Microeconomics

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Lecture 4: May 10, 2006

Abstract

In this lecture we continue consumer theory, specifically consumer preferences.

Midterm Examination

Tentative plan:

Time: May 24, 2006, 2d period. **Place:** 8A108.

Content: Basic concepts, the market for part-time workers, consumer behavior, budgets, preferences, and demand curves.

Preparation: Sample exam questions and answers to homework and sample exam questions will be posted to the home page soon (at latest Tuesday, May 16).

Textbook coverage: Chapters **1–6** and **15**.

Language: **Questions** will be composed in *English*; if I have time I will translate to Japanese *also*. **Answers** may be submitted in either English or Japanese.

Introduction to Consumer Theory

- We gave a sorting argument for upward sloping supply. Similarly demand would slope downward. But this depends on each agent trading one unit. What about agents who trade multiple units, or even in continuous domains?
- We implicitly discuss tradeoffs of work and money for leisure. But this is not very satisfactory, we would like an explicit theory.
- In the apartment case, supply was vertical. This is perhaps plausible. But vertical demand is not. Eventually you run out of money! We need to integrate this constraint with consumer preferences.
- Sometimes the sorting condition fails (*e.g.*, minimum wage).

Goals and Methods of Consumer Theory

- The goal of consumer theory is to relate consumers' directly unobservable tastes/goals to their mostly observable resources and constraints, to market prices, and to decisions.
- We carefully define the budget, which is the constraint defined by the consumer's income and prices.
- The budget defines tradeoffs in the set of feasible *bundles*, that is, consumption plans.
- Consumers' tastes also define tradeoffs among bundles, and therefore implicitly of one good for another.
- We relate preferences to prices through *marginal analysis* of the consumer's *optimization problem*.

Basic Behavior of Consumers

- All economic agents buy and sell goods on markets.
- Consumers own resources. Their goals are defined by what they like. They do not produce outputs from inputs. (Firms differ.)
- There are many goods the consumer might want: meats and sheets, coats and boats, houses and blouses. We bundle them up, listing quantities of all goods that consumer might conceivably buy. If needed, explicitly write zero. Also called a “vector”.
  - Typically we use 2 goods. (Mathematically we can handle any number. Graphs of more than 2-good bundles are hard.)
  - Anything not spent on goods explicit in the bundle is thrown away (not “saved”). To account for saving, put “future consumption” in the bundle.

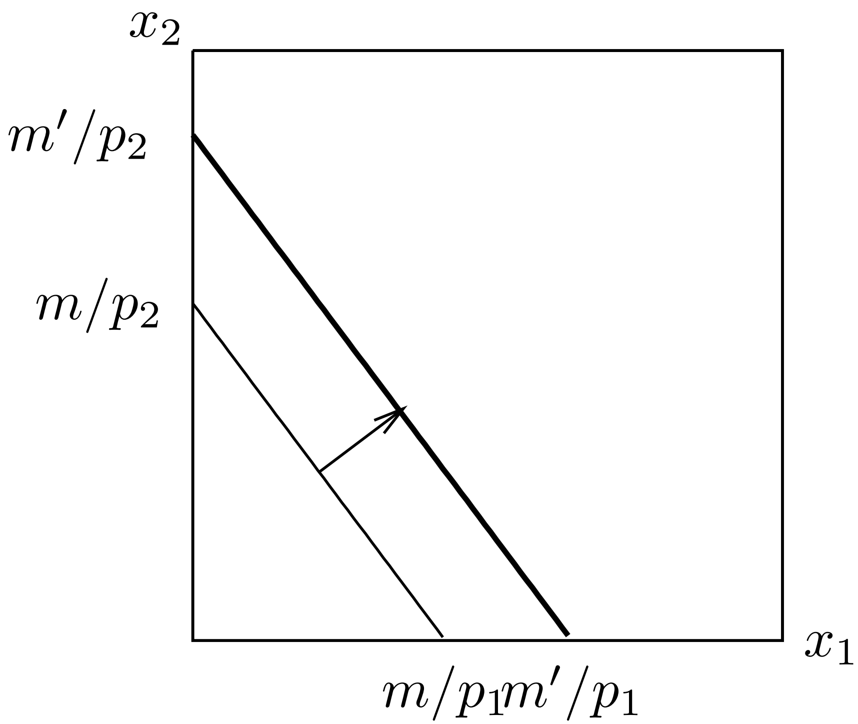
Budget Constraint Summary

- The consumer is *endowed* with *income* or *wealth*.
- The consumer buys goods in the market, at the price in the market for each good, and expenditure must not exceed income:

