Final Exam, Economics 205B
Answer any two (2) questions.

1. Consider a *RBC* model. The representative household maximizes

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i u(c_{t+i}, 1 - n_{t+i}),$$

where c is consumption and n is time spent in production. The household has a constant returns to scale (CES) technology for producing output, given by

$$y_t = e^{z_t} F(n_t, k_t),$$

where $z_t = \rho z_{t-1} + e_t$, and faces a budget constraint given by

$$k_{t+1} = (1 - \delta) k_t + y_t - c_t.$$

   (a) Write down the equilibrium conditions for this economy.

   (b) Assume

$$u(c_t, 1 - n_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \Psi \frac{n_t^{1+\eta}}{1+\eta}.$$

   and $F(n_t, k_t) = n_t^a k_t^{1-a}$. For each of the unknown parameters $(a, \rho, \delta, \beta, \sigma, \eta, \Psi)$, briefly discuss how you might calibrate them.

   (c) Three characteristics of actual business cycles 1) output displays persistent fluctuations, 2) employment and output are highly correlated, 3) real wages are very weakly related to output. Are there parameter values for which the model of this question can account for these business cycle "facts"? If so, are these reasonable values for the parameters (i.e., are they the ones you would obtain from the calibrations described in part c)? If they are not, how might you modify the model to better match these three stylized facts?

2. Suppose the representative household maximizes

$$\mathrm{E}_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[u(C_{1,t}, C_{2,t}) - v(N_{t+i}) \right] \right\}$$

subject to a nominal budget constraint of the form

$$W_t N_t + i_t B_t + M_t + T_t - P_t \left(C_{1,t} + C_{2,t} \right) - M_{t+1} \geq 0.$$

and a cash-in-advance constraint of the form

$$P_t C_{1,t} \leq M_t + T_t - B_t,$$

where $C_{1,t}$ is a "cash" good and $C_{2,t}$ is a "credit" good (there is no capital). Both consumption goods are produced by the same constant returns to scale technology. W_t is the nominal wage the household earns on the labor N_t it supplies, M_t is the stock of nominal money the household enters period t with, B_t are purchases of bonds at the start of period t, T_t is a lump-sum transfer, i_t is the nominal interest rate, and P_t is the price level. (Be sure you understanding the notation and timing.)

(a) Write Bellman's equation for the household's problem.

(b) Derive the first order conditions for the optimal choices of $C_{1,t}$, N_t, B_t, and M_{t+1}, and the economy's equilibrium conditions after eliminating the value function and any Lagrangian multipliers. Explain in words what each condition means.

(c) What distortions are caused by a non-zero nominal rate of interest?

(d) What is the optimal rate of inflation in this economy? Explain.

3. Assume that there are a large number of workers $\bar{L}$, and that each worker maximizes expected discounted utility, given by

$$U = \int_0^\infty e^{-\rho t} u(t) dt$$

$$u(t) = \begin{cases} w(t) - e(t) \text{ if employed} \\ b \text{ if unemployed} \end{cases}$$

where e is effort: e is either equal to 0 or equal to $\bar{e} > 0$. The firm catches a worker who is shirking (i.e., for whom $e = 0$) with probability q per unit of time. Unemployed workers find new jobs with probability a per unit of time and receive unemployment benefit b per unit of time while unemployed. There are a large number of firms N. Firms maximize expected discounted profits given by

$$\pi(t) = F(\bar{e}L(t)) - w(t)[L(t) + S(t)],$$

where L is the number of working employees and S is the number of shirking employees. Firms choose w and L at each point in time to maximize the instantaneous flow of profits.There is an exogenous probability of job loss given by δ per unit of time.

(a) Workers can be either working and exerting effort (E), working and shirking (S), or unemployed (U). Write the valuation equations for each of these three states.

(b) What is the flow into unemployment? What is the flow out of unemployment? If the two flows are equal, show how a is affected by a change in δ for a given level of total employment NL. Explain intuitively why a is affected as δ changes.

(c) What is the no shirking condition? Explain how the no shirking condition is affected by the wage w and by the level of unemployment benefits b.