Overview, Outline:

A (minimalist?) route to the classification of simple compact Lie groups.

1. Representation theory generalities. The main tools: averaging, Schur's lemma, characters. A representation is determined by its character. Peter-Weyl theorem statement – used to insure the list of irreps of a G is complete.

2. Representation theory for T a torus.

3. Definitions of the maximal torus, of weights. Roots = weights for the adjoint representation. Fact that a rep is determined by (a) its characters, or (b) its weights. Fact that T is a slice for AD.

4. The (finite!) Weyl group W, which is $N(T)/T$ where $N(T)$ = Normalizer of T, and the action of W on $Lie(T)^*$.

5. Representation theory for $SU(2)$.

6. The α-string through β.

7. The Weyl group forms a Coxeter group. Root system axiomatics.

8. Combinatorial classification of root systems.

9. The list.

Asides along the way: $G/AD = T/W$, $Lie(G)/Ad = Lie(T)/W$,

The classification of compact simple Lie groups agrees exactly with the classification of the complex simple Lie algebras. See Serre's beautiful short book on that latter subject, and the last chapter thereof for this isomorphism between categories.