Notes on the open economy NK model

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1 The open economy

Open economy issues

- Analysis so far has been conducted within the context of a closed economy.
- Need to incorporate open economy aspects
  - How does the transmission process of monetary policy differ in the open economy?
  - How are the objectives of policy affected?
  - How are conclusions about policy implementation affected?

Differences between open and closed economies

Primary differences between open and closed economies

- Domestic production and demand can differ;
- Additional relative price – terms of trade;
- Interest rate linkages;
- Price of domestic output and consumer price index can differ.

Open economy issues

How does this affect the three components of the basic New Keynesian model?

- Aggregate demand depends on terms of trade and foreign income;
- Aggregate inflation adjustment may depend on terms of trade via imported inputs and wedge between domestic output price and consumer price index;
- Policy objectives might depend on consumer price inflation or product real wage.

Foreign and domestic goods
• Distinguish between home production and foreign production.
• Can do so on the consumption side (utility depends on both foreign and domestically produced goods).
• Can do so on the production side (use of foreign goods as inputs).

2 Clarida, Galí, and Gertler (AER, May 2001)

• Clarida, Galí, and Gertler modify their Science of Monetary Policy model to apply it to the case of a small open economy.
• The basic components of the model are
  − nominal price stickiness
  − consumption goods are traded, with the foreign produced and domestic produced goods being imperfect substitutes in utility
  − domestic good is a composite of a continuum of differentiated goods each produced by a monopolistically competitive domestic firm
  − domestic economy is small – treats foreign price level and interest rate as given
  − consumption risk is shared internationally.

2.1 Households

Household utility

• Households consume a CES composite of home and foreign goods:

\[ C_t = \left[ \left( \frac{1 - \gamma}{\gamma} \right)^{\frac{1}{a-1}} C^{h}_t \right]^{\frac{a-1}{a}} + \left( C^{f}_t \right)^{\frac{a-1}{a}} \]

for \( a > 1 \) and goods in the interval \([0, \gamma)\) are foreign produced and in \([\gamma, 1]\) are domestically produced.
• This means that in terms of percentage deviations, we will have

\[ c_t = (1 - \gamma)c^h_t + \gamma c^f_t \]

for foreign \((c^f)\) and domestic \((c^h)\) goods.

Demand

• As before, relative demands depend on relative price:

\[ \frac{C^h_t}{C^f_t} = \left( \frac{1 - \gamma}{\gamma} \right) \left( \frac{P^h_t}{P^f_t} \right)^{-\alpha} \]
• Hence,

\[ c_t^f = -a \left( p_t^f - p_t^h \right) + c_t^h \]  

(1)

**Law of one price**

- Law of one price implies

\[ P_t^f = S_t P_t^* \]

where \( P_t^* \) is the domestic currency price of foreign produced goods and \( E \) is the nominal exchange rate (price of foreign currency in terms of domestic currency).

  - Assume all foreign goods sell for price \( P_t^f \).
  - Assumes complete pass-through.

**Aggregate price index and terms of trade**

- The aggregate CPI price index \( P_t \):

\[ p_t^i = (1 - \gamma)p_t^h + \gamma p_t^f = p_t^h + \gamma (c_t + p_t^* - p_t^h) \]  

(2)

and \( p_t^h \) is the average price of domestically produced goods.

- Define the terms of trade as

\[ \delta_t = p_t^f - p_t^h = c_t + p_t^* - p_t^h \]

- Then

\[ p_t^i = p_t^h + \gamma \delta_t \]

**Aggregate price index and terms of trade**

- Combining expression of relative demands with definition of the CPI,

\[ c_t^f - c_t^h = -a \left( p_t^f - p_t^h \right) = -a \delta_t \]  

(3)

- Hence,

\[ c_t = (1 - \gamma)c_t^h + \gamma c_t^f = c_t^h - \gamma a \delta_t. \]

Defining \( \pi_t^h = p_t^h - p_{t-1}^h \),

\[ \pi_t^c = p_t^c - p_{t-1}^c = \pi_t^h + \gamma (\delta_t - \delta_{t-1}). \]  

(4)

**Labor supply**

- Households equate marginal rate of subsitution between consumption and leisure to real wage.

- Assume this takes the form

\[ \eta p_t + \sigma c_t = w_t - p_t^c = w_t - p_t^h - \gamma \delta_t. \]
2.2 Production

Domestically produced goods

- Domestically produced goods are sold to domestic residents and to foreigners:
  \[ y_t = (1 - \gamma)c^h_t + \gamma c^k_t \]
  where \( c^k_t \) is foreign consumption of home produced goods.
  - This assumes the same preferences for the home and foreign households.

Production and inflation

- The production function takes the form
  \[ y^h_t = n_t + \varepsilon_t. \]  \hspace{1cm} (5)
- Marginal cost is
  \[ mc_t = w_t - p^h_t - \varepsilon_t. \]  \hspace{1cm} (6)
- The inflation rate for the price index of domestically produced goods is
  \[ \pi^h_t = \beta E_t \pi^h_{t+1} + \kappa mc_t, \]  \hspace{1cm} (7)
  where \( \kappa = (1 - \omega)(1 - \beta \omega) / \omega. \)
  - But,
    \[ mc_t = w_t - p^c_t + (p^c_t - p^h_t) - \varepsilon_t \]
    \[ = w_t - p^c_t + \gamma \delta_t - \varepsilon_t \]
- Will use these later.

