Lectures and Reading, by week

Week 1. Sept 26...

Lecture 1. Basic examples. Basic Definitions. IFTs. Embedded (a la Milnor) vs Intrinsically defined manifolds.

Lecture 2. Functorialities: products of manifolds, open subsets of, gluing. Bump functions. Partitions of unity. Cor of partitions of unity : Simplest version of Whitney Embedding theorem. Existence of Riemannian metrics. Tangent space at a point, start (embedded and level set case)

Week 2. Oct 3 Lect 3: **Presentation Fund Thm of Algebra (ch 1, Milnor).** Sard. Tangent space at a point: (1) as an affine subspace for embedded mfds (2) as derivations, (3) as curves , (4) as "n-vectors transforming contravariantly under change of coordinates". Also: via ker of differential for manifolds as level sets [IFT] . IFTs revisited. const. rank thm (Warner?) . **Reading:** p.18-21 of Barden-Thomas. Ch. 2, Milnor. Warner: p. 11-22. Auslander-MacK : chapter 4 and 102-105;

Lect 4. Wed... **Presentation: computing tangent spaces.** $O(n)$ case; vector fields, start...

Week 3. Oct 10.

Lect 5. Vector fields and their flows. Lie bracket: three or 4 defs– via derivations, coord computation, deviation from commutativity; pullback formula. Straightening lemma, aka "flow-box theorem".

Lect 6. **Presentation: 1. Vector Fields HW. 2. Lie bracket for linear vector fields.** Vector fields, the linear case. Associated bracket. Linearizing at a fixed pt.

Week 4. OCT 17: We are not going as fast as these predicted by this lecture schedule. What I have not covered so far, and do need to cover: MISSED, TO COVER: Immersion, submersion. Embedding. Submanifolds. IFTs revisited. Partitions of Unity. Existence and meaning of Riem metrics.

ALSO: : Lie bracket. Functoriality of Lie bracket and tangent and cotangent bundle. Tangent and cotangent bundle as manifolds. . Straightening Lemma. Other Normal forms lemmas.

EXAMPLES of MANIFOLDS to discuss. Products. Grassmannians. Other quotients. Connect Sum. Lie groups. Veronese ("pure state" and "mixed state" embeddings) of Grassmannians. Flag manifolds.

TOOLS needed: straightening lemma. Lie groups.

thus – the following lectures probably will have to be pushed to future weeks ... ! –

Lect 7. Vector fields as defining isotopies. Manifolds with boundary.

Lect 8. **Presentations: 1. Hopf fibration (Sharpe; see esp figure). 2. Brouwer fixed pt thm (ch. 2 Milnor).** Fiber bundles. Vector bundles. Tangent space and bundle revisited.

Week 5. Oct 24

Lect 9. Orientation. Degree in the oriented case. Orientation as a restriction on nature of overlap maps. Other pseudogroups– complex, symplectic, affine, flat (Euclidean), ...

Lect 10. **Presentations** on Orientation: Mobius strip, Klein bottle, Projective plane . Other pseudogroups, ct'd: expressing existence of various geometries (pseudogroups) as existense of some tensor: vol, J, ω; TENSORS, start .... : intrinsic linear algebra

week 6. Oct 31

Lect 11. **Presentation:** from HW... Tensors: intrinsic linear algebra. Rep ring. Operations on vector bundles. Applied to tangent bundle. Various geometries defined via specifying a tensor, revisited.

Lect 12. Poincare-Hopf theorem. Euler characteristic. **Reading:** Milnor ch 6. Guillemin-Pollack.

week 7. Nov 7 Lect 13 . **Presentation:** invariance of Euler characteristic: surface case. Frobenius. Lie groups, beginning.

lect 14. **Presentation:** the pseudogroup defined by a foliation. Lie groups ct'd.

week 8. Nov 14

Lect 15. **Presentation:** $SU(2) = Sp(1) = S^3$**. 2. The 2:1 map** $SU(2) \to SO(3)$**.** More Lie groups, Frobenius, invariant tensors, as time permits. Homogeneous spaces. Equivariance.

lect 16. **Presentation: HW – embedding projective space into the Euclidean space of symmetric matrices:k equivariance w.r.t to GL(n). Presentation: Grassmanians, and their natural equivariant embeddings into matrix spaces.** Transversality. Isotopy.

Week 9- 10. Nov 21, Nov 28; Take a breath. Fill in the places we rushed through or skipped. Peek into Manifolds II: Intro to differential forms; and what I use manifolds for: mechanics on manifolds, 'intrinsic' thinking, dynamics on manifolds, quantum mechanics on manifolds...