Economics 205B, Winter 2008
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Advanced Macroeconomic Theory
Assignment 1: Answers

Questions 1-3 are straightforward data exercises. See me if you had any problems with these. The solutions for the problem from Romer will be put in your mailboxes.

4. A Traditional Keynesian model with sticky nominal wages: Consider the following model (all variables except interest rates expressed in log form, superscripts d and s refer to demand and supply).

$$y_t^d = y_0 - \sigma\left[i_t - \left(E_t p_{t+1} - p_t\right)\right] + u_t \qquad (1)$$

$$m_t^d - p_t = y_t - \gamma i_t + v_t \qquad (2)$$

$$y_t^s = a n_t + e_t \qquad (3)$$

$$n_t^d = n_0 - \alpha\left(w - p_t\right) \qquad (4)$$

$$m_t^s = \bar{m} \qquad (5)$$

$$y_t^d = y_t^s$$

$$m_t^d = m_t^s$$

$$n_t = n_t^d$$

where y, p, i, m, n, w denote output, the price level, the nominal interest rate, nominal money balances, employment, and the nominal wage respectively. The nomnal wage w is fixed. u, v, and e are white noise disturbance terms (i.e., mean zero, serially uncorrelated stochastic shocks).

   (a) Briefly explain what each of these equations represents. *The first equation represent an aggregate spending or demand equation in which demand depends negatively on the real rate of interest. The disturbance u captures other factors such as fiscal policy. An increase in the real interest rate leads households and firms to postpone spending so current demand falls. The second equation represents the real demand for money, assumed to be a function of total output and the opportunity cost of money, measured by i_t. The third equation is the aggregate production function relating output to employment and a productivity shock e. The fourth equation gives the demand for labor as a decreasing function of the real wage. The fifth equation makes the supply of money equal to an exogenous constant, while the next two equations are the equilibirium conditions for the goods and money markets. Finally, the last equation specifies that employment is equal to labor demand.*

(b) Why does y_t^d depend on $i_t - (E_t p_{t+1} - p_t)$ while m_t^d depends on i_t? *The demand equation reflects decisions about real consumption and investment and therefore depends on the rate at which individuals can trade real consumption today for real consumption tomorrow. This tradeoff is measured by the real rate of interest. The demand for money depends on opportunity cost of holding money instead of interest earning bonds. The real return on bonds is* $i_t - (E_t p_{t+1} - p_t)$, *the real return on money is* $E_t p_{t+1} - p_t$ *since money pays no explicit interest. Therefore, the opportunity cost of holding money is* $[i_t - (E_t p_{t+1} - p_t)] - (E_t p_{t+1} - p_t) = i_t$.

(c) Find expressions for equilibrium output and the equilibrium price level as functions of the various constants $(n_0, \bar{m}, y_0)$, the nominal wage w, the shocks u, v, and e, and $E_t p_{t+1}$. What is $\partial y_t / \partial \bar{m}$? What is $\partial p_t / \partial \bar{m}$? *Let the equilibrium values of output and money be denoted by y and m. Then from (2) and (5),*

$$\bar{m} = m_t^s = p_t + y_t - \gamma i_t + v_t$$

or

$$i_t = \frac{p_t - \bar{m} + y_t + v_t}{\gamma}.$$

Substitute this into (1) to obtain

$$\begin{aligned} y_t &= y_0 - \sigma \left[i_t - (E_t p_{t+1} - p_t) \right] + u_t \\ &= y_0 - \sigma \left(\frac{p_t - \bar{m} + y_t + v_t}{\gamma} \right) + \sigma (E_t p_{t+1} - p_t) + u_t \\ &= \frac{\gamma y_0 - \sigma (p_t - \bar{m} + v_t) + \gamma \sigma E_t p_{t+1} - \gamma \sigma p_t + \gamma u_t}{\gamma + \sigma}. \end{aligned} \tag{6}$$