$$p_1x_1 + p_2x_2 \leq m.$$

- The triple (p_1, p_2, m) and the triangular budget set on the graph are actually the same thing.
- It's not enough to summarize by "total budget" (income); prices also affect what the consumer can afford.
- A change in exogenous variables represents an increase in "real" income if the new budget set contains the old one.

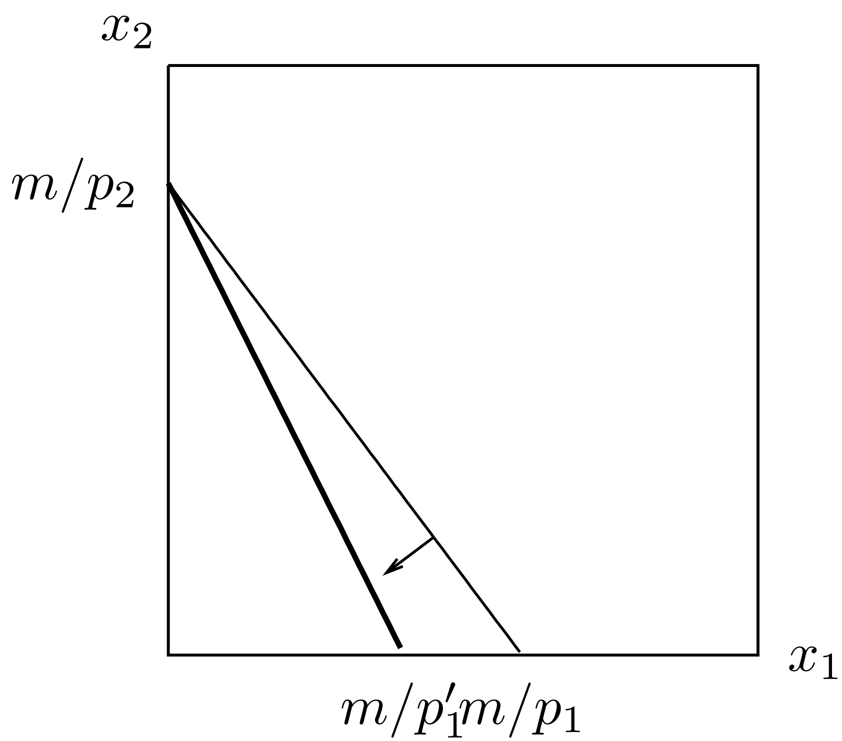
Increasing Income



Since $x_2 = \frac{m}{p_2} - \frac{p_1}{p_2}x_1$, an increase in income leaves slope unchanged and enlarges the budget set.

Increasing Price of Good 1

Since $x_2 = \frac{m}{p_2} - \frac{p_1}{p_2}x_1$, an increase in the price of good 1 leaves the x_2-intercept unchanged and gives a steeper slope.



Applications of Changes in the Budget

- Simultaneous changes in more than one variable (inflation, perfect inflation).
- *Excise (quantity-based) taxes* and *ad valorem (value-based) taxes* can be reduced to changes in *price*.
- *Lump-sum taxes* subtract from income, but do not change the trade-off between goods.
- *Subsidies* are negative taxes, and come in the same varieties that taxes do.
- *Price* and *income* changes result in a triangular budget set, but *rationing* and various complex policies can generate non-triangular budget sets.

Consumer Preferences

- The basic assumptions about consumer choice are
  - The consumer chooses *bundles*;
  - The feasible bundles are limited by the *budget constraint*;
  - Given any two bundles (not necessarily feasible) the consumer's basic action is to say which one is better;
  - This ranking of two bundles is called *preference*;
  - The idea of rationality is that the consumer knows her preference and follows it when making choices.