2.3 The foreign country

The foreign country

- Assume it is large.
  - Don’t need to distinguish between CPI inflation and domestic inflation.
  - Domestic output and consumption are equal.
- Foreign demand for home produced goods depends on terms of trade:
  \[ c^k_t = a \delta_t + y^*_t \]  \hspace{1cm} (8)
  where \( y^*_t \) is foreign income.
Euler condition for foreign households implies

\[ y_t^f = E_y y_{t+1}^f - \left( \frac{1}{\sigma} \right) \left( i_t^f - E_i \pi_{t+1}^f \right), \]

or

\[ \rho_t^f = i_t^f - E_i \pi_{t+1}^f = \sigma \left( E_y y_{t+1}^f - y_t^f \right). \] (9)

2.4 Equilibrium

Equilibrium conditions

- Home production equal the consumption of the domestically produced good:
  \[ y_t = (1 - \gamma)c_t^h + \gamma c_t^h. \] (10)

- Using earlier results, this can be written as
  \[ y_t = (1 - \gamma)c_t^h + \gamma c_t^h = (1 - \gamma) [c_t + \gamma a \delta_t] + \gamma a \delta_t + \gamma y_t^f \]
  \[ = (1 - \gamma)c_t + (2 - \gamma) \gamma a \delta_t + \gamma y_t^f. \] (11)

Uncovered interest parity

- Uncovered interest parity implies
  \[ i_t = i_t^f + E_t s_{t+1} - s_t, \]

- In real terms,
  \[ \rho_t^h = i_t - E_t \pi_{t+1}^h = \rho_t^f + (E_t \delta_{t+1} - \delta_t) \Rightarrow \rho_t^h = \rho_t^f + (E_t \delta_{t+1} - \delta_t). \] (12)

- So
  \[ i_t - E_t \pi_{t+1}^f = \rho_t^h - \gamma (E_t \delta_{t+1} - \delta_t) = \rho_t^f + (1 - \gamma) (E_t \delta_{t+1} - \delta_t). \]

Euler condition

- The Euler condition for optimal consumption becomes
  \[ c_t = E_t c_{t+1} - \left( \frac{1}{\sigma} \right) \left[ \rho_t^f + (1 - \gamma) (E_t \delta_{t+1} - \delta_t) \right] \]
  \[ = E_t c_{t+1} - \left( \frac{1}{\sigma} \right) \left[ (1 - \gamma) \rho_t^h + \gamma \rho_t^f \right]. \]
The relationship between domestic output and consumption (11) can be used to eliminate $c_t$ from the Euler condition, yielding

$$y_t = E_t y_{t+1} - \left( \frac{1+w}{\sigma} \right) \left[ \rho^h_t - \left( \frac{w}{1+w} \right) \rho^f_t \right].$$  (13)

where

$$w = (2-\gamma)\gamma(\sigma\alpha - 1).$$

• Or

$$y_t = E_t y_{t+1} - \left( \frac{1+w}{\sigma} \right) \left[ i_t - E_t \pi^h_{t+1} - \left( \frac{w}{1+w} \right) \rho^f_t \right]$$

Equation (13) implies

$$\rho^h_t = \left( \frac{\sigma}{1+w} \right) (E_t y_{t+1} - y_t) + \frac{w}{1+w} \rho^f_t.$$  (14)

• But from (12), $\rho^h_t = \rho^f_t + (E_t \delta_{t+1} - \delta_t)$, so (14) becomes

$$\rho^h_t = \rho^f_t + (E_t \delta_{t+1} - \delta_t) = \left( \frac{\sigma}{1+w} \right) (E_t y^h_{t+1} - y^h_t) + \frac{w}{1+w} \rho^f_t,$$

or

$$E_t \delta_{t+1} - \delta_t = \left( \frac{\sigma}{1+w} \right) (E_t y^h_{t+1} - y^h_t) - \frac{1}{1+w} \rho^f_t.$$

• Since $\rho^f_t = \sigma \left( E_t y^f_{t+1} - y^f_t \right)$, this last equation can be written as

$$E_t \Delta \delta_{t+1} = \left( \frac{\sigma}{1+w} \right) \left( E_t \Delta y^h_{t+1} - E_t \Delta y^f_{t+1} \right).$$  (15)

• From (11),

$$y_t = (1-\gamma)(c_t - y^h_t) + (1-\gamma)y^h_t + (2-\gamma)\gamma a \delta_t + \gamma y^f_t.$$

• Then, we end up with

$$y_t = c_t + \left( \frac{w+\gamma}{\sigma} \right) \delta_t + \gamma \left( \frac{1+w}{\sigma} \right) E_t \delta_{t+1} - \gamma \left( E_t y^h_{t+1} - E_t y^f_{t+1} \right).$$  (16)

