Lectures and Reading, by week

Week 1. Basic examples: smooth hypersurfaces; Projective space, Riemann surfaces. Basic definitions. Embedded Manifolds : special subsets of a Euclidean space (Milnor). Abstract topological and smooth manifolds: Vassiliev; Sharpe Warner, ... Fund Thm of Alg via differential topology (Milnor). The IFTs (Inverse and Implicit Function Theorems). **Reading:** Ch 1 Milnor. p.1-6 of Sharpe. ... Warner, pp 30-34, ; p. 1-15: Barden-Thomas. ch 1 and 2 Auslander-MacK. Vassiliev: ch. 7.

Week 2. **Presentation***: Fund Thm of Algebra (ch 1, Milnor). Bump functions. Partitions of unity: Embedding theorems and existence of Riemannian metrics. . Sard. Tangent space at a point: four ways: as an affine subspace in the embedded case; via derivations (abstract algebraic), as equivalence classes of curves (intrinsic geometric), as "n-vectors depending on coordinate choice that transform a particular way under coordinate transformations" (often most computationally useful but geometrically bone-headed) . IFTs revisited. const. rank thm (Warner?) . **Reading:** p.18-21 of Barden-Thomas. Ch. 2, Milnor. Warner: p. 11-22. Auslander-MacK : chapter 4 and 102-105;

: (*) NB: 'Presentation': means students sign up for topic AND present it in class.

Week 3. **Presentation:** Sard (ch 2, Milnor) . Vector fields and their flows. Lie bracket: three or 4 defs– via derivations, coord computation, deviation from commutativity; pullback formula. Straightening lemma, aka "flow-box theorem". Existence of Riem metrics via partitions of unity. **Reading:** Barden-Thomas: Section 2.6, 2.7. Sharpe: sec. 1.4. Warner pp. 8-10; p. 69-72. Auslander-MacK ch. 4. Vassiliev: ch. 7.

Week 4. **Presentation:** Vector Field HW. Manifolds with boundary. Fiber bundles. Vector bundles. Tangent space and bundle. **Reading:** Sharpe: section 1.3 Barden-Thomas, ch 3. Ch 2. Milnor. Auslander-MacK: ch . 9. Vassiliev: ch. 6.

Week 5. **Presentations:** 1. Hopf fibration (Sharpe; see esp figure). 2. Brouwer fixed pt thm (ch. 2 Milnor). Presentation: Lie bracket for linear vector fields.

Week 6.. Orientation. Degree. Charts whose overlaps lie in other pseudogroups to yield manifolds with additonal structure: complex manifolds, symplectic manifolds, locally Euclidean manifolds, hyperbolic manifolds, ...; **Reading:** Milnor, ch. 5. p. 6-10 of Sharpe. Vassiliev: ch. 8.

Week 7. **Presentation:**1. on the non-orientability of the Möbius strip, the Klein bottle. 2. (HW) - and 'every other' projective space. Poincare-Hopf theorem. **Reading:** Milnor ch 6. Guillemin-Pollack.

Week 8. **Presentation:** invariance of Euler characteristic: surface case. Frobenius. Lie groups, beginning. **Reading:** Sharpe: p. 12-13; 63-64.Barden-Thomas: ch. 8.

Week 9. **Presentation:** $SU(2) = Sp(1) = S^3$, $SO(3)$. The 2:1 map $SU(2) \to SO(3)$. Presentation: HW – embedding projective space into the Euclidean space of symmetric matrices:k equivariance w.r.t to GL(n). Presentation: Grassmanians, and their natural equivariant embeddings into matrix spaces. Transversality. Isotopy.

Week 10: Peek into Manifolds II: Intro to differential forms; and what I use manifolds for: mechanics on manifolds, 'intrinsic' thinking, dynamics on manifolds, quantum mechanics on manifolds...