Preference, Utility, and Abstraction

- Economists often discuss *utility functions*, which assign a numerical value to the outcome of a person's behavior. As if success in real life were like the score you get in a video game!
- Of course, no real person knows their utility function.
- And if we have a numerical scale, we'd expect to have a list of the *Fortunate 500*, who have the 500 highest utilities in the world.
- So utility functions aren't "real"—economists admit that. However, it turns out that by looking at behavior based on *preference*, we can *construct* a person's utility function. This is similar to the physicist's construction of "wave equations" for electronics.

Notation

- With two goods, a bundle is a vector (x_1, x_2)
- When bundle (x_1, x_2) is strictly better than another bundle (y_1, y_2) write $(x_1, x_2) \succ (y_1, y_2)$
  - Don't compare x_1 to y_1 or to x_2; the whole X (X abbreviates (x_1, x_2)) bundle is compared to the whole Y bundle
  - "Strictly better" means that the consumer would always switch from Y to X and refuse to switch from X to Y when offered those choices
- The consumer is *indifferent* between X and Y if she would not refuse to switch either for the other. We write $(x_1, x_2) \sim (y_1, y_2)$

Weak Preference

- We have a notation for the case where the consumer does not prefer Y to X; we say the consumer *weakly prefers* X and write $(x_1, x_2) \succeq (y_1, y_2)$
  - We assume the consumer knows given a particular X and Y, but one or both may be variables, and *e.g.*, in a set of bundles $\{X : X \succeq Y\}$
- $X \succeq Y$ iff $X \succ Y$ or $X \sim Y$
- $X \succeq Y$ iff not $Y \succ X$
- Weak preference is useful in the logical analysis, and makes many equations shorter, just as numerical weak inequality ($\geq$) does.

Examples of Preference Notation

- Offered the choice between a vanilla ice cream cone and a chocolate ice cream cone, the consumer prefers the vanilla. Then the bundle is the number of vanilla ice cream cones and the number of chocolate ice cream cones $X = (x_v, x_c)$:

$$(1, 0) \succ (0, 1)$$

- The consumer likes more ice cream (x_i) but doesn't care about sprinkles(x_s):

$$(x_i', x_s) \succ (x_i, x_s)$$

$$(x_i, x_s') \sim (x_i, x_s)$$

where $x_i' > x_i$ and $x_s' > x_s$. Note the goods included in the bundle changed! This is a *different model.*

Rational Preferences

Complete $(x_1, x_2) \succeq (y_1, y_2)$ or $(y_1, y_2) \succeq (x_1, x_2)$ or both ("both" implies indifference).

Reflexive $(x_1, x_2) \succeq (x_1, x_2)$. We can deduce that $(x_1, x_2) \sim (x_1, x_2)$.

Transitive If $(x_1, x_2) \succeq (y_1, y_2)$ and $(y_1, y_2) \succeq (z_1, z_2)$, then $(x_1, x_2) \succeq (z_1, z_2)$.

Interpreting Rational Preferences

- *Completeness* says the consumer knows what they like, at least in a side-by-side comparison. It is hard to object to this.
- *Reflexivity* is trivial, as long as you accept that only the quantities in the bundles matter. But by positing that any difference the consumer can perceive that is non-quantitative makes a different good, we can accept this.
- Although *transitivity* is not logically necessary, it is a natural extension of completeness to choice from sets of more than two bundles. Something like it is necessary for a theory assuming the consumer makes the "best choice."

Economic Selfishness

- Consider the trivial case of a one-good bundle, say "cake," $X = (x_1)$, x_1 is the fraction of the cake the consumer receives.
- When Mom gets (0.500001) and Akane gets (0.499999), Akane gets mad because Mom's is bigger.
- In economics, we usually assume consumers are "selfish." However, Akane is not "selfish" in this sense. Selfish to an economist means you don't care at all about what others have—but Akane does. We cannot express Akane's preferences!
- We can extend the model to be non-selfish. Let x_1 be "cake Akane holds" and x_2 be "cake Mom holds", then we have $(0.500001, 0.499999) \succ (0.499999, 0.500001)$.

