{k}_t^w; \tag{24}$$

- The job posting condition:

$$q_t = \left[\frac{\gamma\,(1-\eta k_t^w)}{(1-\eta)k_t^f}\right] - h \tag{25}$$

or

$$\gamma\,(1-\eta k_t^w) = (1-\eta)k_t^f(q_t+h).$$

Hence,

$$\gamma\left[1-\eta k^w(1+\hat{k}_t^w)\right] = (1-\eta)k^f(1+\hat{k}_t^f)(q+q\hat{q}_t+h)$$

But $(1-\eta)k^f = \gamma\,(1-\eta k_t^w)\,/(q+h)$ (from 25), so

$$\gamma\left[1-\eta k^w(1+\hat{k}_t^w)\right] = \left[\frac{\gamma\,(1-\eta k_t^w)}{q+h}\right](1+\hat{k}_t^f)(q+q\hat{q}_t+h)$$

or

$$\left[1-\eta k^{w}(1+\hat{k}_t^{w})\right]=\frac{(1-\eta k_t^{w})}{q+h}\left[q+h+q\hat{q}_t+(q+h)\hat{k}_t^{f}\right].$$

Rearranging,

$$\left[\frac{1-\eta k^{w}(1+\hat{k}_t^{w})}{(1-\eta k_t^{w})}\right]=1+\left(\frac{q}{q+h}\right)\hat{q}_t+\hat{k}_t^{f}$$

which reduces to

$$\hat{k}_t^{f}=-\left(\frac{\eta k^{w}}{1-\eta k^{w}}\right)\hat{k}_t^{w}-\left(\frac{q}{q+h}\right)\hat{q}_t; \tag{26}$$

- The output equation:

$$Y_t=Q_t-\gamma V_t=z_t\left[\int_{\tilde{a}}a_{it}\frac{d\mathrm{F}(a)}{1-\mathrm{F}(\tilde{a})}\right](1-\rho_t)N_t-\gamma V_t.$$

$$Y_t=Q_t-\gamma V_t=z_t G(\tilde{a}_t)\varphi_t N_t-\gamma V_t.$$

where

$$H(\tilde{a}_t)=\int_{\tilde{a}}a_{it}\frac{d\mathrm{F}(a)}{1-\mathrm{F}(\tilde{a})}$$

Hence,

$$Y_t=Y(1+\hat{y}_t)=G(\tilde{a})\varphi N(1+\hat{z}_t+e_{H,a}\hat{a}_t+\hat{\varphi}_t+\hat{n}_t)-\gamma V(1+\hat{v}_t).$$

$$Y\hat{y}_t=G(\tilde{a})\varphi N(\hat{z}_t+e_{H,a}\hat{a}_t+\hat{\varphi}_t+\hat{n}_t)-\gamma V(\hat{v}_t),$$

or

$$\hat{y}_t=\left(\frac{Q}{Y}\right)(e_{H,a}\hat{a}_t+\hat{\varphi}_t+\hat{n}_t+z_t)-\left(\frac{\gamma V}{Y}\right)\hat{v}_t; \tag{27}$$

- The Euler condition from the household's optimization problem equation canbe approximated as

$$0=\mathrm{E}_t\hat{y}_{t+1}-\hat{y}_t-\left(\frac{1}{\sigma}\right)(\hat{r}_t-\mathrm{E}_t\hat{\pi}_{t+1}); \tag{28}$$

- The inflation equation from the retail firms' pricing decisions equations (14) and (15) :

$$0=\beta\mathrm{E}_t\hat{\pi}_{t+1}-\hat{\pi}_t-\kappa\hat{\mu}_t. \tag{29}$$

- The present value condition for matches:

$$
\begin{aligned}
q_t &= (1-\rho^x)(1-\eta k_t^w)\beta \mathrm{E}_t\left(\frac{u'_{t+1}}{u'_t}\right)\left[\int_{\tilde{a}_{t+1}}^{\bar{a}} s_{t+1} f(a)da\right] - h \\
&= (1-\rho^x)(1-\eta k_t^w)\beta \mathrm{E}_t\left(\frac{u'_{t+1}}{u'_t}\right)\left(\frac{z_{t+1}}{\mu_{t+1}R_{t+1}}\right)\left(\int_{\tilde{a}_{t+1}} a_{t+1} f(a)da\right) \\
&\quad +(1-\rho^x)(1-\eta k_t^w)\beta \mathrm{E}_t\left(\frac{u'_{t+1}}{u'_t}\right)(q_{t+1}-A)\left[1-F(\tilde{a}_{t+1})\right] - h \\
&= (1-\eta k_t^w)\beta \mathrm{E}_t\left(\frac{u'_{t+1}}{u'_t}\right)\varphi_{t+1}\left[\left(\frac{z_{t+1}}{\mu_{t+1}R_{t+1}}\right)H(\tilde{a}_{t+1}) + q_{t+1} - A\right] - h,
\end{aligned}
$$

where

$$
H(\tilde{a}) \equiv \int_{\tilde{a}_{t+1}} a_{t+1}\frac{f(a)}{1-F(\tilde{a}_{t+1})}da.
$$

Define

$$
\begin{aligned}
e_{H,a} &= \frac{\partial H}{\partial a}\frac{a}{H} \\
e_{F,a} &= \frac{\partial F}{\partial a}\frac{a}{F}.
\end{aligned}
$$

In the steady-state,

$$
\begin{aligned}
q &= (1-\eta k^w)\beta\varphi\left[\left(\frac{H(\tilde{a})}{\mu R}\right) + q - A\right] - h \\
&= \frac{(1-\eta k^w)\beta\varphi\left[\left(\frac{H(\tilde{a})}{\mu R}\right) - A\right] - h}{1-(1-\eta k^w)\beta\varphi}.
\end{aligned}
$$

Let

$$
X_{1t+1} = \mathrm{E}_t\left(\frac{u'_{t+1}}{u'_t}\right).
$$

Then

$$
q_t = (1-\eta k_t^w)\beta \mathrm{E}_t X_{1t+1}\varphi_{t+1}\left[\left(\frac{z_{t+1}}{\mu_{t+1}R_{t+1}}\right)H(\tilde{a}_{t+1}) + q_{t+1} - A\right] - h
$$

$$
\begin{aligned}
q(1+\hat{q}_t) &= (1-\eta k^w)\beta\varphi\left(\frac{H(\tilde{a})}{\mu R}\right)\mathrm{E}_t(1+\hat{x}_{1t+1}+\hat{\varphi}_{t+1}+\hat{z}_{t+1}-\hat{\mu}_{t+1}-\hat{R}_{t+1}+e_{H,a}\hat{a}_{t+1}) \\
&\quad +(1-\eta k^w)\beta q\varphi \mathrm{E}_t(1+\hat{x}_{1t+1}+\hat{\varphi}_{t+1}+\hat{q}_{t+1}) \\
&\quad -(1-\eta k^w)\beta\varphi \mathrm{E}_t(1+\hat{x}_{1t+1}+\hat{\varphi}_{t+1})A \\
&\quad -\eta\hat{k}_t^w k^w\beta\varphi\left[\left(\frac{H(\tilde{a})}{\mu R}\right) + q - A\right] - h.
\end{aligned}
$$

Subtracting

$$q = (1-\eta k^w)\,\beta\varphi\left[\left(\frac{H(\tilde{a})}{\mu R}\right)+q-A\right]-h$$

from both sides,

$$\begin{aligned} q\hat{q}_t &= (1-\eta k^w)\,\beta\varphi\left(\frac{H(\tilde{a})}{\mu R}\right)\mathrm{E}_t(\hat{x}_{1t+1}+\hat{\varphi}_{t+1}+\hat{z}_{t+1}-\hat{\mu}_{t+1}-\hat{R}_{t+1}+e_{H,a}\hat{a}_{t+1}) \\ &\quad +(1-\eta k^w)\,\beta q\varphi\mathrm{E}_t(\hat{x}_{1t+1}+\hat{\varphi}_{t+1}+\hat{q}_{t+1}) \\ &\quad -(1-\eta k^w)\,\beta\varphi\mathrm{E}_t(\hat{x}_{1t+1}+\hat{\varphi}_{t+1})A \\ &\quad -\eta\hat{k}_t^w k^w\beta\varphi\left[\left(\frac{H(\tilde{a})}{\mu R}\right)+q-A\right]. \end{aligned}$$

