

## Week 1: Readings and HW

**Readings** any or all of:

Flanders [Fl]: ch 1, ch 2, ch 3.1, 3.2

Guillemin and Pollack [G-P]: 4.1, and 4.2

Sjamaar [Sj] : chapter 2.

extra readings:

Burke. Div Grad curl are dead. ch 2.

Darling: chapters 1 and 2.

**HW Exercises:**

Flanders, ch 2. 1-4.

G-P: 4.2: 1-4. All starred exercises.

Sj: 2.1-2.5 [p. 27]

**presentation: exterior algebra**

## Week 2. Pullbacks. Exterior derivative

**Reading:**

Fl ch 3.

Sj. ch 3.

G-P: 4.3, 4.4

**Exercises:**

Fl: ch 3: 1,2,3

G-P: 4.4: 8, 9 \* ,

Sj 3.17, 3,18 \*

(\*) = prioritize these \* ed exercises.

**presentation: Sjamaar p 25-26 translating div grad curl to d**

**Presentation: Sjamaar: Exer 2.23 on p 30 converting Maxwell's eqns to Forms**

## week 3. Integration. Stokes' theorem. Cartan's Magic Formula.

**Reading:** G-P. 4.5, 4.7; Lee or Abraham-Marsden for Cartan formula.

Fl : 5.5-5.8

GAME PLAN: Integration and Stokes: Tu. Cartan: Thurs. (May be too ambitious)

**Exercises:**

G-P: 4.7: 7 \* , 8, 9, 2; 4.5: 2\*, 1 ; 4.4: 12\*, 10,11,13, 14.

Sj. p. 28: 2.1-2.11

Thursday: Cartan's magic formula  $L_X = di_X + i_X d$ . Lie derivatives of tensors and forms.

**Presentations:**

1. Poincare lemma w proof, case of two-forms in  $R^n$ .

2. any HW from 4.7.

## week 4. de Rham theorem; Hodge thm (?)

**Readings.**

Fl 5.6-5.8. Readings under 'Hodge' in index.

G-P : 4.8.

Sj. 10.3; Look up Hodge \*

**Exercises on  $L_X$  and  $i_X$ .** Prove identities 9, 10, 13, 16 \* in table 2.4.1 of p. 121 of Abraham -Marsden (see course web site for copy). The starred 'mandatory' identity 16 is actually five separate identities. Prove these five identities using any identity earlier than the identities 16, or the definitions, or by coordinate computations.

**week 5. = Feb 2, 4. Degree of a map. ‘Degree Formula’. Frobenius via forms.**

**Readings:** GP: 4.8. Fl 6.1-6.2

*Additional Readings:* Milnor, Topology from the Differentiable Viewpoint. *a phenomenal and very short book!*

**Exercises:** 4.8: 2\* , 3, 6,7 \*,8 \*

missing here: HW on Frobenius integrability via forms and its opposite, Chow-Rashevsky. To be assigned week 9-10.

**week 6. Lie groups and algebras**

**Readings** Fl. ch 9.

local structure: via forms  $d\theta^i = \sum_{j,k} c_{jk}^i \theta^j \wedge \theta^k$ . Dually: vector fields:  $[X_i, X_j] = \sum_k c_{ij}^k X_k$

**Exercises:** See ‘HWs’ on web page. Do “HW6LieGroups”. This HW is doubly important. Additional: Fl. Ch 9, 2, 5, 7 \*.

**week 7 = Feb 16, 18. Mechanics on Manifolds. Lagrangian, Hamiltonian, Legendre Transformation. Canonical one-form and two-form.**

**Readings:** Fl. ch. 10.

**Exercises:** ?? either ones I write, or from Arnol’d or from Ab-Mars.

**week 8.** Surface theory, a bit of Riem geometry; via forms.

**Readings:** GP 4.9.

*Additional readings.* Any undergrad book on differential geometry a.k.a: “curves and surfaces”. O’Neill, and Singer-Thorpe are good, and the later in particular uses forms.

**Exercises:** GP: 4.9: 8 \*, 10 \*

**week 9-10. ??**

**Exercises:** on Frobenius and Chow, via forms.

**Possible Presentation topics:**

Fl: 6.3: the Hopf invariant

Fl 6.4 : linking number via forms

Hamilton’s eqns as yielding symplectic flow.

the Kepler problem a la Stephanie Singer’s book.

**Additional topics I’d like to cover:**

Moser’s method: proof of Darboux, pf that any two vol. forms on a cpt manifold w same total vol. are isotopic.

Isotopy lemma:  $Diff(M)$  acts transitively on  $M$

Real and Complex line bundles. Vector bundles.

More geometric mechanics, in particular Momentum Maps.

Hodge star

Sj p. 29: 2.22-2.23

Cohomology of semisimple Lie groups using  $(\Lambda^*(LieG))^G$

‘By how much does a rigid body...’ ?

**Some Additional Readings:**

Arnol’d.

Abraham-Marsden, ch 4.

Guillemin-Sternberg: Symp Tech