Reflexivity Example

- With small children, an interesting thing can happen. When Mom gets half and Akane gets half, Akane gets mad because Mom's is "bigger." If they trade, she's still mad!
- Then we have $(\frac{1}{2}, \frac{1}{2}) \succ (\frac{1}{2}, \frac{1}{2})$, and Akane's preferences are *irreflexive*! Unfortunately, it is not possible to resolve this logical paradox until Akane grows up.
- Related "adult" example: splitting chemically identical beers into two goods on the basis of the label name.

Transitivity Example

- Condorcet Voting Paradox (a political science example using rational choice)
- A club has three members $\{1, 2, 3\}$, and makes decisions by majority rule.
- Suppose for the three alternatives $\{A, B, C\}$, 1 ranks them $A \succ B \succ C$, 2 ranks them $B \succ C \succ A$, and 3 ranks them $C \succ A \succ B$.

The Club's Decision Rule

- We compute by counting votes:

	A v. B	B v. C	A v. C
1	A	B	A
2	B	B	C
3	A	C	C
Club	$A \succ B$	$B \succ C$	$C \succ A$

- The club's preferences are *not* transitive!

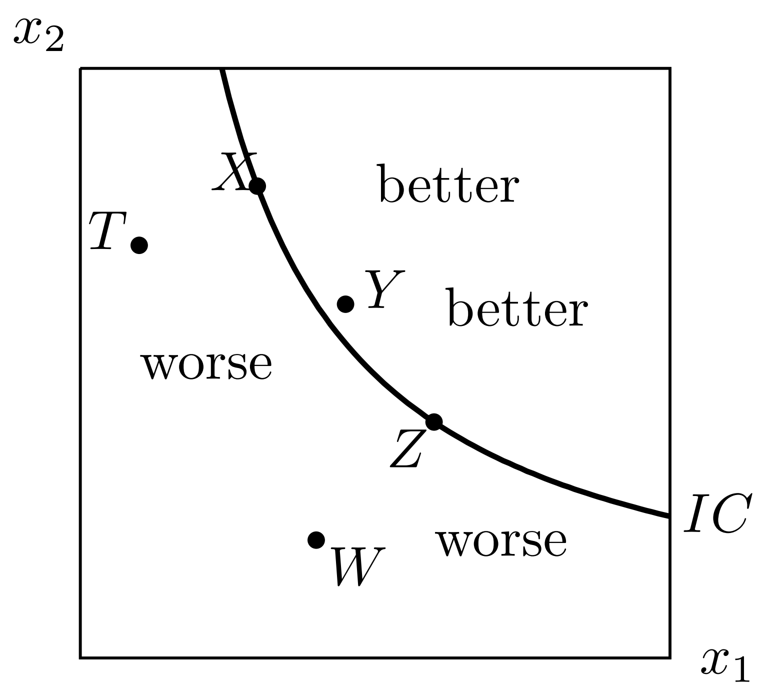
Indifference Curves

- Graphical presentation of preference
- $\{Y : Y \succeq X\}$ is the *weakly preferred set* (aka the "upper contour set") of X
- $\{Y : X \succeq Y\}$ is the *weakly less preferred set* (aka the "lower contour set") of X
- $\{Y : Y \sim X\}$ is the *indifference curve* (aka the "indifference set") through X
- $\{Y : Y \succ X\}$ is the *strictly preferred set* (aka the "strict upper contour set") of X
- For an optimal choice X, "X is feasible and the strictly preferred set of X is disjoint from the feasible set," or "X is feasible and the feasible set is contained in the lower contour set of X."

