Economics 205B
Winter 2008

Advanced Macroeconomic Theory
Assignment 2: Answers

1. Consider a simple RBC model consisting of a representative household whose utility is given by

$$E_t \sum_{i=0}^{\infty} \beta^i u(c_{t+i}, 1 - n_{t+i})$$

where c is consumption and n is time spend in production. The household has a constant returns to scale technology for producing output given by

$$y_t = e^{z_t} F(n_t, k_t).$$

and faces budget constraint given by

$$k_{t+1} = (1 - \delta)k_t + y_t - c_t,\ 0 < \delta < 1.$$

(a) Write down the social planners problem for this economy. What are the first order conditions? What are the equilibrium conditions for this economy?

$$V(k_t, z_t) = \max_{c_t, n_t} \left\{ u(c_t, 1 - n_t) + \beta E_t V\left[(1 - \delta)k_t + e^{z_t} F(n_t, k_t) - c_t, z_{t+1}\right] \right\}.$$

First order conditions are

$$u_c(t) - \beta E_t V_k(t + 1) = 0$$

$$-u_l(t) + \beta e^{z_t} F_N(t) E_t V_k(t + 1) = 0$$

$$V_k(t) = \beta \left[1 - \delta + e^{z_t} F_k(t)\right] E_t V_k(t + 1)$$

Equilibrium conditions are these first order conditions plus

$$e^{z_t} F(n_t, k_t) = c_t + k_{t+1} - (1 - \delta)k_t.$$

(b) Assume

$$u(c_{t+i}, 1 - n_{t+i}) = \frac{c_t^{1-\sigma}}{1 - \sigma} - \Psi \frac{n_t^{1+\eta}}{1 + \eta},\ \sigma, \eta > 0$$

and

$$F(n_t, k_t) = n_t^a k_t^{1-a}$$

Rewrite the first order condition you derived in part (a) using these functional forms.

$$c_t^{-\sigma} = \beta E_t V_k(t + 1)$$

$$\Psi n_t^\eta = \beta a \left(\frac{y_t}{n_t}\right) E_t V_k(t+1)$$

$$V_k(t) = \beta \left[1 - \delta + (1-a)\left(\frac{y_t}{k_t}\right)\right] E_t V_k(t+1)$$

These imply

$$\Psi n_t^\eta = a \left(\frac{y_t}{n_t}\right) c_t^{-\sigma}$$

$$c_t^{-\sigma} = \beta \left[1 - \delta + (1-a)\left(\frac{y_{t+1}}{k_{t+1}}\right)\right] c_{t+1}^{-\sigma}$$

(c) Assume $z_t = \rho z_{t-1} + e_t$ where $\mid \rho \mid < 1$ and e is a mean zero, white noise process. Derive the set of linear equations that approximate the behavior of the percentage deviations around the steady-state equilibrium. *Done in class.*

$$\eta \hat{n}_t = \hat{y}_t - \hat{n}_t - \sigma \hat{c}_t$$

$$\hat{c}_t = -\sigma^{-1}\beta(1-a)\left(\frac{Y}{K}\right)\left(\hat{y}_{t+1} - \hat{k}_{t+1}\right) + E_t \hat{c}_{t+1}$$

(d) For each of the unknown parameters that appear in your linear approximation, discuss how you would calibrate their values. *One could use long-run data on labor's share of total income to calibrate a; use the average real return R and calibrate β as $1/R$. Then, since*

$$\beta \left[1 - \delta + (1-a)\left(\frac{y}{k}\right)\right] = 1,$$

$$\frac{y}{k} = \left(\frac{1}{1-a}\right)\left(\frac{1}{\beta} - 1 + \delta\right),$$

$$\frac{y}{k} = \left(\frac{n}{k}\right)^a \Rightarrow \frac{n}{k} = \left(\frac{y}{k}\right)^{1/a} = \left(\frac{1}{1-a}\right)^{\frac{1}{a}} \left(\frac{1}{\beta} - 1 + \delta\right)^{\frac{1}{a}}$$

and

$$\frac{y}{n} = \frac{y}{k}\frac{k}{n} = \left(\frac{n}{k}\right)^{a-1} = \left(\frac{1}{1-a}\right)^{\frac{a-1}{a}} \left(\frac{1}{\beta} - 1 + \delta\right)^{\frac{a-1}{a}}.$$

Now use the FOC for the labor-leisure choice,

$$\Psi n_t^\eta = a \left(\frac{y}{n}\right) c^{-\sigma}$$

to obtain

$$\Psi = a \left(\frac{y}{n}\right)^{1-\sigma} \left(\frac{c}{y}\right)^{-\sigma} n^{-(\eta+\sigma)}.$$

Now use

$$\frac{c}{y} = 1 - \frac{I}{y} = 1 - \delta\left(\frac{k}{y}\right)$$

and $n = 1/3$ to find Ψ. The depreciate rate could be found from the long-run average value of investment to capital, and ρ could be obtained by using a and the production to generate an estimated time series of z_t.

2. Using the model you obtained in Question 1 and the calibrated values you chose in Part (d) of that questions, write a program based on Harald Uhlig's methods to solve the model. (Hint: see Uhlig's sample program Exampl0.m).

   (a) Using your program, what does the model imply for the ratio of the standard deviation of consumption to the standard deviation of output? The ratio of the standard deviation of investment to the standard deviation of output? *Setting* $\sigma = 0.5$, $\sigma_c/\sigma_y = 0.28$, $\sigma_i/\sigma_y = 3.62$.

   (b) Simulate the model for $\sigma = 0.5$, 1, and 2. How do σ_y, σ_c, and σ_i vary with σ (where σ_x is the standard deviation of the variable x and i i investment)? Explain. *I set* $\eta = 1$ *(eta in Basic_rbc.m) while varying* σ. *With a larger* σ, *households are less willing to substitute consumption over time, so consumption varies less.*

	$\boldsymbol{\sigma}$		
	0.5	**1.0**	**2.0**
$\boldsymbol{\sigma_y}$	**1.30**	**1.22**	**1.13**
$\boldsymbol{\sigma_c}$	**0.37**	**0.31**	**0.24**
$\boldsymbol{\sigma_i}$	**4.71**	**4.26**	**4.03**

   (c) Replace your utility function with

$$u(c_t, 1 - n_t) = \ln c_t + \Psi \ln(1 - n_t).$$

   Given this new utility function, modify the linear approximation from Question 1 and repeat parts (a) and (b). *The only equation affected is the labor market equilibrium expression. This now becomes*

$$\frac{\Psi}{1 - n_t} = \frac{\Psi}{l_t} = a\left(\frac{y_t}{n_t}\right) c_t^{-1} \Rightarrow -\hat{l}_t = \hat{y}_t - \hat{n}_t - \hat{c}_t.$$

   Note that

$$\hat{l}_t = -\left(\frac{n}{1 - n}\right)\hat{n}_t$$

   so the equilibrium condition becomes

$$\left[1 + \left(\frac{n}{1 - n}\right)\right]\hat{n}_t = \hat{y}_t - \hat{c}_t.$$

   This corresponds to $\sigma = 1$ and $\eta = n/(1-n)$. *One finds that* $\sigma_c/\sigma_y = 0.25$, $\sigma_i/\sigma_y = 3.51$.