Assignment 4.

1. Consider the sphere $S^n = \{q \in \mathbb{R}^{n+1} : \langle q, q \rangle = 1\}$ In class we asserted that the geodesic equations on this sphere can be written in the form

$$\ddot{q} = \lambda(t) q(t) \tag{1}$$

where $\lambda(t)$ is a scalar. Derive that $\|\dot{q}(t)\|$ and $\lambda(t)$ are constant along any solution to this equation, provided the solution $q(t)$ indeed lies on the sphere. HINT: repeatedly use the relation $d/dt\langle A(t), B(t)\rangle = \langle \dot{A}, B\rangle + \langle A, \dot{B}\rangle$ for differentiating dot products.

2. The Poincaré metric on the upper half plane $y > 0$ is $ds^2 = (dx^2 + dy^2)/y^2$. Derive the geodesic equations. Verify that half circles orthogonal to the ideal $(y = 0)$ are geodesics.