From (4) and (3),

$$y_t = a \left[n_0 - \alpha (w - p_t) \right] + e_t. \tag{7}$$

Equating these two expressions for output,

$$\frac{\gamma y_0 - \sigma (p_t - \bar{m} + v_t) + \gamma \sigma E_t p_{t+1} - \gamma \sigma p_t + \gamma u_t}{\gamma + \sigma} = a \left[n_0 - \alpha (w - p_t) \right] + e_t,$$

which determines the equilibrium price level. Collecting terms

$$\begin{aligned} \left[(\gamma + \sigma) a \alpha + \sigma (1 + \gamma) \right] p_t &= \gamma y_0 + \sigma \bar{m} - \sigma v_t + \gamma \sigma E_t p_{t+1} + \gamma u_t \\ &\quad - (\gamma + \sigma) a n_0 + (\gamma + \sigma) a \alpha w - (\gamma + \sigma) e_t, \end{aligned}$$

$$\begin{aligned} p_t &= \frac{\gamma \sigma E_t p_{t+1}}{\gamma (a \alpha + \sigma) + \sigma (a \alpha + 1)} + \frac{\gamma y_0 + \sigma \bar{m} - (\gamma + \sigma) a n_0 + (\gamma + \sigma) a \alpha w}{\gamma (a \alpha + \sigma) + \sigma (a \alpha + 1)} \\ &\quad + \frac{\gamma u_t - \sigma v_t - (\gamma + \sigma) e_t}{\gamma (a \alpha + \sigma) + \sigma (a \alpha + 1)} \end{aligned} \tag{8}$$

Hence

$$\frac{\partial p_t}{\partial \bar{m}} = \frac{\sigma}{\gamma(a\alpha + \sigma) + \sigma(a\alpha + 1)} < 1$$

and, using (7),

$$\frac{\partial y_t}{\partial \bar{m}} = a\alpha \left(\frac{\partial p_t}{\partial \bar{m}} \right) = \frac{a\alpha\sigma}{\gamma(a\alpha + \sigma) + \sigma(a\alpha + 1)} < 1.$$

(d) Find expressions for the aggregate demand (AD) and aggregate supply (AS) curves for this economy. Which variables shift the AD? Which variables shift the AS? *The AD curve is given by (6); changes in y_0, $\bar{m}$, u_t, v_t, and $E_t p_{t+1}$ shift it. The aggregate supply curve is given by (7); changes in n_0, w, or e_t shift it.*

(e) Suppose $n_t^s = \bar{n}$ and that $w = p_t + \alpha^{-1}(n_0 - \bar{n})$[1] each period (i.e., the wage equilibrates labor demand and labor supply.

i. Show that output is independent of the demand side of the economy (i.e., independent of equations 1 and 2). *Given that $w = p_t + \alpha^{-1}(n_0 - \bar{n})$, labor demand and labor supply are equal, so employment equals $n^s = \bar{n}$. Substituting this into the production function (3), $y_t = a\bar{n} + e$ which is independent of the demand side of the economy.*

ii. Find an expression for the equilibrium price level as a function of the shocks, constants, and $E_t p_{t+1}$. *From (8) we had*

$$\begin{aligned} p_t \quad = \quad & \frac{\gamma\sigma E_t p_{t+1}}{\gamma(a\alpha + \sigma) + \sigma(a\alpha + 1)} + \frac{\gamma y_0 + \sigma\bar{m} - (\gamma + \sigma)a n_0 + (\gamma + \sigma)a\alpha w}{\gamma(a\alpha + \sigma) + \sigma(a\alpha + 1)} \\ & + \frac{\gamma u_t - \sigma v_t - (\gamma + \sigma)e_t}{\gamma(a\alpha + \sigma) + \sigma(a\alpha + 1)}. \end{aligned}$$

Using the expression for the nominal wage, this becomes

$$\begin{aligned} p_t \quad = \quad & \frac{\gamma\sigma E_t p_{t+1}}{\gamma(a\alpha + \sigma) + \sigma(a\alpha + 1)} + \frac{\gamma y_0 + \sigma\bar{m} - (\gamma + \sigma)a n_0 + (\gamma + \sigma)a\left(\alpha p_t + n_0 - \bar{n}\right)}{\gamma(a\alpha + \sigma) + \sigma(a\alpha + 1)} \\ & + \frac{\gamma u_t - \sigma v_t - (\gamma + \sigma)e_t}{\gamma(a\alpha + \sigma) + \sigma(a\alpha + 1)}. \end{aligned}$$

Collecting p terms and canceling the n_0 terms,

$$p_t = \frac{\gamma\sigma E_t p_{t+1}}{\sigma\left(1 + \gamma\right)} + \frac{\gamma y_0 + \sigma\bar{m} - (\gamma + \sigma)a\bar{n}}{\sigma\left(1 + \gamma\right)} + \frac{\gamma u_t - \sigma v_t - (\gamma + \sigma)e_t}{\sigma\left(1 + \gamma\right)}. \tag{9}$$

iii. Suppose $m_t^s = \bar{m} + \eta_t$, where η_t is white noise. Guess that the equilibrium price level can be expressed as

$$p_t = A_0 + A_1\eta_t + A_2 e_t + A_3 u_t + A_4 v_t.$$

[1] Note the α^{-1} was missing in the problem set – the critical statement is that the wage equates labor demand and supply.