$$\begin{aligned} q\hat{q}_t &= (1-\eta k^w)\,\beta\varphi\left[\left(\frac{H(\tilde{a})}{\mu R}\right)+q-A\right]\mathrm{E}_t\left(\hat{x}_{1t+1}+\hat{\varphi}_{t+1}\right) \\ &\quad (1-\eta k^w)\,\beta\varphi\left(\frac{H(\tilde{a})}{\mu R}\right)\mathrm{E}_t(\hat{z}_{t+1}-\hat{\mu}_{t+1}-\hat{R}_{t+1}+e_{H,a}\hat{a}_{t+1}) \\ &\quad +(1-\eta k^w)\,\beta q\varphi\mathrm{E}_t\hat{q}_{t+1} \\ &\quad -\eta k^w\beta\varphi\left[\frac{q+h}{(1-\eta k^w)\,\beta\varphi}\right]\hat{k}_t^w \end{aligned}$$

$$\begin{aligned} q\hat{q}_t &= (q+h)\,\mathrm{E}_t\left(\hat{x}_{1t+1}+\hat{\varphi}_{t+1}\right) \\ &\quad +(1-\eta k^w)\,\beta\varphi\left(\frac{H(\tilde{a})}{\mu R}\right)\mathrm{E}_t(\hat{z}_{t+1}-\hat{\mu}_{t+1}-\hat{R}_{t+1}+e_{H,a}\hat{a}_{t+1}) \\ &\quad +(1-\eta k^w)\,\beta q\varphi\mathrm{E}_t\hat{q}_{t+1} \\ &\quad -\left[\frac{\eta k^w}{(1-\eta k^w)}\right](q+h)\,\hat{k}_t^w \end{aligned}$$

$$\begin{aligned} \hat{q}_t &= \left(\frac{q+h}{q}\right)\mathrm{E}_t\left(\hat{x}_{1t+1}+\hat{\varphi}_{t+1}\right) \\ &\quad +AB\mathrm{E}_t(\hat{z}_{t+1}-\hat{\mu}_{t+1}-\hat{R}_{t+1}+e_{H,a}\hat{a}_{t+1}) \\ &\quad +(1-\eta k^w)\,\beta\varphi\mathrm{E}_t\hat{q}_{t+1} \\ &\quad -\left[\frac{\eta k^w}{(1-\eta k^w)}\right]\left(\frac{q+h}{q}\right)\hat{k}_t^w \end{aligned}$$

where

$$AB = \frac{(1-\eta k^w)\,\beta\varphi}{q}\left(\frac{H(\tilde{a})}{\mu R}\right)$$

Since

$$X_{1t+1} = \mathrm{E}_t\left(\frac{u'_{t+1}}{u'_t}\right) = \mathrm{E}_t\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma},$$

$$\hat{x}_{1t+1} = -\sigma\left(\mathrm{E}_t\hat{c}_{t+1} - \hat{c}_t\right) = -\left(\hat{R}_t - \mathrm{E}_t\pi_{t+1}\right).$$

Hence,

$$\begin{aligned}
\hat{q}_t &= AB(\mathrm{E}_t\hat{z}_{t+1} - \mathrm{E}_t\hat{\mu}_{t+1} - \mathrm{E}_t\hat{R}_{t+1} + e_{H,a}\mathrm{E}_t\hat{a}_{t+1}) \\
&\quad + \left(\frac{q+h}{q}\right)\mathrm{E}_t\hat{\varphi}_{t+1} - \left(\frac{q+h}{q}\right)\left(\hat{R}_t - \mathrm{E}_t\pi_{t+1}\right) \\
&\quad - \left(\frac{\eta k^w}{1-\eta k^w}\right)\left(\frac{q+h}{q}\right)\hat{k}_t^w + \left(1-\eta k^w\right)\beta\varphi\mathrm{E}_t\hat{q}_{t+1}
\end{aligned}$$

which corrects one error in the published version of the paper (the coefficient on $\mathrm{E}_t\hat{\varphi}_{t+1}$).