2.5 The flexible-price equilibrium

Flex-price output

• Assume that the productivity and wage markup disturbances are mean zero, white noise disturbances. Let $z^p_t$ denote the flexible-price equilibrium value of a variable $z_t$ (still expressed as percentage deviations around the steady state).
From the marginal cost condition and the production function with flexible prices,

\[ \eta(y_t^o - \varepsilon_t) + \sigma c^o_t = w_t^o - p_t^c = w_t^o - p_t^h - \gamma \delta_t^o = \varepsilon_t - \gamma \delta_t^o, \]

or

\[ \eta y_t^o + \sigma c_t^o = (1 + \eta) \varepsilon_t - \gamma \delta_t^o. \]

Rewrite this as

\[ (\eta + \sigma) y_t^o + \sigma (c_t^o - y_t^o) = (1 + \eta) \varepsilon_t - \gamma \delta_t^o. \]

Further manipulation yields

\[ y_t^o = \left( \frac{(1 + w)(1 + \eta)}{(1 + w)\eta + \sigma} \right) \varepsilon_t - \left[ \frac{\sigma w}{(1 + w)\eta + \sigma} \right] y_t^f. \quad (17) \]

The flex-price equilibrium

Then, the flexible-price equilibrium (with serially uncorrelated shocks) satisfies

\[ y_t^o = \left( \frac{(1 + w)(1 + \eta)}{(1 + w)\eta + \sigma} \right) \varepsilon_t - \left[ \frac{\sigma w}{(1 + w)\eta + \sigma} \right] y_t^f. \quad (18) \]

\[ y_t^o = \left[ \frac{(1 + w)(1 + \eta)}{(1 + w)\eta + \sigma} \right] \varepsilon_t - \left[ \frac{\sigma w}{(1 + w)\eta + \sigma} \right] y_t^f. \quad (19) \]

\[ \rho_t^o = i_t - E_t \pi_t^{h+1} = \rho_t^f - \delta_t^o \quad (20) \]

\[ \delta_t^o = \left( \frac{\sigma}{1 + w} \right) (y_t^o - y_t^f). \quad (21) \]

2.6 The sticky price equilibrium

Deviations from the flexible price equilibrium

• When prices are sticky, the real wage can deviate from the marginal product of labor, but with flexible wages, it is still equal to the marginal rate of substitution between leisure and consumption.

• Define the output gap \( x_t \) as

\[ x_t = y_t - y_t^o. \]

• Real marginal cost, given by (6), is equal to the gap between the real product wage and the marginal product of labor. When prices are sticky, the real wage can deviate from the marginal product of labor, but with flexible wages, the real consumption wage is still equal to the marginal rate of substitution between leisure and consumption. Thus,

\[ w_t - p_t^h - \varepsilon_t = (\eta m_t + \sigma c_t + \mu_t^w + \gamma \delta_t) - \varepsilon_t. \]
From the production function and (16),
\[ n_t + \sigma c_t + \mu_t + \gamma \delta_t = \eta (y^h_t - \varepsilon_t) + \sigma \left( y^h_t - \left( \frac{w + \gamma}{\sigma} \right) \delta_t \right) + \mu_t + \gamma \delta_t. \]

In terms of deviations from the flex-price equilibrium,
\[ mc_t = (\eta + \sigma) x_t - \omega (\delta_t - \delta_t^\omega) + \mu_t. \]

From (15), \( \delta_t - \delta_t^\omega = (\sigma / (1 + w)) y_t^\omega \), so
\[ mc_t = \left[ \eta + \sigma - \left( \frac{\sigma w}{1 + w} \right) \right] x_t + \mu_t. \]

Hence, using (7) implies that domestic inflation is\(^1\)
\[ \pi_t^h = \beta E_t \pi_{t+1}^h + \kappa \left[ \sigma + \eta - \left( \frac{\sigma w}{1 + w} \right) \right] x_t + \kappa \mu_t. \]  \hspace{1cm} (22)

From (13),
\[ x_t = E_t x_{t+1} - \left( \frac{1 + w}{\sigma} \right) \left[ i_t - E_t \pi_{t+1}^h - \left( \frac{w}{1 + w} \right) \rho_t^\omega \right] + E_t y_{t+1}^\omega - y_t^\omega \]
\[ = E_t x_{t+1} - \left( \frac{1 + w}{\sigma} \right) \left[ i_t - E_t \pi_{t+1}^h - \rho_t^\omega \right], \]  \hspace{1cm} (23)

where \( \rho_t^\omega \) is the equilibrium real interest rate under flexible prices, given by (21).

**Equilibrium with sticky prices**

- Collecting results, model can be written as
\[ x_t = E_t x_{t+1} - \left( \frac{1 + w}{\sigma} \right) \left( i_t - E_t \pi_{t+1}^h - \rho_t^\omega \right) \]  \hspace{1cm} (24)
\[ \pi_t^h = \beta E_t \pi_{t+1}^h + \lambda w x_t + \kappa \mu_t \]  \hspace{1cm} (25)
\[ \delta_t = \left( \frac{\sigma}{1 + w} \right) x_t + \delta_t^\omega \]  \hspace{1cm} (26)

where \( \omega = \gamma (\sigma \eta - 1)(2 - \gamma) \) and \( \lambda_w = \kappa \left( \sigma + \eta - \frac{\sigma w}{1 + w} \right) \).