Typical Indifference Curve

In the graph at right

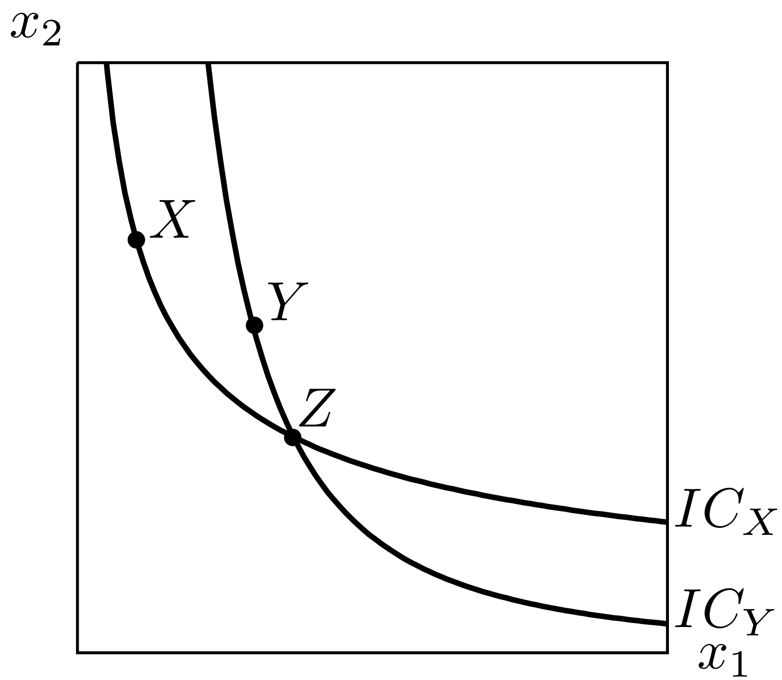
- $Z \sim X$, $Y \succ X$, and $X \succ W$ (by position relative to IC)
- $Y \succ T$ (by transitivity)
- *This* IC does not show how to compare W and T, but both W and T have their own indifference curves (not shown)



Indifference Curves May Not Cross

- Suppose there is a bundle X and another bundle Y, such that not $X \sim Y$.
- Then X is not on Y's indifference curve, and Y is not on X's indifference curve.
- Suppose they cross. Then there exists Z on both indifference curves (see following graph).
- By definition of indifference, we have $X \succeq Z \succeq Y$ and $Y \succeq Z \succeq X$.
- Transitivity and the definition of indifference give $X \sim Y$, so the assumption that IC_X and IC_Y cross is not compatible with the assumption that they are different ICs.

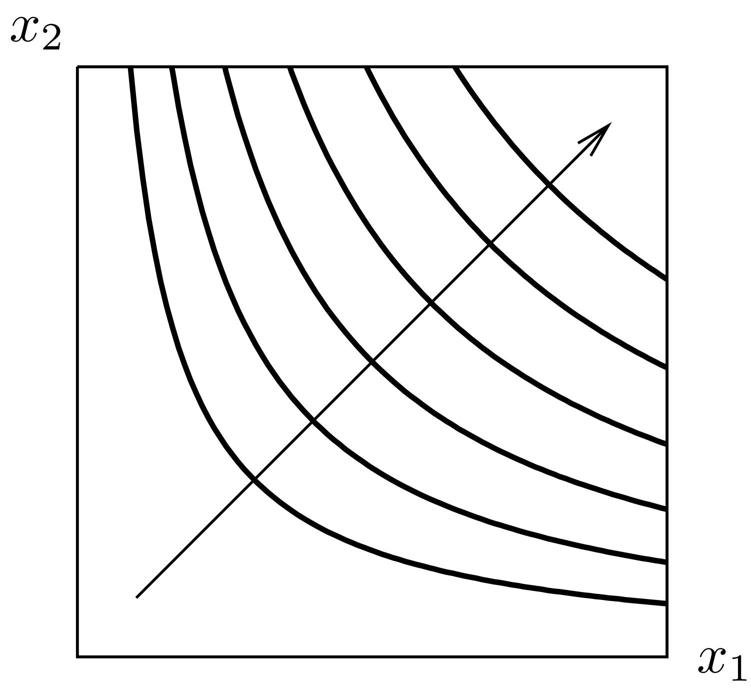
Crossing Indifference Curves?



