

Material.

The definition of a smooth manifold. Examples: spheres, orthogonal groups, projective spaces, spaces of lines, Grassmannians, ... The IFTs. Regular and critical points and values for maps. Sard. The tangent bundle. Vector fields, their flows and Lie brackets. Partitions of Unity. Whitney embedding thm. We will cover Chapters 1-5, 8, 9 of the textbook and some parts of Chapters 6, 7 and 10.

Lectures and Reading, by week

Week 1. Sept 26...

Lecture 1. Examples. Spheres. The Rotation group. Basic Definitions. Topological vs Smooth Mfds.

HW1: the cross.

Break

Mfd embedded in Euclidean space. Statements of IFTs [Inverse and Implicit Function Theorems]; first for curves in the plane or maps $\mathbb{R}^2 \rightarrow \mathbb{R}^2$, then $\mathbb{R}^m \rightarrow \mathbb{R}^n$. Statement of Whitney embedding thm.

L2. Examples: The spaces of lines in the plane = TS^1 ; in \mathbb{R}^n as - . Oh! TS^{n-1} . TM , generally. Embedded case; intrinsic case. Vector fields and tgt spaces. The tgt space at a pt: via curves, via derivations, via coordinates (n-vectors transforming contravariantly). Notation for v-flds.

Problem: Describe a vector field on the 2-sphere.

On the space of lines.

Problem/Example: Stereo proj and resulting charts. \mathbb{RP}^1 , \mathbb{RP}^n .

PROBLEM: on the property of agreeing to 1st order; to kth order, in charts.

\mathbb{RP}^1 : as a cptification of \mathbb{R} .

Problem: when we view \mathbb{RP}^1 : as a cptification of \mathbb{R} , does the vector field $\frac{\partial}{\partial x}$ smoothly extend to ∞ ? If so, is ∞ a singular point of this vector field?

Problem verify that $X(g) = gE_3$ defines a vector field on the rotation group $SO(3)$,

Problem: identifying the tgt space to a point ℓ of projective space \mathbb{RP}^{n-1} as $Hom(\ell, \mathbb{R}^n/\ell)$.

FOR Next week: Students assigned something on the IFTs: either prove one of them, or the equivalence of the two.

Week 2. Oct 3 IFTs and critical pts revisited

L3. Pfs of part of , Euclidean case. [a student!]

Application. $O(n)$ and $SO(N)$, with $F(A) = AA^T$.

**** BREAK ****

Problem: If $F : S^2 \rightarrow \mathbb{R}^2$ then F has critical points.

The IFTs for maps between mfds.

EXAMPLES. The real and complex projective spaces.

L4.

Def, regular, critical, immersion, submersion, embedding.

Problem: $F : SO(3) \rightarrow S^2$ by $F(A) = Ae_3$.

Problem: $F : SO(2) \rightarrow SO(3)$ by $F(\lambda) = g_0\lambda$.

Embedded vs immersed submfds. Examples.

PROBLEM. Compute the critical pts for $f([v]) = \langle Av, v \rangle / \langle v, v \rangle$ $[v]$ homog coord of a pt in projective space.

Week 3. Flows.

L5. Vector fields as autonomous 1st order ODEs. The corresponding flow. Fundamental thm of ODEs: existence, uniqueness, and cts dep on i.c.s for solutions of. Flows as one-parameter diffeos.

L6; problem of blow-up; irrelevance of blow-up for ‘small time’ defs”. Lie bracket as a commutator of flows. of derivations.

Flow box thm, aka ‘Straightening lemma’ . Normal form for v-flds.

Week 4. Pull-back and push-forward. By maps. By diffeos. Functorialities.

Intrinsic linear algebra perspective. Computational perspective.

Lie Derivatives covered, in some detail ...

week of Nov 13

Nov 13: Vector bundles. Normal bundles. Sheaves of sections. Tensors I. Maybe the Euler class.

Nov 15. Unit quaternions. $SU(2) = Sp(1) = \tilde{SO}(3)$ as the 3-sphere. The unit tangent bundles of the simply connected constant curvature surfaces. A bit on geodesic flow. A bit on uniformization.

Nov 17. Tensors II. Tensor algebra(s). (r, s) -tensors are r times covariant, s times contravariant. Basic examples of geometric structures defined by tensors. $(1, 0)$ tensors: one forms, $(0, 1)$ tensors: vector fields. Almost complex structures J is a particular type of $(1, 1)$ tensor. A Riemannian metric is a particular type of $(2, 0)$ tensor. A symplectic form is a particular type of skew-symmetric $(2, 0)$ tensor. A k -form is a totally antisymmetric $(k, 0)$ tensor. A Poisson tensor is a particular type of skew-symmetric $(0, 2)$ tensor. Some words on the debauch of the indices.

SIGN UP for in-class presentations. Plan on 20 to 30 min each. Beginning next week.

A topic of your choice, related to the class.

Manifold Learning. (Rui?)

Fund thm of alg (Milnor)

Borsuk-Ulam

Space filling curves or Cantor set onto any set

For cpt Hausdorff : $1-1$ + onto implies homeomorphism

ANY HWs you had to struggle with.

More on Euler Class.

Laplacian on a Manifold

week of Nov 20 = Thx Giving week.

Nov 20. Transversality. Sard. The Euler class, maybe, maybe again. The Poincare-Hopf theorem

Nov 22. Quotient spaces. Covering spaces. Homogeneous spaces.

Thanksgiving week = week of Nov 27

Nov 27: Riemannian metrics; geodesic flows. Lagrangian and Hamiltonian formalism. Examples. Mixing between.

Student Presentation: curves and surfaces to Gauss-Bonnet (?).

Nov. 29. Hmmm.. More Riemannian geom and Mechanics on Manifolds' Overview of faculty working on or with manifolds

Last week = week of Dec 4

Dec 4 : Student presentation: Milnor: fund thm of alg.

Student Presentation: Guillemin-Pollack exercise on Morse functions.

Dec 6. Conclusion.