**Policy implications**

\(^1\)Clarida, Galí, and Gertler (2002) assume that the stochastic wage markup \( \mu_t^w \) does not affect the flex-price equilibrium, so it appears in as a disturbance in the inflation equation. If \( \mu_t^w \) is viewed as a taste disturbance, it should be incorporated into the definition of the flexible-price equilibrium and, in this case, it would not enter the inflation equation independently of the output gap variable.
• Resulting equations look just like a closed economy model.
  
  – This implies no new issues arise in the open economy case as long as the policy objectives are defined in terms of $x_t$ and $\pi_t^h$.
  
  – Shifts in $r_t^e$, the flexible price equilibrium real rate of interest will affect the optimal nominal interest rate setting (the instrument rule) but will not affect $x$ or $\pi^h$.
  
  – The stochastic mark-up term $\mu^w_t$ corresponds to the cost shock that generates policy trade-offs.
  
  – Policy trade-offs only arise from inflation shocks as long as GDP inflation and not CPI inflation enters the objective function of the central bank.
  
  – If CPI inflation matters, trade-offs are more complicated. An appreciation reduces firms’ marginal costs and reduces GDP inflation.
  
  – In face of a positive shock to aggregate spending, the central bank must raise the nominal interest rate to stabilize the output gap. But this leads to an appreciation of the exchange rate and a decline in CPI inflation.

3 Open economy issues: imported inputs

• McCallum and Nelson (2000) have proposed a somewhat different open economy model:
  
  – imported goods are used as inputs into the production of the domestic good;
  
  – households consume only the domestically produced good.

• The basic model is summarized by the following ten equations (ignoring constants and simplifying the dynamics) for consumption, output, imports, exports, employment, the flex-price output, the real exchange rate, the nominal exchange rate, the price level, and the nominal rate of interest:

\[
\begin{align*}
  c_t &= E_t c_{t+1} - b_1 (R_t - E_t \pi_{t+1}) + v_t \\
  y_t &= \omega_1 c_t + \omega_2 y_t + \omega_3 x_t \\
  im_t &= y_t - \sigma s_t \\
  x_t &= y_t + \sigma^s s_t \\
  s_t &= c_t - p_t + p_t^* \\
  y_t &= (1 - \alpha)(n_t + \varepsilon_t) + \alpha im_t \\
  \bar{y}_t &= \left( \frac{\sigma \alpha}{1 - \alpha} \right) s_t + \varepsilon_t 
\end{align*}
\]
\[ \pi_t = \frac{1}{2} E_t \pi_{t+1} + \frac{1}{2} \pi_{t-1} + \phi_2 (y_t - \bar{y}_t) + u_t \]
\[ R_t = R_t^* + E_t e_{t+1} - e_t + \xi_t \]
\[ R_t = (1 - \mu_3) [\mu_0 - \mu_1 \pi^* + (1 + \mu_1) \pi_t + (1 - \mu_3) \mu_2 \bar{y}_t] + \mu_3 R_{t-1} \]

3.1 Reducting the model to an isomorphic NK one

- This model can be reduced to the following three equations:

\[ \pi_t = \frac{1}{2} E_t \pi_{t+1} + \frac{1}{2} \pi_{t-1} + \phi_2 \bar{y}_t + u_t \]
\[ \bar{y}_t = E_t \bar{y}_{t+1} - \tilde{b}_1 (R_t - E_t \pi_{t+1}) + \delta_t \]
\[ R_t = (1 - \mu_3)(1 + \mu_1) \pi_t + (1 - \mu_3) \mu_2 \bar{y}_t + \mu_3 R_{t-1} \]

(27)

where

\[ \delta_t \equiv (\rho_a - 1) \varepsilon_t + v_t + \left( \frac{\sigma \alpha}{1 - \alpha} + \omega_3 \sigma^* \right) (r_t^* + \xi_t) \]
\[ + (1 - \rho_y) \omega y_t + (1 - \rho_{y^*}) \omega y_t^* \]

What have we gained?

- So what has the open economy aspect gained for us?

1. The coefficient on the real interest rate in the spending equations is increased. For baseline calibration,

\[ b_1 = 0.4 \]

while

\[ \tilde{b}_1 = \left( \omega_1 b_1 + \frac{\sigma a}{1 - \alpha} + \omega_3 \sigma^* \right) = 0.47 \]

2. The “demand shock” is generated by real productivity disturbances, by government spending shocks, by foreign income shocks, by foreign real interest rate shocks, and by the risk premium shock \( \xi \) that captures deviations from uncovered interest parity.

- Otherwise the model is identical to a closed economy model.

- Because of the assumption that the exchange rate affects domestic inflation only through the output gap, the McC-N open economy model misses out on the more complicated policy choices facing a central bank in the open economy.