More About Indifference Curves

- We cannot (yet) rule out a "4-armed" shape for the indifference "curve" through Z, but in that case $X \sim Y$.
- This implies that indifference curves are locally well-ordered as on a topographical contour map (see following graph).

Ordered Indifference Curves



Economic Rationality

- So far we have discussed a very basic idea of rationality. While there are unrealistic aspects to it, the idea of *complete*, *reflexive*, and *transitive* preference should apply in many non-economic contexts, such as political choice.
- In economics there are three more conditions we usually use. They are plausible, but outside of economics may not be defined.

Continuity If $X \succ Y$, then there is a bundle Z such that $X \succ Z \succ Y$. Furthermore, for all Z close enough to X, $Z \succ Y$.

Monotonicity A bundle containing more of all goods is preferred to one with less.

Convexity If X and Y are bundles such that $X \sim Y$, then for all $0 < \alpha < 1$, $\alpha X + (1 - \alpha)Y \succ Y$.

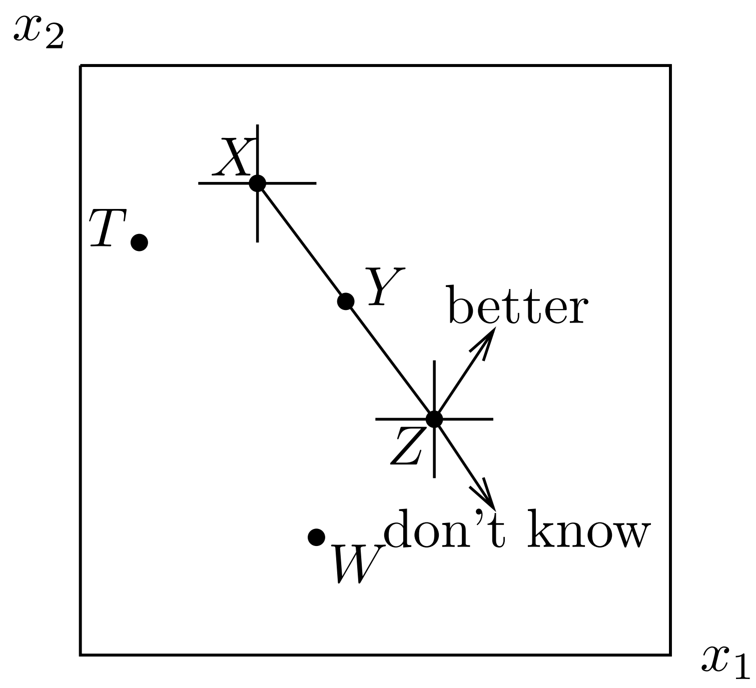
Implications of Economic Rationality

- *Continuity* is basically a mathematical condition that ensures we can solve the consumer's problem. It also ensures that the indifference curves are actually curves.
- *Monotonicity* is easy to understand. More is better.
- *Convexity* implies consumers like "averages" in some sense. The "happy medium."

Economic Preferences on the Graph

In the graph at right

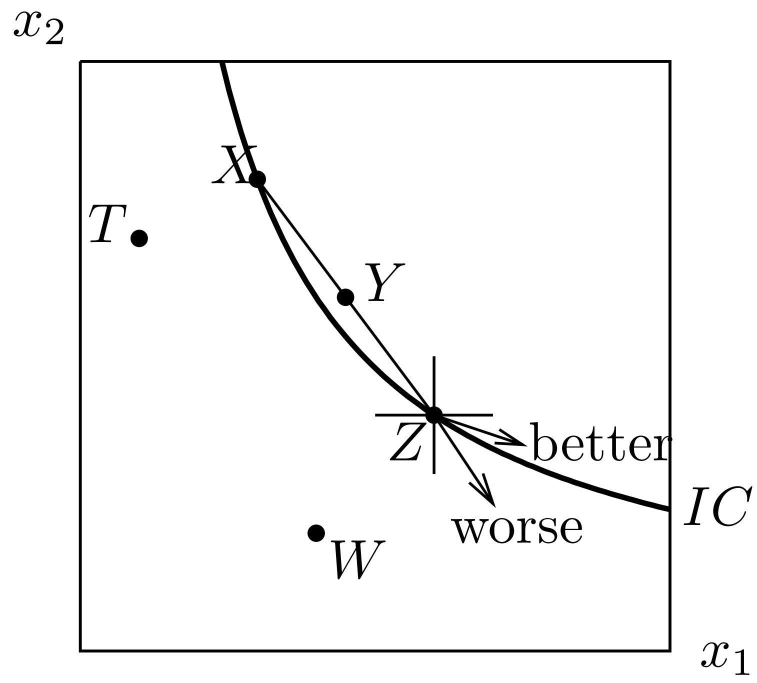
- $X \succ T$, and $Z \succ W$ (by monotonicity)
- $Y \succ X \sim Z$ (by convexity)
- monotonicity is not sufficient to classify "don't know"