## 4.2 Simulations

### 4.2.1 Flexible prices

Let $x_t = (\Theta_t, \hat{R}_t, \hat{n}_t, \hat{\varphi}_t, \hat{y}_t, \hat{a}_t, \hat{v}_t, \hat{k}_t^f, \hat{k}_t^w, \hat{q}, \hat{u}_t, \hat{\pi}_t)'$ be the vector of endogenous variables. The equilibrium conditions of the model can be written as

$$\bar{A}\mathrm{E}_t x_{t+1} + \bar{B}x_t + \bar{C}\mathrm{E}_t\psi_{t+1} + \bar{D}\psi_t = 0, \tag{30}$$

where $\psi_t = (\phi_t, z_t)'$ and

$$\psi_t = N\psi_{t-1} + \varepsilon_t.$$

where $\varepsilon_t = (\phi_t, \varphi_t)'$.

Uhlig (1999) provides a complete discussion of the methods used to solve systems such as (30). In his formulation, the equations in (30) are divided into those that involve forward looking variables and those that do not:

$$0 = Ax_{1t} + Bx_{1t-1} + Cx_{2t} + D\psi_t$$

and

$$0 = F\mathrm{E}_t x_{1t} + Gx_{1t} + Hx_{1t-1} + J\mathrm{E}_t x_{2t} + Kx_{2t} + L\mathrm{E}_t\psi_{t+1} + M\psi_t,$$

where $x_{1t}$ is a vector of endogenous state variables and $x_{2t}$ is a vector of other endogenous variables, $A$, $B$, etc. are conformal matrices, and $C$ is of full column rank. Stacking these equations,

$$\begin{aligned}
0 &= \begin{bmatrix} 0 & 0 \\ F & J \end{bmatrix}\begin{bmatrix} \mathrm{E}_t x_{1t} \\ \mathrm{E}_t x_{2t} \end{bmatrix} + \begin{bmatrix} A & C \\ G & K \end{bmatrix}\begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} \\
&\quad + \begin{bmatrix} B & 0 \\ H & 0 \end{bmatrix}\begin{bmatrix} x_{1t-1} \\ x_{2t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ L \end{bmatrix}\mathrm{E}_t\psi_{t+1} + \begin{bmatrix} D \\ M \end{bmatrix}\psi_t.
\end{aligned}$$

The exogenous disturbances are given by

$$\psi_t = N\psi_{t-1} + \varepsilon_t.$$

In the flexible-price version of the model, let $x_{1t} = (\Theta_t, \hat{R}_t, \hat{n}_t, \hat{\varphi}_t, \hat{y}_t)'$ and $x_{2t} = (\hat{a}_t, \hat{v}_t, \hat{k}_t^f, \hat{k}_t^w, \hat{q}, \hat{u}_t, \hat{\pi}_t)'$. With flexible prices, the mark-up $\mu$ is constant, equal to $\theta/(\theta-1)$, and equation (29) is dropped. Thus, the flex-price version of the model is

$$\Theta_t = \rho_m \Theta_{t-1} + \phi_t;$$

$$\hat{y}_t - \hat{y}_{t-1} + \hat{\pi}_t - \Theta_t = 0;$$

$$\hat{n}_{t+1} - \varphi \hat{n}_t - \varphi \hat{\varphi}_t - \left(\frac{vk^f}{N}\right) \hat{v}_t - \left(\frac{vk^f}{N}\right) \hat{k}_t^f = 0;$$

$$\hat{r}_t - \left(\frac{\mu R q}{\tilde{a}}\right) \hat{q}_t - \hat{a}_t - \hat{z}_t = 0;$$