Shocks to UIP
The previous open economy models reduced to structures identical with a closed economy model.

Key difference:
- CGG allow trade in final goods but assume UIP holds exactly;
- McN assume only inputs imported but deviations from UIP are allowed.

Consider modifying CGG’s model to allow a disturbance to the uncovered interest parity condition:
\[ i_t - E_t \pi^h_{t+1} = E_t \delta_{t+1} - \delta_t + r^*_t + \xi_t \] (29)

How is model affected?
- Equation for terms of trade becomes
  \[ \delta_t - \delta^o_t = \left( \frac{\sigma}{1 + w} \right) x_t + \xi_t \]
- Marginal costs becomes
  \[ m_c = \left[ \phi + \sigma - \left( \frac{\sigma w}{1 + w} \right) \right] x_t + \mu^w_t + \gamma \xi_t \]
- Domestic inflation becomes
  \[ \pi^h_t = \beta E_t \pi^h_{t+1} + \kappa \left[ \phi + \sigma - \left( \frac{\gamma w}{1 + w} \right) \right] x_t + \kappa \mu^w_t + \delta \gamma \xi_t \]

Deviations from UIP now becomes a source of inflation shocks and pose policy tradeoff problems.

A positive \( \xi \) produces a depreciation for given \( x \), and this increases the CPI, leading to an increase in the nominal wage. This raises marginal costs and pushes up domestic goods inflation.

Critical issue is how the flexible price equilibrium output level is measured. CGG exclude \( \mu^w \), in this last model, the \( \xi \) disturbance is excluded. Why?

Another critical issue is whether \( \pi^h \) or \( \pi^c \) is the correct inflation measure in the policy objective function.

4 Deviations from law of one price: Monacelli (2005)

Monacelli (2005)
- Standard models assume complete pass through.
- Empirical evidence strongly rejects complete pass through.
4.0.1 The model

Households

- Standard conditions:

\[ c_t = E_t c_{t+1} - \left( \frac{1}{\sigma} \right) (i_t - E_t \pi_{t+1} - r_t^m) \]

\[ \eta w_t + \sigma c_t = w_t - p_t. \]

- Inflation here is CPI inflation.

Complete pass through

- Earlier we assumed complete pass through of foreign prices into domestic prices:

\[ p_t^f = p_t^s + s_t. \]

- The domestic price level is

\[ p_t = (1 - \gamma)p_t^h + \gamma p_t^f = p_t^h + \gamma \left( p_t^f - p_t^h \right) = p_t^h + \gamma \delta_t, \]

where \( \delta_t \equiv p_t^f - p_t^h \) is the terms of trade.

- This means

\[ \pi_t = \pi_t^h + \gamma (\delta_t - \delta_{t-1}). \]

- Previously, we assumed \( \delta_t = p_t^s + s_t - p_t^h. \)

Incomplete pass through

- Law of one price does not hold.

- Define \( s_t \) as the nominal exchange rate (domestic currency price of one unit of foreign currency)

- The real exchange rate in log terms is

\[
x_t = s_t + p_t^s - p_t = (s_t + p_t^s - p_t^f) + p_t^f - p_t = \psi_t + (1 - \gamma)\delta_t \]

where \( \psi_t \equiv s_t + p_t^s - p_t^f \) is the deviation from the law of one price.

Domestic producers

- Continuum of monopolistic competitive firms.
Production technology:
\[ y_t(i) = n_t(i) + z_t. \]

Real marginal costs is
\[ mc_t = (w_t - p^h_t) - z_t. \]

Domestic producers

- Inflation adjustment for domestic goods based on Calvo:
  \[ \pi_t^h = \beta E_t \pi_{t+1}^h + \kappa_h mc_t. \]

- This takes the same form as employed in previous models.

Imported goods prices

- Inflation adjustment for imported goods: two extremes for short run
  - local currency pricing – domestic currency prices of imports unresponsive to exchange rate
  - producer currency pricing – domestic currency prices of imports completely responsive to exchange rate

- Evidence rejects both extremes.

- Assume retailers import consumer goods and adjust the domestic currency price at which they sell the goods according to a Calvo mechanism.

- Inflation of the domestic price of imported goods will be equal to
  \[ \pi_t^f = \beta E_t \pi_{t+1}^f + \kappa_f \psi_t. \]

- Notice that the law of one price gap \( \psi_t \) serves as the marginal cost variable.

- Importers pay \( p_t^* + s_t \) for foreign goods and sell them at \( p_t^f \), so retailer will want to raise price if \( \psi_t > 0 \) and lower them if \( \psi_t < 0 \).