Typical Economic Indifference Curve

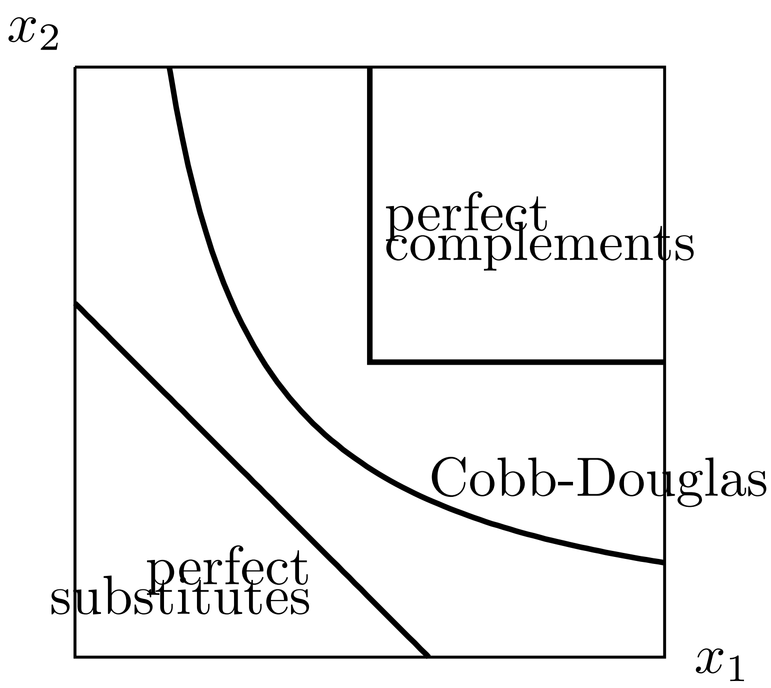
In the graph at right

- the curve passes below Y (by convexity)
- the curve is continuous in the usual sense (by continuity)
- we now can classify one downward arrow as "worse," the other as "better"



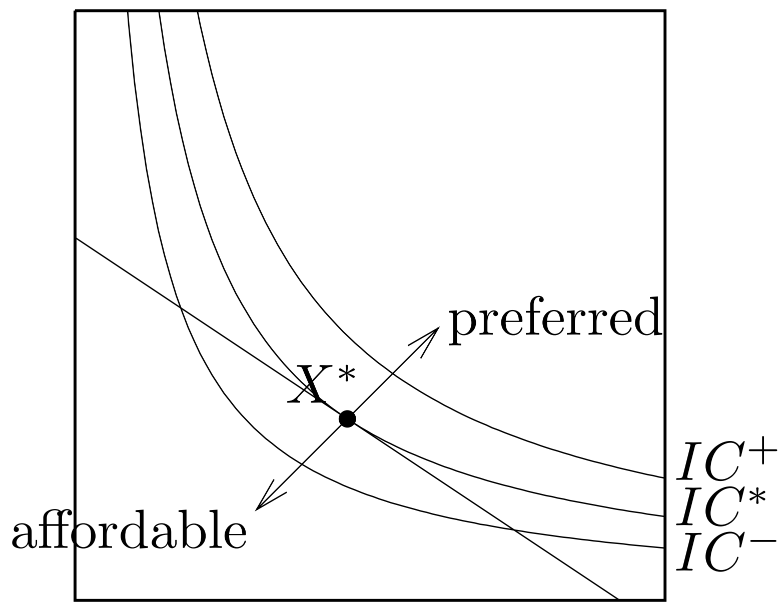
Indifference Curve Examples

Try to understand the meaning of *perfect substitutes* (economically, the same good) and *perfect complements* (economically, a composite good with fixed proportions of the components) from the indifference curves.

