Lecture schedule. Riemann Surfaces. Wtr 2022. Tentative.

Abbreviations: RSs = Riemann surfaces. Of references: FK = Farkas-Kra. D = Donaldson.

Week 1. Definition. Examples. Genus g as topological classifier. Refs: . Ch 1 FK and D

Meromorphic functions as maps to $\mathbb{CP}^1$. An analogue to Morse theory: one non-constant meromorphic function encodes the topology of the RS: D: sec: 4.1. Cor 1, p 45 Riemann-Hurwitz formula (FK, Thm p 21) multiplicity, degree, branching number of a point rel. a function or map. FK: Prop. on p 12,

Uniformization theorem [UT] (statement, not proofs!) The three models, their automorphism groups and relations to Riemannian geometry. Rational, elliptic, hyperelliptic. $g = 0, 1, > 1$. Curvature $1, 0, -1$. Refs: My notes. Index of D.

The shape sphere of the three-body problem as the Riemann Sphere. Application of Riemann-Hurwitz to the regularized shape sphere. Sources: My notes. My paper, 'The three-body problem and the shape sphere' available on my web page.

Week 2. The great synthesis as explained by Mumford. Going back and forth between algebraic and quotient models. Ref: Mumford.

First steps towards Teichmuller and moduli space via UT and the character variety of $PSL(2, \mathbb{C})$. Dimension $3g - 3$ over $\mathbb{C}$.

Hyperbolic pants and $1/r^2$ potentials in the three body problem. My paper: "Hyperbolic pants fit a three-body problem" -available on my web site.

Week 3. Holomorphic and meromorphic one-forms. The canonical divisor K and other line bundles. Sources: FK: 1.3, 1.8.

Divisors, their corresponding line bundles and the sub-vector spaces of the field of meromorphic functions they define. Statement one of Riemann-Roch [RR]

Multivalued mero functions arising as indefinite integrals of mero one-forms. D: ch 5 and 6.

Refs: Miranda: ch 6, 7. D. ch 11. FK: p 69 – and index. Consequences of RR: Miranda ch 7. D. ch 12. FK: all over (see index).

Week 4. Periods. The Jacobian of a curve. Abel-Jacobi map. Canonical embedding as the differential of the AJ map.

Refs: Miranda, ch 8. D. ch. 12. FK. Chapter 3. Mumford.

Week 5. Hurwitz surfaces. The $84(g - 1)$ count. The Klein quartic, firstly as the modular curve $X(7)$. The modular curve $X(1)$ as the Riemann sphere. The $X(N)$'s. $X(5)$. $X(7)$.

Refs: Elkies. Donaldson 4. ... ; 'The Eightfold Way'.

Week 6. (?) More on RR. $H^{p,q}$. Refs: D ch 11. Chern: Complex Manifolds without Potential Theory.

Week 7. (?) The Picard variety: the moduli space of topologically trivial, holomorphically non-trivial line bundles.

Refs: D. Miranda. FK.

Week 8. (?) Strebel differentials. Flat structures. Rational billiards.

Week 9; (?) How many cpt RSs are there for a given genus g? Ans (Teich.; ...) a complex $3g - 3$ dimensional family. Tangent space to Teich space. ..

Week 10. Return to the great synthesis.