Rest of model

- Complete markets assumption for risk sharing implies
  \[ c_t = c_t^* + \frac{1}{\sigma} q_t = c_t^* + \frac{1}{\sigma} [\psi_t + (1 - \gamma) \delta_t]. \]

- Uncovered interest parity:
  \[ r_t = r_t^* + E_t (s_{t+1} - s_t). \]
Flex-price output

- Combining the marginal rate of substitution between consumption and leisure with the definition of marginal cost,
  \[ mc_t = (w_t - p^h_t) - z_t = \eta n_t + \sigma c_t + \gamma \delta_t - z_t. \]
  
- This can be rewritten as
  \[ mc_t = \eta (y_t - z_t) + \sigma \left\{ c_t + \frac{1}{\sigma} \left[ \psi_t + (1 - \gamma) \delta_t \right] \right\} + \gamma \delta_t - z_t. \]
  
- Since \( \psi_t + \delta_t = s_t + p^1_t - p^f_t + p^h_t - p^f_t = s_t + p^1_t - p^h_t \), a rise in the domestic currency price of imported goods raises the wage (by raises the CPI) and increases marginal costs.

Demand

- Demand for domestic goods:
  - from domestic residents: \( c^h_t = c_t + \gamma a \delta_t \);
  - from foreign residents: \( c^h_t = c^*_t + a (\psi_t + \delta_t) = c^*_t + a (s_t + p^1_t - p^h_t) \)

- Goods market clearing:

\[
y_t = (1 - \gamma) c^h_t + \gamma c^h_t = (1 - \gamma) (c_t + \gamma a \delta_t) + \gamma [c^*_t + a (\psi_t + \delta_t)]
\]

\[
= (1 - \gamma) \left[ c_t - \frac{1}{\sigma} [\psi_t + (1 - \gamma) \delta_t] + \gamma a \delta_t \right] + \gamma [c^*_t + a (\psi_t + \delta_t)]
\]

\[
= c^*_t + \frac{1}{\sigma} [(1 - \gamma) \psi_t + (1 - \gamma) (1 - \gamma) \delta_t + (1 - \gamma) \gamma a \sigma \delta_t] + \gamma a \psi_t + \gamma a \delta_t
\]

\[
= c^*_t + \frac{1}{\sigma} \left\{ [1 + \gamma (a \sigma - 1)] \psi_t + \frac{1}{\sigma} \right\} \left\{ [1 - \gamma] (1 - \gamma) + (1 - \gamma) \gamma a \sigma + \gamma a \sigma \right\} \delta_t
\]

\[
= c^*_t + \frac{1}{\sigma} \left\{ [1 + \gamma (a \sigma - 1)] \psi_t + \frac{1}{\sigma} \right\} \left\{ [1 - (2 - \gamma)] \gamma + (2 - \gamma) \gamma a \sigma \right\} \delta_t
\]

\[
= y^*_t + \frac{1}{\sigma} \left\{ [1 + (2 - \gamma) \gamma (\sigma a - 1)] \delta_t + [1 + \gamma (\sigma a - 1)] \psi_t \right\}
\]

so

\[
y_t = y^*_t + \frac{1}{\sigma} \left\{ \omega_\delta \delta_t + \omega_\psi \psi_t \right\}
\]

where

\[
\omega_\delta = 1 + (2 - \gamma) \gamma (\sigma a - 1)
\]

\[
\omega_\psi = 1 + \gamma (\sigma a - 1)
\]

Supply

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• Assume foreign economy follows flex-price equilibrium.
• Than in terms of gaps, \( x_t = y_t - y_t^0 \), and
  \[
  x_t = \left( \frac{1}{\sigma} \right) \left\{ \omega_\delta \hat{\delta}_t + \omega_\psi \psi_t \right\},
  \]
  where \( \hat{\delta} = \delta - \delta^0 \).
• Real marginal cost is
  \[ m_c_t = \eta x_t + \psi_t + \hat{\delta}_t. \]
• Combining,
  \[
  m_c_t = \eta x_t + \psi_t + \frac{\sigma}{\omega_\delta} \left[ x_t - \frac{\omega_\psi}{\sigma} \psi_t \right] = \left( \eta + \frac{\sigma}{\omega_\delta} \right) x_t + \left( 1 - \frac{\omega_\psi}{\omega_\delta} \right) \psi_t.
  \]

**Inflation**

• We can now write inflation in domestic produced goods as
  \[
  \pi_t^h = \beta E_t \pi_{t+1}^h + \kappa_h \left( \eta + \frac{\sigma}{\omega_\delta} \right) x_t + \kappa_h \left( 1 - \frac{\omega_\psi}{\omega_\delta} \right) \psi_t.
  \]
• In domestic prices of foreign produced goods,
  \[
  \pi_t^f = \beta E_t \pi_{t+1}^f + \kappa_f \psi_t.
  \]
• And in the CPI,
  \[
  \pi_t^c = \beta E_t \pi_{t+1}^c + (1 - \gamma) \kappa_h \left( 1 - \frac{\omega_\psi}{\omega_\delta} \right) x_t + \left[ (1 - \gamma) \kappa_h \left( 1 - \frac{\omega_\psi}{\omega_\delta} \right) + \gamma \kappa_f \right] \psi_t.
  \]