This implies $E_t p_{t+1} = A_0$. Using this guess in the expression you obtained in part d.ii, find the equilibrium values of the A_i coefficients. *From (9) (adjusted to reflect the assumption now that m^s contains a stochastic component and the guess*

$$\begin{aligned} p_t &= A_0 + A_1\eta_t + A_2 e_t + A_3 u_t + A_4 v_t \\ &= \frac{\gamma\sigma A_0}{\sigma(1+\gamma)} + \frac{\gamma y_0 + \sigma(\bar{m}+\eta_t) - (\gamma+\sigma)a\bar{n}}{\sigma(1+\gamma)} \\ &\quad + \frac{\gamma u_t - \sigma v_t - (\gamma+\sigma)e_t}{\sigma(1+\gamma)}. \end{aligned}$$

For this equation to hold for all realizations of u, v, and e, it must hold that

$$A_0 = \frac{\gamma\sigma A_0}{\sigma(1+\gamma)} + \frac{\gamma y_0 + \sigma\bar{m} - (\gamma+\sigma)a\bar{n}}{\sigma(1+\gamma)} \Rightarrow A_0 = \frac{\gamma y_0 + \sigma\bar{m} - (\gamma+\sigma)a\bar{n}}{\sigma};$$

$$A_1 = \frac{1}{1+\gamma};$$

$$A_2 = -\left[\frac{\gamma+\sigma}{\sigma(1+\gamma)}\right];$$

$$A_3 = \left[\frac{\gamma}{\sigma(1+\gamma)}\right];$$

$$A_4 = -\left[\frac{1}{1+\gamma}\right].$$

6. Here is a MATLAB program that answers the questions posed in problem 6..

```
% ps1_205b.m
% generates simple impulse responses
clear all
A = eye(4);
B = zeros(4);
A(2,3) = -0.11;
A(2,4) = -0.12;
B(1,3) = 1;
B(2,2) = 0.92;
B(3,1) = -0.16;
B(3,2) = -0.12;
B(3,3) = 1.04;
B(3,4) = 0.12;
B(4,3) = 0.06;
B(4,4) = 0.85;
disp('The matrix A is equal to ')
```

```
A
disp('hit any key to continue')
pause
disp('The matrix B is equal to ')
B
disp('hit any key to continue')
pause
disp('The matrix Abar is equal to ')
Abar = inv(A)*B
disp('hit any key to continue')
% Impulse responses
N = 40;
shk = 'd' ;
% program is set up to calculate impulse responses to demand, supply, and
policy shocks.
j = 0;
while j < 3;
    IMP=zeros(N,3);
    if shk == 'd'; % demand shock
        v1 = [0 0 1 0]';
        else
            if shk == 's'; % inflation shock
                v1 = [0 0 0 1]';
            else
                % policy shock
                v1 = [0 1 0 0]';
            end
        end
% Defining selection matrix
disp('The selection matrix:')
sel = [zeros(3,1) eye(3)]
disp('hit any key to continue')
pause
t=1;
while t<=N;
    s=t-1;
    IMP(t,:) = (sel*(Abar^s)*(v1))';
    t=t+1;
end
% Defining axis for plots
ax=1:N;
ax=ax';
% Plotting impulse response functions
    if shk == 'd'
        figure (1)
        plot(ax,IMP)
```

```
        title('Impulse responses to a demand shock')
        xlabel('periods')
        legend('i','output gap','inflation')
    else
        if shk == 's'
            figure(2)
            plot(ax,IMP)
            title('Impulse responses to an inflation shock')
            xlabel('periods')
            legend('i','output gap','inflation')
        else
            figure(3)
            plot(ax,IMP)
            title('Impulse responses to a policy shock')
            xlabel('periods')
            legend('i','output gap','inflation')
        end
    end
j = j+1;
% Redefining shock to get next set of impulse response functions
    if shk == 'd'
        shk = 's'
    else
        if shk == 's'
            shk = 'm'
        end
    end
end % while j loop (loop over shocks).
```