$$\hat{\varphi}_t + \left(\frac{\rho^n}{1-\rho^n}\right) e_{F,a} \hat{a}_t = 0;$$

$$\hat{u}_t + \left(\frac{\varphi N}{u}\right) \hat{n}_t + \left(\frac{\varphi N}{u}\right) \hat{\varphi}_t = 0;$$

$$\hat{k}_t^f - a\hat{u}_t + (1-\xi)\hat{v}_t = 0;$$

$$\hat{v}_t + \hat{k}_t^f - \hat{u}_t - \hat{k}_t^w = 0;$$

$$\hat{k}_t^f + \left(\frac{\eta k^w}{1-\eta k^w}\right) \hat{k}_t^w + \left(\frac{q}{q+h}\right) \hat{q}_t = 0;$$

$$\hat{y}_t - \left(\frac{Q}{Y}\right) (e_{E,a}\hat{a}_t + \hat{\varphi}_t + \hat{n}_t + z_t) + \left(\frac{\gamma V}{Y}\right) \hat{v}_t = 0;$$

$$\mathrm{E}_t \hat{y}_{t+1} - \hat{y}_t - \left(\frac{1}{\sigma}\right) \hat{r}_t + \left(\frac{1}{\sigma}\right) \mathrm{E}_t \hat{\pi}_{t+1} = 0;$$

$$AB \left(e_{H,a}\mathrm{E}_t\hat{a}_{t+1} - \mathrm{E}_t\hat{R}_{t+1} + \mathrm{E}_t z_{t+1}\right) + \left(\frac{q+h}{q}\right) \mathrm{E}_t\hat{\varphi}_{t+1} - \left(\frac{q+h}{q}\right) \left(\hat{R}_t - \mathrm{E}_t\hat{\pi}_{t+1}\right)$$
$$- \left(\frac{\eta k^w}{1-\eta k^w}\right) \left(\frac{q+h}{q}\right) \hat{k}_t^w + (1-\eta k^w)\beta\varphi \mathrm{E}_t\hat{q}_{t+1} - q_t = 0$$

$$z_t = \rho_z z_{t-1} + g e_t.$$

The first ten equations of the form

$$0 = Ax_{1t} + Bx_{1t-1} + Cx_{2t} + D\psi_t$$

are given by

$$0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & -\varphi & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{\varphi N}{u} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{(1-\rho^x)NH}{Y} & 1 \end{bmatrix} \begin{bmatrix} \Theta_t \\ \hat{R}_t \\ \hat{n}_{t+1} \\ \hat{\varphi}_t \\ \hat{y}_t \end{bmatrix} + \begin{bmatrix} -\rho_m & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -\varphi & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\varphi N}{u} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{(1-\rho^x)NH}{Y} & 0 & 0 \end{bmatrix} \begin{bmatrix} \Theta_{t-1} \\ \hat{R}_{t-1} \\ \hat{n}_t \\ \hat{\varphi}_{t-1} \\ \hat{y}_{t-1} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & -\frac{k^f V}{N} & -\frac{k^f V}{N} & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & \frac{\mu R q}{\tilde{a}} & 0 & 0 \\ \frac{\rho^n e_{Fa}}{1-\rho^n} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & \xi - 1 & -1 & 0 & 0 & \alpha & 0 \\ 0 & 1 & 1 & -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & \frac{\eta k^w}{1-\eta k^w} & \frac{q}{q+h} & 0 & 0 \\ -\frac{(1-\rho^x)NHe_{Ha}}{Y} & \frac{\gamma V}{Y} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{a}_t \\ \hat{v}_t \\ \hat{k}_t^f \\ \hat{k}_t^w \\ \hat{q}_t \\ \hat{u}_t \\ \hat{\pi}_t \end{bmatrix}$$

$$+ \begin{bmatrix} -1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -\frac{(1-\rho^x)NH}{Y} \end{bmatrix} \begin{bmatrix} \phi_t \\ z_t \end{bmatrix}$$