**Demand**

• The Euler condition can be written now as
  \[
  c_t = E_t c_{t+1} - \left( \frac{1}{\sigma} \right) (i_t - E_t \pi_{t+1})
  \]
• From
  \[
  y_t = y_t^* + \left( \frac{1}{\sigma} \right) \left\{ \omega_\delta \delta_t + \omega_\psi \psi_t \right\}
  \]
  and
  \[
  c_t = c_t^* + \left( \frac{1}{\sigma} \right) [\psi_t + (1 - \gamma) \hat{\delta}_t] \]
We have

\[ y_t = y_t^* + \left( \frac{1}{\sigma} \right) \{ \omega_s \delta_t + \omega_t \psi_t \} \]

\[ = y_t^* + \left( \frac{1}{\sigma} \right) \omega_s \left[ \frac{\sigma c_t - \sigma c_t^* - \psi_t}{1 - \gamma} \right] + \left( \frac{1}{\sigma} \right) \omega_t \psi_t \]

\[ = y_t^* + \left[ \frac{\omega_s c_t - \omega_s c_t^*}{1 - \gamma} \right] - \frac{\omega_s}{\sigma(1 - \gamma)} + \left( \frac{1}{\sigma} \right) \omega_t \psi_t \]

\[ = \left( 1 - \frac{\omega_s}{1 - \gamma} \right) y_t^* + \left( \frac{\omega_s}{1 - \gamma} \right) c_t + \left( \frac{1}{\sigma} \right) \left( \omega_t - \frac{\omega_s}{1 - \gamma} \right) \psi_t \]

since

\[ \omega_s = 1 + (2 - \gamma)\gamma(\sigma a - 1) \]

\[ \omega_t = 1 + \gamma(\sigma a - 1) \]

\[ \omega_t - \frac{\omega_s}{1 - \gamma} = 1 + \gamma(\sigma a - 1) - \frac{1 + (2 - \gamma)\gamma(\sigma a - 1)}{1 - \gamma} \]

\[ = \frac{1 - \gamma + (1 - \gamma)\gamma(\sigma a - 1) - 1 - (2 - \gamma)\gamma(\sigma a - 1)}{1 - \gamma} \]

\[ = \frac{-\gamma + (1 - \gamma)\gamma \sigma a - (1 - \gamma)\gamma - (2 - \gamma)\gamma \sigma a + (2 - \gamma)\gamma}{1 - \gamma} \]

\[ = \frac{(1 - \gamma)\gamma \sigma a - (2 - \gamma)\gamma - (2 - \gamma)\gamma \sigma a + (2 - \gamma)\gamma}{1 - \gamma} \]

\[ = \frac{(1 - \gamma)\gamma \sigma a - (2 - \gamma)\gamma \sigma a}{1 - \gamma} = -\gamma \sigma a \frac{1}{1 - \gamma} \]

So

\[ y_t = \left( 1 - \frac{\omega_s}{1 - \gamma} \right) y_t^* + \left( \frac{\omega_s}{1 - \gamma} \right) c_t - \left( \frac{\gamma a}{1 - \gamma} \right) \psi_t \]

So

\[ c_t = \left( \frac{1 - \gamma}{\omega_s} \right) y_t - \left( \frac{1 - \gamma}{\omega_s} \right) \left( 1 - \frac{\omega_s}{1 - \gamma} \right) y_t^* + \left( \frac{1 - \gamma}{\omega_s} \right) \left( \frac{\gamma a}{1 - \gamma} \right) \psi_t \]

\[ = \left( \frac{1 - \gamma}{\omega_s} \right) y_t + \left( 1 - \frac{1 - \gamma}{\omega_s} \right) y_t^* + \left( \frac{\gamma a}{\omega_s} \right) \psi_t \]
Hence

\[
\left(\frac{1 - \gamma}{\omega_\delta}\right) y_t = -\left(1 - \frac{1 - \gamma}{\omega_\delta}\right) y_t^* + \left(\frac{\gamma a}{\omega_\delta}\right) \psi_t \\
+ \left(\frac{1 - \gamma}{\omega_\delta}\right) E_t y_{t+1} + \left(1 - \frac{1 - \gamma}{\omega_\delta}\right) E_t y_{t+1}^* + \left(\frac{\gamma a}{\omega_\delta}\right) E_t \psi_{t+1} \\
- \left(\frac{1}{\sigma}\right) (i_t - E_t \pi_{t+1}) \\
= \left(\frac{1 - \gamma}{\omega_\delta}\right) E_t y_{t+1} + \left(1 - \frac{1 - \gamma}{\omega_\delta}\right) E_t \Delta y_{t+1}^* + \left(\frac{\gamma a}{\omega_\delta}\right) E_t \Delta \psi_{t+1} \\
- \left(\frac{1}{\sigma}\right) (i_t - E_t \pi_{t+1})
\]

- Multiplying by \(\omega_\delta/(1 - \gamma)\),

\[
y_t = E_t y_{t+1} + \left(\frac{\omega_\delta}{1 - \gamma}\right) \left(1 - \frac{1 - \gamma}{\omega_\delta}\right) E_t \Delta y_{t+1}^* + \left(\frac{\gamma a}{1 - \gamma}\right) E_t \Delta \psi_{t+1} \\
- \left(\frac{1}{\sigma}\right) \left(\frac{\omega_\delta}{1 - \gamma}\right) (i_t - E_t \pi_{t+1})
\]

\[
y_t = E_t y_{t+1} - \left(\frac{\omega_\delta}{\sigma(1 - \gamma)}\right) (i_t - E_t \pi_{t+1}) \\
+ \left(\frac{\gamma a}{1 - \gamma}\right) E_t \Delta \psi_{t+1} + \left(\frac{\omega_\delta}{1 - \gamma} - 1\right) E_t \Delta \psi_{t+1}^*
\]

