We use our knowledge of the representations of a torus to understand and organize the representations of a compact Lie group, focussing on the case $SU(3)$.

We will use the following facts regarding representations of a torus T:

- Every complex irrep is one-dimensional.
- Every character $\chi : T \to S^1$ defines a one-dimensional representation $V_\chi \cong \mathbb{C}$ by having T act on $\mathbb{C}$ by $tz = \chi(t)z$.
- The above exhausts the irreps: the space $\hat{T}$ of irreps is in natural bijection with the space of characters $\hat{T} = Hom(T, S^1)$.
- We can canonically identify $\hat{T}$ with a lattice $\Lambda^* \subset \mathfrak{t}^*$ by setting $\chi(t) = exp(2\pi i f(X))$ for $t = e^X$, $f \in \mathfrak{t}^*$.
- The lattice $\Lambda^* \subset \mathfrak{t}^*$ is the dual lattice to the lattice $\Lambda = ker(exp) \subset \mathfrak{t}$, where $exp : \mathfrak{t} \to T$ is the exponential map. Here 'dual' means that $f \in \Lambda^*$ iff $f(\lambda) \in \mathbb{Z}$ for all $\lambda \in \Lambda$.
- Any unitary representation $\rho : T \to U(\mathbf{V})$ breaks up into an orthogonal direct sum of these one-dimensional representations :

$$\rho = \Sigma_{\lambda \in \hat{T}} m_\chi V_\chi$$

  where $m_\chi V_\chi$means take $m_\chi \in \mathbb{N}$ copies of the representation V_χ

Definition 1. *For T a torus and $\rho : T \to U(V)$ a unitary representation, the set of $\chi \in \Lambda^*$ occuring in the direct sum (*) will be called the "weights" of ρ, while the integers $m_\chi > 0$ will be called the "multiplicities" of those weights.*

Definition 2. *For G a compact Lie group, and $\rho : G \to U(V)$ a unitary representation, the "weights" of ρ are the weights of the representation $\rho|_T : T \to U(V)$, where T is (any) maximal torus of G.*

The "roots" of G are the weights of the adjoint representation $Ad : G \to U(\mathfrak{g})$.

The "weight lattice" of G is the set of all weights obtained as weights of finite-dimensional unitary representations of G and is a sublattice of Λ^*

We proceed to the details of the $SU(3)$ case...

We can think of this $\mathbb{Z}^2$ lying concretely inside $\mathfrak{t}^* = \mathbb{R}^2$ as follows. An element of $\mathfrak{t}^*$ is a linear function $f : \mathfrak{t} \to \mathbb{R}$.