Economics 205B
Winter 2006

Final Exam: Answer any three (3) questions

1. Suppose the central bank's objective is to minimize

$$V = \frac{1}{2}\left[(\pi - \pi^*)^2 + \lambda u^2\right],$$

where π is the inflation rate and u is the unemployment rate. The economy is described by

$$u = u_n - a(\pi - \pi^e) + v,$$

where u_n is the natural rate of unemployment and π^e is expected inflation. Expectations are formed by the public before observing the disturbance v. The central bank can set inflation after observing v.

   (a) What is the equilibrium rate of inflation under discretion? What is the equilibrium unemployment rate?

   (b) What is the equilibrium rate of inflation under commitment? What is the equilibrium unemployment rate?

   (c) How does inflation differ under discretion and under commitment? Explain intuitively how this difference is affected by λ and by a.

2. Consider a simple *RBC* model. The representative household maximizes

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i u(c_{t+i}, 1 - n_{t+i}),$$

where c is consumption and n is time spent in production. The household rents capital and sells labor services to firms and faces a budget constraint given by

$$k_{t+1} = w_t n_t + r_t k_t + (1 - \delta)k_t - c_t.$$

Firms maximize profits, subject to a constant returns to scale (*CES*) technology for producing output, given by

$$y_t = e^{z_t} F(n_t, k_t),$$

where $z_t = \rho z_{t-1} + e_t$.

   (a) Write down the equilibrium conditions for this economy (assume all markets are perfectly competitive).

(b) Assume

$$u(c_t, 1-n_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \Psi \frac{n_t^{1+\eta}}{1+\eta}.$$

and $F(n_t, k_t) = n_t^a k_t^{1-a}$. How would you calibrate the parameters a, β, ρ, σ_e, η, and Ψ?

(c) Three characteristics of actual business cycles are 1) output displays persistent fluctuations, 2) employment and output are highly correlated, 3) real wages are very weakly related to output. Are there parameter values for which the model of this question can account for these business cycle "facts"? If so, are these reasonable values for the parameters (i.e., are they the ones you would obtain from the calibrations described in part b)? If they are not, briefly discuss how might you modify the model to better match these three stylized facts?

3. Assume the utility of the representative agent is given by

$$\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{(1+\xi_t)N_t^{1+\eta}}{1+\eta}.$$

The aggregate production function is $Y_t = Z_t N_t$. The notation is: C is consumption, ξ is a stochastic shock to "tastes," N is time spent working, Y is output, and $Z_t = (1 + z_t)$ is a stochastic aggregate productivity disturbance; σ and η are constants. Both ξ and z have zero means. Assume a standard model of monopolistic competition with Calvo pricing.

(a) Derive an expression for the flexible-price equilibrium output y_t^f for this economy, expressed as a percentage deviation from steady state output. *Explain* how a positive realization of ξ_t would affect y_t^f.

(b) Assuming a zero steady-state rate of inflation, the inflation adjustment equation can be written as

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \mu_t,$$

where μ_t is real marginal cost (expressed as a percent deviation around the steady-state). Derive an expression for μ_t in terms of an output gap.

(c) Does the taste shock affect the output gap? Does it affect inflation? Explain.

4. Suppose the representative household maximizes

$$\mathrm{E}_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[u(C_{1,t+i}, C_{2,t+i}) - v(N_{t+i}) \right] \right\}$$

subject to a nominal budget constraint of the form

$$W_t N_t + i_t B_t + M_t + T_t - P_t\left(C_{1,t} + C_{2,t}\right) - M_{t+1} \geq 0.$$

and a cash-in-advance constraint of the form

$$P_t C_{1,t} \leq M_t + T_t - B_t.$$

$C_{1,t}$ is a "cash" good and $C_{2,t}$ is a "credit" good (there is no capital). Both consumption goods are produced by the same constant returns to scale technology. W_t is the nominal wage the household earns on labor N_t it supplies, M_t is the nominal money stock the household enters period t with, B_t are purchases of bonds at the start of period t, T_t is a lump-sum transfer, i_t is the nominal interest rate, and P_t is the price level. (Bonds B_t purchased at the begining of t mature at the end of the period, yielding an interest payment of $i_t B_t$ – i.e., they are *within* period bonds.)

(a) Write Bellman's equation for the household's problem.

(b) Derive the first order conditions for the optimal choices of $C_{1,t}$, $C_{2,t}$, N_t, B_t, and M_{t+1}, and the economy's equilibrium conditions after eliminating the value function and any Lagrangian multipliers. Explain in words what each condition means.

(c) What distortions are caused by a non-zero nominal rate of interest? What is the optimal rate of inflation in this economy? Explain.

5. Consider an economy in which job matches are determined by an aggregate matching function of the form

$$m(v,u) = m_0 v^{1-\alpha} u^{\alpha},$$

where v is the number of job vacancies and u is the number of unemployed workers. The exogenous rate at which jobs (worker-firm matches) break-up is δ. Firms face a constant (per period) cost of c if they post a vacancy. Each filled job produces output of y per period. Let w denote the worker's wage. The real interest rate is r.

(a) Derive the Beveridge curve between u and $\theta = v/u$ along which u is constant. Explain why it implies a negative relationship between θ and u.

(b) What is the equilibrium value of an unfilled vacancy? Why? Write down the value equation for a vacancy and derive an equilibrium condition for the value of a filled job (J) in terms of θ and c.

(c) Write down the value equation for a filled job (J) , and express J in terms of $y - w$, r, and δ.

(d) Use your results in (b) and (c) to obtain a relationship between w and θ. *Explain intuitively* what this equilibrium condition implies about the relationship between wages and labor market tightness.