- so

\[
x_t = E_t x_{t+1} - \left(\frac{\omega_\delta}{\sigma(1 - \gamma)}\right) (i_t - E_t \pi_{t+1}) + \left(\frac{\gamma a}{1 - \gamma}\right) E_t \Delta \psi_{t+1}
\]

- Next, rewrite in terms of \(\pi_t^h\) using the fact that \(\pi_t = \pi_t^h + \gamma \Delta \delta_t\) and

\[
x_t = \left(\frac{1}{\sigma}\right) \left\{\omega_\delta \delta_t + \omega_\psi \psi_t\right\},
\]

or \(\delta_t = (\sigma/\omega_\delta)x_t - \omega_\psi \psi_t\), to obtain

\[
\pi_t = \pi_t^h + \left(\frac{\gamma \sigma}{\omega_\delta}\right) \Delta x_t - \gamma \omega_\psi \Delta \psi_t
\]

and

\[
x_t = E_t x_{t+1} - \left(\frac{\omega_\delta}{\sigma(1 - \gamma)}\right) (i_t - E_t \pi_{t+1}^h) - \left(\frac{\gamma \sigma}{\omega_\delta}\right) E_t \Delta x_{t+1} + \gamma \omega_\psi E_t \Delta \psi_{t+1} + \left(\frac{\gamma a}{1 - \gamma}\right) E_t \Delta \psi_{t+1}
\]
or

\[ x_t = E_t x_{t+1} - \left( \frac{\omega_\delta}{\sigma(1-\gamma)} \right) (i_t - E_t \pi^h_{t+1}) - \left( \frac{\omega_\delta}{(1-\gamma)} \right) \left( - \left( \frac{\gamma \sigma}{\omega_\delta} \right) E_t \Delta x_{t+1} + \gamma \omega_\psi E_t \Delta \psi_{t+1} \right) + \left( \frac{\gamma}{1-\gamma} \right) \]

\[ = E_t x_{t+1} - \left( \frac{\omega_\delta}{\sigma(1-\gamma)} \right) (i_t - E_t \pi^h_{t+1}) + \left( \frac{\omega_\delta}{\sigma(1-\gamma)} \right) \left( \frac{\gamma \sigma}{\omega_\delta} \right) E_t \Delta x_{t+1} + \left( \frac{\gamma}{1-\gamma} \right) \left( \frac{1}{\sigma} \right) [\sigma a - \omega_\delta \omega_\psi] E_t \Delta \psi_{t+1} \]

\[ \left( \frac{1}{1-\gamma} \right) x_t = \left( \frac{1}{1-\gamma} \right) E_t x_{t+1} - \left( \frac{\omega_\delta}{\sigma(1-\gamma)} \right) (i_t - E_t \pi^h_{t+1}) + \left( \frac{\gamma}{1-\gamma} \right) \left( \frac{1}{\sigma} \right) [\sigma a - \omega_\delta \omega_\psi] E_t \Delta \psi_{t+1} \]

\[ \omega_\delta = 1 + (2 - \gamma) \gamma (\sigma a - 1) \]

\[ \omega_\psi = 1 + \gamma (\sigma a - 1) \]

\[ \sigma a - \omega_\delta \omega_\psi = \sigma a - 1 - \gamma (\sigma a - 1) + (2 - \gamma) \gamma (\sigma a - 1) + \gamma (\sigma a - 1) \]

\[ = (\sigma a - 1) [1 - \gamma + (2 - \gamma) \gamma + (2 - \gamma) \gamma^2 (\sigma a - 1)] \]

**Collecting equations**

- The model has been reduced to two basic equations in domestic price inflation and the output gap:

\[ \pi^h_t = \beta E_t \pi^h_{t+1} + \kappa_h \left( \eta + \frac{\sigma}{\omega_\delta} \right) x_t + \kappa_h \left( 1 - \frac{\omega_\psi}{\omega_\delta} \right) \psi_t. \]

\[ x_t = E_t x_{t+1} - \left( \frac{\omega_\delta}{\sigma} \right) (i_t - E_t \pi^h_{t+1}) + \Gamma \psi E_t \Delta \psi_{t+1}. \]

- Deviations from law of one price show up like a cost shock.

- But \( \psi \) is endogenous.

\[ \psi_t = \psi_{t-1} + \Delta s_t - \pi^f_t \]

\[ E_t s_{t+1} - s_t = i_t - i^*_t \]

- These imply

\[ E_t \Delta \psi_{t+1} = E_t \Delta s_{t+1} - E_t \pi^f_{t+1} = i_t - i^*_t - E_t \pi^f_{t+1} \]