The next two equations of the form

$$0 = F\mathrm{E}_t x_{1t} + Gx_{1t} + Hx_{1t-1} + J\mathrm{E}_t x_{2t} + Kx_{2t} + L\mathrm{E}_t \psi_{t+1} + M\psi_t,$$

are given by

$$0 = \begin{bmatrix} 0 & -AB & 0 & 0 & \frac{q+h}{q} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathrm{E}_t\Theta_{t+1} \\ \mathrm{E}_t\hat{R}_{t+1} \\ \mathrm{E}_t\hat{n}_{t+2} \\ \mathrm{E}_t\hat{\varphi}_{t+1} \\ \mathrm{E}_t\hat{y}_{t+1} \end{bmatrix} + \begin{bmatrix} 0 & -\frac{q+h}{q} & 0 & 0 & 0 \\ 0 & -\frac{1}{\sigma} & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \Theta_t \\ \hat{R}_t \\ \hat{n}_{t+1} \\ \hat{\varphi}_t \\ \hat{y}_t \end{bmatrix}$$

$$+ \begin{bmatrix} ABe_{Ha} & 0 & 0 & 0 & (1-\eta k^w)\beta\varphi & 0 & \frac{q+h}{q} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sigma} \end{bmatrix} \begin{bmatrix} \mathrm{E}_t\hat{a}_{t+1} \\ \mathrm{E}_t\hat{v}_{t+1} \\ \mathrm{E}_t\hat{k}^f_{t+1} \\ \mathrm{E}_t\hat{k}^w_{t+1} \\ \mathrm{E}_t\hat{q}_{t+1} \\ \mathrm{E}_t\hat{u}_{t+1} \\ \mathrm{E}_t\hat{\pi}_{t+1} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & -\frac{\eta k^w(q+h)}{(1-\eta k^w)q} & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{a}_t \\ \hat{v}_t \\ \hat{k}^f_t \\ \hat{k}^w_t \\ \hat{q}_t \\ \hat{u}_t \\ \hat{\pi}_t \end{bmatrix} + \begin{bmatrix} 0 & AB \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathrm{E}_t\phi_{t+1} \\ \mathrm{E}_t z_{t+1} \end{bmatrix}$$

Finally, the exogenous processes are given by

$$\begin{bmatrix} \phi_t \\ z_t \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \rho_z \end{bmatrix} \begin{bmatrix} \phi_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \phi_t \\ \varepsilon_t \end{bmatrix}.$$

If an equilibrium solution to this system of equations exists, it takes the form of stable laws of motion given by

$$\begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} = Px_{1t-1} + Q\psi_t.$$

### 4.2.2 Sticky prices

With sticky-prices, $\mu_t$ is an endogenous variable and two equations are affected. The equation for $\hat{a}_t$ becomes

$$\hat{a}_t = \hat{R}_t + \hat{\mu}_t - \left(\frac{\mu R q}{\tilde{a}}\right)\hat{q}_t - \hat{z}_t,$$

and the equation for $\hat{q}_t$ becomes

$$\begin{aligned}\hat{q}_t &= AB\left(e_{H,a}\mathrm{E}_t\hat{a}_{t+1} - \mathrm{E}_t\hat{\mu}_{t+1} - \mathrm{E}_t\hat{r}_{t+1} + \mathrm{E}_t z_{t+1}\right) + \left(\frac{q+h}{q}\right)\mathrm{E}_t\hat{\varphi}_{t+1} \\ &\quad - \left(\frac{q+h}{q}\right)\left(\hat{R}_t - \mathrm{E}_t\hat{\pi}_{t+1}\right) - \left(\frac{\eta k^w}{1-\eta k^w}\right)\left(\frac{q+h}{q}\right)\hat{k}^w_t + (1-\eta k^w)\beta\varphi\mathrm{E}_t\hat{q}_{t+1}\end{aligned}$$

With one new element of $x_{2t}$ (i.e., $\mu_t$), there is one additional equation which is given by

$$0 = \beta \mathrm{E}_t \hat{\pi}_{t+1} - \hat{\pi}_t - \kappa \hat{\mu}_t.$$

This is then added to the block of equations containing future expectations.