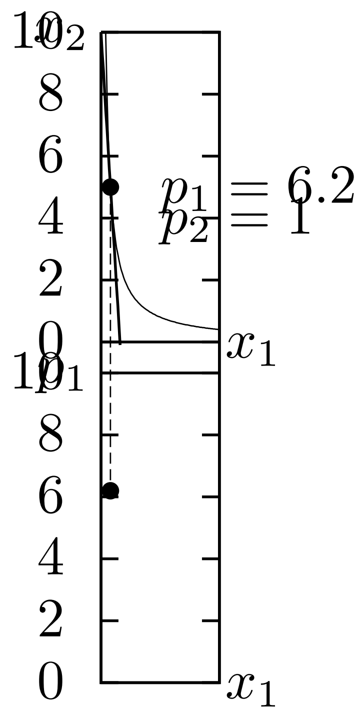


The Consumer's Optimum Graphically

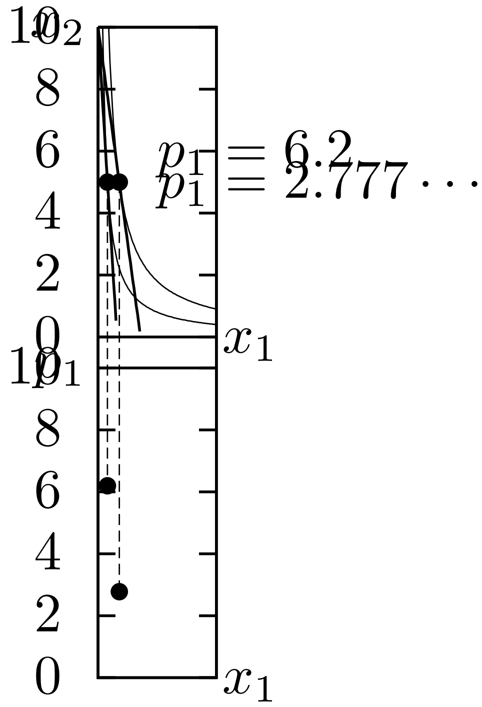
The point X^* divides the plane into 3 areas: *affordable* (the budget set), *strictly preferred* to X^* (the upper contour set), and "neither." Since nothing is affordable *and* strictly preferred, X^* is optimal.



Deriving the Demand Curve: 1

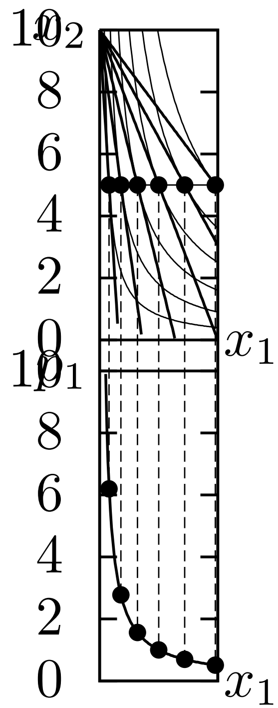


Deriving the Demand Curve: 2



Deriving the Demand Curve: 3

- The set of optimal points for different budget lines defined by various prices of good 1 is called the *offer curve.*
- The graph of prices and quantites of good 1 corresponding to the offer curve is the *demand curve.*



Deriving the Demand Curve

1. Pick p_2 and m.
2. Pick a new p_1.
3. Plot the budget set on top.
4. Find the tangent IC and optimum.
5. Plot p_1 and x_1^* on the demand curve.
6. Return to step 2.