Manifolds 1. **Syllabus , tentative class structure** Meets: Fall 2016. MW. 09:50AM - 11:25AM. Professor: Richard Montgomery.

Evaluations are to be based on HW, class presentations, a midterm and a take-home final.

Overview: Smooth manifolds are the most general objects on which you can do calculus. They are central to dynamical systems, modern mechanics, general relativity and swaths of theoretical physics. In our department Ginzburg, Lewis, Lin, and Qing work on a day-to-day basis with manifolds. The subject has deep interactions with algebraic geometry, stemming in large part from the fact that a smooth algebraic variety over $\mathbb{C}$ or $\mathbb{R}$ is an example of a smooth manifold, but the underpinnings of the two fields inform each other. Roughly speaking, in the first quarter we learn how to differentiate on manifolds (vector fields) and in the second quarter we learn how to integrate (differential forms).

General course goals. To learn what a manifold is, and is not. To be able to do some calculus on manifolds, primarily using vector fields. (Next quarter is forms.) To begin to think in the three modalities of pictures, and coordinates, and algebraically, and to switch back and forth between these modes of thinking.

Concrete goals: To know and be able to use the IFTs. To know what a tangent vector is. To gain proficiency with vector fields, their flows, and Lie derivatives. To get a rough sense of Lie groups as smooth transformation groups, and of some fiber bundles. To learn some of the basic theorems of differential topology as espoused by Milnor (Fundamental theorem of algebra, Brouwer fixed point theorem, Poincaré Hopf theorem, ...)

REFERENCES. The references I will primarily be assigning reading from are:

Milnor's ' Topology from the Differentiable Viewpoint" [Milnor]

Auslander and MacKenzie: Introduction to Differentiable Manifolds. [Auslander-MacK]

($ 12.95 through Dover !)

Sharpe's "Differential Geometry: Cartan's generalization of Klein's Erlangen Program", [Sharpe]

Frank Warner's "Introduction to Differentiable Manifolds" – [Warner]

Barden and Thomas' : An Introduction to Differential Manifolds' , [Barden-Thomas]

Guillemin and Pollack's Differential Topology (described by authors as an expanded version of Milnor)

Vassiliev: Introduction to Topology [Vassiliev]

There are hundreds, probably thousands , of introductory texts on smooth manifolds. They vary a lot in content and outlook from the topological to the analytic. Some people like encyclopaedic or extremely chatty books. I am not one of these people. But you might be!

ENCYCLOPAEDIC TREATMENTS:

John Lee: Introduction to Smooth Manifolds.

Spivak (5 volumes) ; mostly starting with vol. 1. A comprehensive introduction to differential geometry.

Abraham-Marsden-Ratiu: Manifolds, Tensor analysis, and applications,

Abraham-Marsden: Foundations of Mechanics, ch.1-3.

A CONCISE TREATMENT: Serge Lang's Differentiable Manifolds (particularly the 1st edition). (suffers from almost no examples.)

Books others have recommended to me but which I cannot personally vouch for.

Loring Tu's "An Introduction to Manifolds". (Available on line)

Manifolds and Differential Geometry, by Jeffrey Lee. (not Jack or John ..)

Modern Geometry, Methods and Applications. By Dubrovin, Fomenko, and Novikov.

Finally, you might enjoy perusing :

Thurston: 'Three dimensional topology and geometry. ' There are earlier, perhaps better versions of these wonderful Thurston notes which are available online via MSRI:

http://library.msri.org/books/gt3m/PDF/3.pdf

(or try 1.pdf , 2.pdf etc. for other chapters)